Induced interaction between impurities from superfluid EFT

Keisuke Fujii

Department of Physics, Institute of Science Tokyo

Symmetry and Effective Field Theory of Quantum Matter

28 November 2024

Collaborators

Tilman Enss

Yukinao Akamatsu

Shimpei Endo

Masaru Hongo

KF, M. Hongo, & T. Enss, PRL. **129**, 233401 (2022)

Y. Akamatsu, S. Endo, KF, M. Hongo, PRA **110**, 033304 (2024)

Plan of this talk

1. **Introduction of the polaron**
    - Polaron in ultracold atoms
    - Induced interaction between polarons

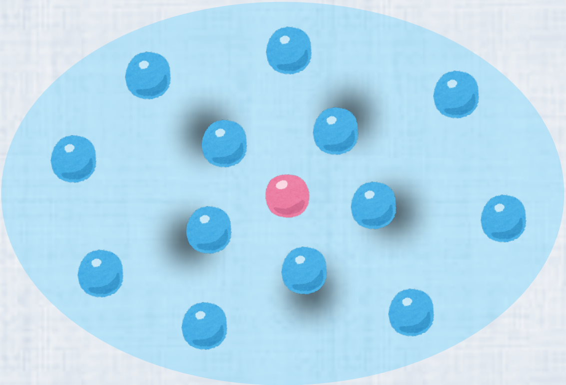
2. **EFT approach to induced interactions**

3. **Complex-valued induced interaction in finite-temperature media**

4. **Summary**

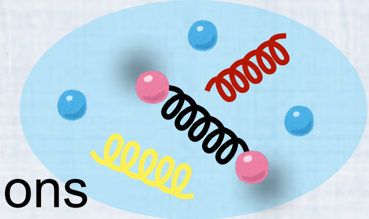
Impurities immersed in quantum gases

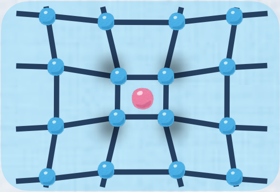
Polaron in ultracold atoms
: an impurity interacting with quantum gas particles



- Ultracold atoms provide a simple and ideal research platform.
  ✔ **High experimental controllability**
- Impurity problem appears across discipline.

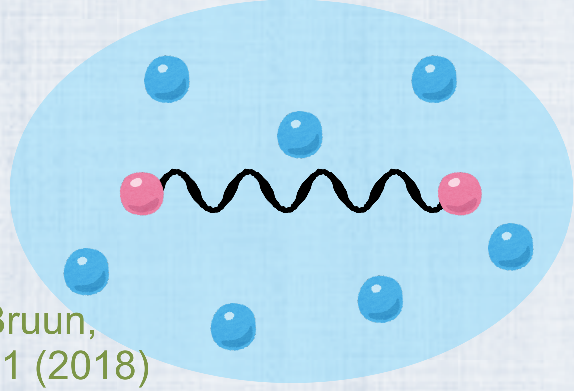
e.g., heavy quarks in QGP,
electrons in lattice phonons





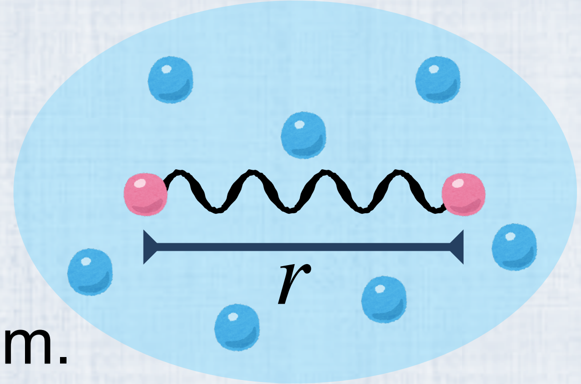
From One to Two

- One impurity problem
  : effective mass, mobility, dressing cloud, etc.
- Two impurity problem
  : induced interaction, bipolaron state, etc.



A. Camacho-Guardian, L. A. Peña Ardila, T. Pohl, and G. M. Bruun, "Bipolarons in a Bose-Einstein Condensate", PRL **121**, 013401 (2018)

Induced Interaction



Theoretical setting

: two test impurities in a medium separated by a distance r

- Medium (quasi-)particles induce interactions $V(r)$ between them.

At long distances, $V(r)$ is dominated by **the low-energy behavior of the mediating (quasi-)particles**.

In superfluids, superfluid phonons (NG mode)

Superfluid EFT can universally predict the long-distance behavior of $V(r)$.

(i) Universal power-law force
(ii) Universal power-law imaginary part in finite-T media

KF, M. Hongo, & T. Enss, PRL **129**, 233401 (2022); Y. Akamatsu, S. Endo, KF, M. Hongo, PRA **110**, 033304 (2024)

Plan of this talk

1. Introduction of the polaron

2. EFT approach to induced interactions
   - Theoretical formulation
   - Van der Waals potential mediated by phonons

3. Complex-valued induced interaction in finite-temperature media

4. Summary

Theoretical formulation of polaron physics

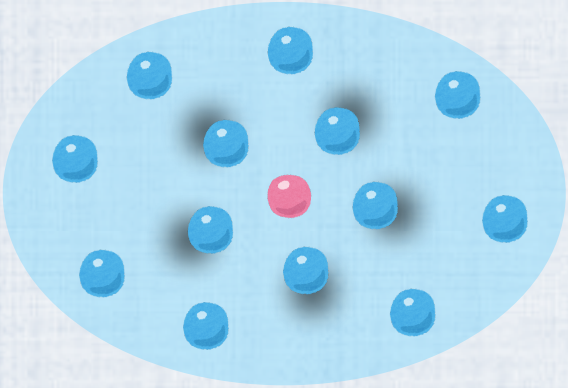
✓ **Microscopic model :**

Medium gas interacting with impurities

$$\mathcal{L}_{\rm micro}(x) = \mathcal{L}_{\rm imp}(x) + \mathcal{L}_{\rm medium}(x) + \mathcal{L}_{\rm int}(x)$$

▸ Impurity-medium interaction in the contact *s*-wave channel

$$\mathcal{L}_{\rm int}(x) = -g_{IM} \underbrace{\Phi^\dagger(x)\Phi(x)}_{\text{Impurity density}} \underbrace{\psi^\dagger(x)\psi(x)}_{\text{Medium density}}$$



✓ **Our problem is to find** $S_{\rm polaron}[\Phi, \Phi^\dagger]$ by integrating out the medium

$$\exp\left[iS_{\rm polaron}[\Phi, \Phi^\dagger]\right] = \int \mathcal{D}(\psi, \psi^\dagger) \exp\left[i \int dt d^3x\, \mathcal{L}_{\rm micro}(x)\right]$$

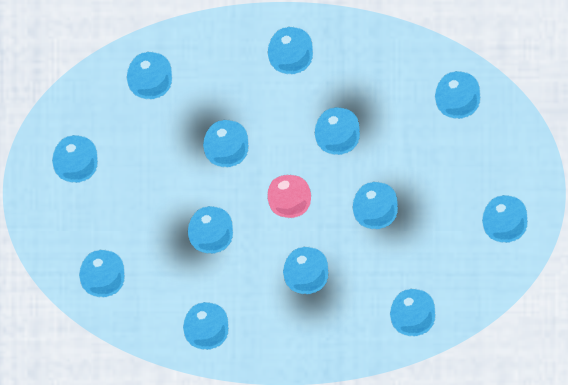
▸ Formally simple, but difficult to perform the integration

Theoretical formulation of polaron physics

✓ Microscopic model :

Medium gas interacting with impurities

$$\mathcal{L}_{\rm micro}(x) = \mathcal{L}_{\rm imp}(x) + \mathcal{L}_{\rm medium}(x) + \mathcal{L}_{\rm int}(x)$$



▸ Impurity-medium interaction in the contact *s*-wave channel

$$\mathcal{L}_{\rm int}(x) = -g_{IM} \underbrace{\Phi^\dagger(x)\Phi(x)}_{\text{Impurity density}} \underbrace{\psi^\dagger(x)\psi(x)}_{\text{Medium density}}$$

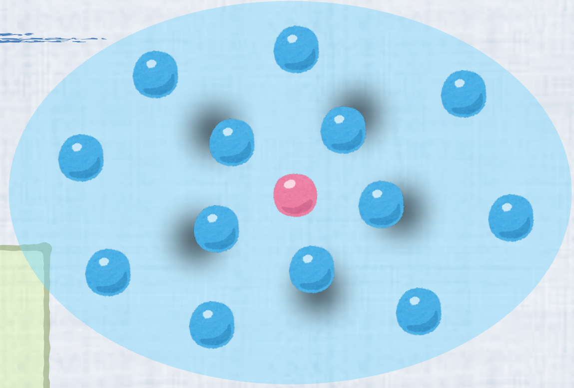
✓ EFT approach

$$\mathcal{L}_{\rm eff}(x) = \mathcal{L}_{\rm imp}(x) + \mathcal{L}_{\rm SF\text{-}EFT}(x) + \mathcal{L}_{\rm int}(x)$$

▸ Our task

- Write down the superfluid EFT $\mathcal{L}_{\rm SF\text{-}EFT}(x)$
- Represent $\mathcal{L}_{\rm int}(x)$ with phonon fields

Superfluid EFT



Medium gas = Non-relativistic gas (cold atomic gas)

✓ Galilean-invariant superfluid EFT

$$\mathcal{L}_{\text{SF-EFT}}(x) = \mathcal{P}(\theta(x))$$

$\mathcal{P}(\mu)$: Pressure as a function of μ

Galilean-invariant combination : $\theta(x) = \mu - \partial_t \phi(x) - \dfrac{(\nabla\phi(x))^2}{2m}$

Superfluid phonon field : $\phi(x)$

M. Greiter, F. Wilczek, & E. Witten (1989); D. T. Son & M. Wingate, (2006).

▸ **Interaction term**

$$\mathcal{L}_{\text{int}}(x) = -g_{IM}\Phi^\dagger(x)\Phi(x)n(\theta(x)) \qquad \text{with} \qquad n(\mu) = \mathcal{P}'(\mu)$$

Effective theory for polarons

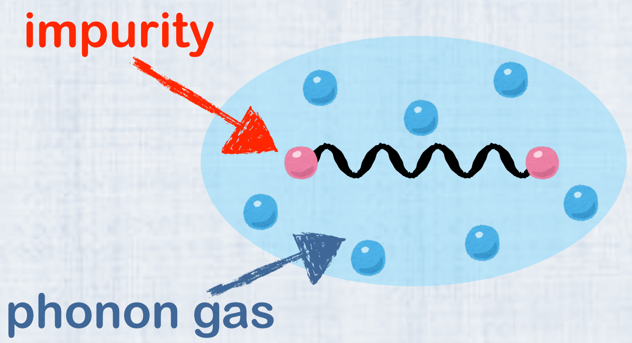
✓ Our effective theory

$$\mathcal{L}_{\rm eff}(x) = \mathcal{L}_{\rm imp}(x) + \mathcal{P}(\theta(x)) - g_{IM}\Phi^\dagger(x)\Phi(x)n(\theta(x))$$

▸ **Galilean invariant combination** $\theta(x) = \mu - \partial_t\phi(x) - \frac{(\nabla\phi(x))^2}{2m}$

▸ Our assumptions are only two:

- **Galilean invariant medium**
- **Contact s-wave impurity-medium coupling**

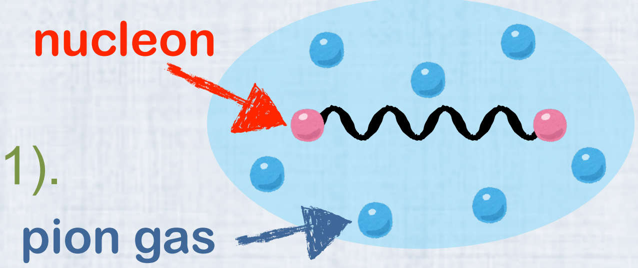


➡ Universal !! : **Independent of the details of the medium**

Our remaining task is **to calculate induced interactions from our effective theory**

cf. nuclear forces are computed from chiral effective field theory

See e.g., R. Machleidt & D. R. Entem, "Chiral effective field theory and nuclear forces," Phys. Rept. **503**, 1 (2011).



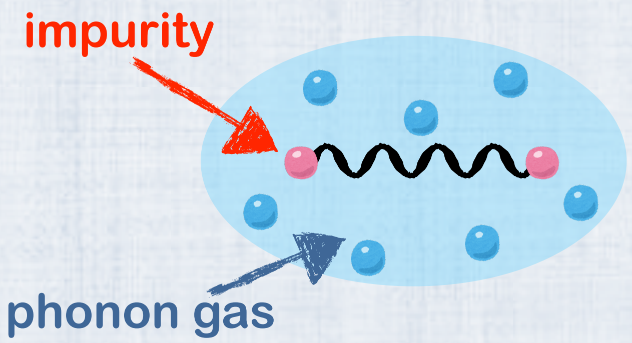
Effective theory for polarons

✓ Our effective theory

$$\mathcal{L}_{\mathrm{eff}}(x) = \mathcal{L}_{\mathrm{imp}}(x) + \mathcal{P}(\theta(x)) - g_{IM}\Phi^\dagger(x)\Phi(x)n(\theta(x))$$

- **Galilean invariant combination** $\theta(x) = \mu - \partial_t\phi(x) - \frac{(\nabla\phi(x))^2}{2m}$

- Our assumptions are only two:
  - **Galilean invariant medium**
  - **Contact s-wave impurity-medium coupling**



⇒ Universal !! **: Independent of the details of the medium**

Our remaining task is **to calculate induced interactions from our effective theory**

At weak 

$$V(r) = -g_{IM}^2 \lim_{\omega\to 0} \mathrm{Re}\left[G^R(\vec{r},\omega)\right]$$

correlation function of the impurity density

Induced interaction mediated by phonons

Expanding $\mathcal{P}(\theta)$ & $n(\theta)$ and keeping the leading terms with rescaling $\varphi = \sqrt{\chi}\phi$

$$\mathcal{L}(x) = \mathcal{L}_{\mathrm{imp}}(x) - g_{IM} n \Phi^\dagger \Phi + \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2} c_s^2 (\nabla \varphi)^2 + g_{IM}\left[\sqrt{\chi}\partial_t \varphi + \frac{(\nabla \varphi)^2}{2m}\right]\Phi^\dagger \Phi + \cdots$$

$\chi = n'(\mu)$: compressibility

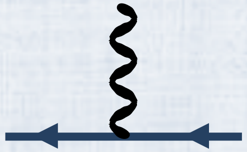
$c_s = \sqrt{n/(m\chi)}$: speed of sound

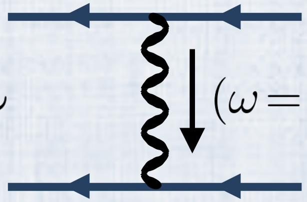
Kinetic term for phonons showing the linear dispersion

✓ Interaction terms between impurities and phonons

$g_{IM}\sqrt{\chi}\partial_t \varphi \Phi^\dagger \Phi$: one-body coupling $\quad$ $g_{IM}\dfrac{(\nabla \varphi)^2}{2m}\Phi^\dagger \Phi$: two-body coupling

- **The coefficients are constrained by the Galilean invariance**
- One-body coupling ➡ One-phonon exchange

$g_{IM}\sqrt{\chi}\partial_t \varphi \Phi^\dagger \Phi \sim$ 

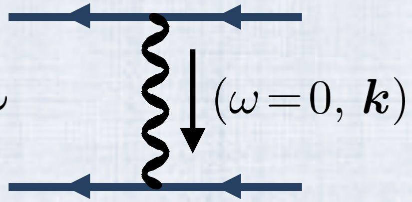
$\tilde{V}(k) \sim$  $(\omega = 0, \boldsymbol{k}) \sim \lim_{\omega \to 0}\left(g_{IK}\sqrt{\chi}\omega\right)^2 \Delta(\omega, \boldsymbol{k}) = 0$

proportional to $\omega = 0$ due to the time-derivative coupling

One-phonon exchange & Yukawa potential

Within our EFT

$\tilde{V}(k) \sim$

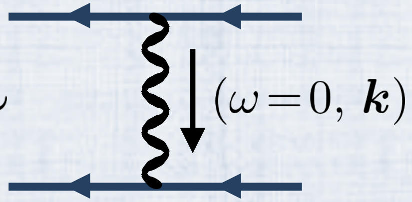


$= 0$ ➡ No potential from one-phonon exchange

Previous study : Induced interaction from the Bogoliubov theory

applicable to weakly-interacting Bose superfluids

$\tilde{V}(k) \sim$



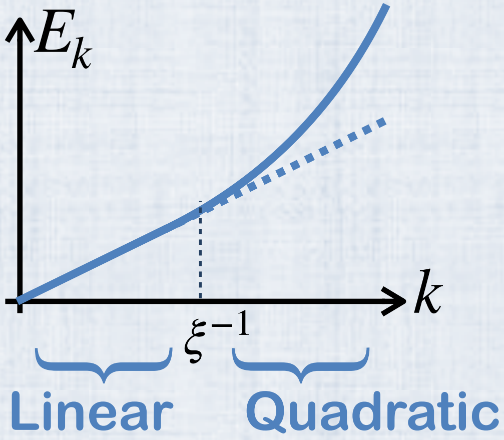
$\sim -g_{IM}^2 \dfrac{1}{\boldsymbol{k}^2/(2m) + 2\mu}$ ➡ Yukawa potential $V_{\text{Yukawa}}(r) \sim -\, g_{IM}^2 \dfrac{e^{-\sqrt{2}r/\xi}}{r}$

(healing length : $\xi = 1/\sqrt{2m\mu}$)

See e.g. Pethick & Smith's text book "Bose-Einstein condensation in Dilute gases"

Bogoliubov dispersion : $E_k = \sqrt{\varepsilon_k(\varepsilon_k + 2\mu)}$

$\varepsilon_k = k^2/(2m)$



- Linear part has NO contribution to the one-phonon exchange.
- Yukawa potential effectively vanishes at long distances

consistent with the result from our EFT

Induced interaction mediated by phonons

Expanding $\mathcal{P}(\theta)$ & $n(\theta)$ and keeping the leading terms with rescaling $\varphi = \sqrt{\chi}\phi$

$$\mathcal{L}(x) = \mathcal{L}_{\text{imp}}(x) - g_{IM} n \Phi^\dagger \Phi + \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}c_s^2(\boldsymbol{\nabla}\varphi)^2 + g_{IM}\left[\sqrt{\chi}\partial_t\varphi + \frac{(\boldsymbol{\nabla}\varphi)^2}{2m}\right]\Phi^\dagger\Phi + \cdots$$

$\chi = n'(\mu)$: compressibility

$c_s = \sqrt{n/(m\chi)}$: speed of sound

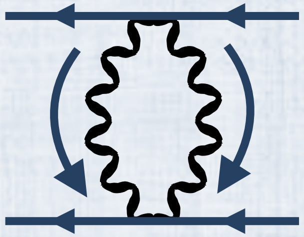
Kinetic term for phonons showing the linear dispersion

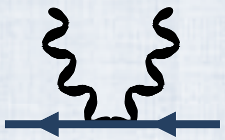
✓ Interaction terms between impurities and phonons

$g_{IM}\sqrt{\chi}\partial_t\varphi\Phi^\dagger\Phi$: one-body coupling

$g_{IM}\frac{(\boldsymbol{\nabla}\varphi)^2}{2m}\Phi^\dagger\Phi$: two-body coupling

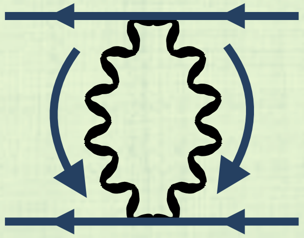
- **The coefficients are constrained by the Galilean invariance**
- One-body coupling ➡ one-phonon exchange **(NO contribution)**
- Two-body coupling ➡ two-phonon exchange



$g_{IM}\frac{(\boldsymbol{\nabla}\varphi)^2}{2m}\Phi^\dagger\Phi \quad \sim$ 

Van der Waals force from two-phonon exchange

✓ **Two-phonon exchange potential** from $g_{IM}\frac{(\nabla\varphi)^2}{2m}\Phi^\dagger\Phi$



At zero temperature

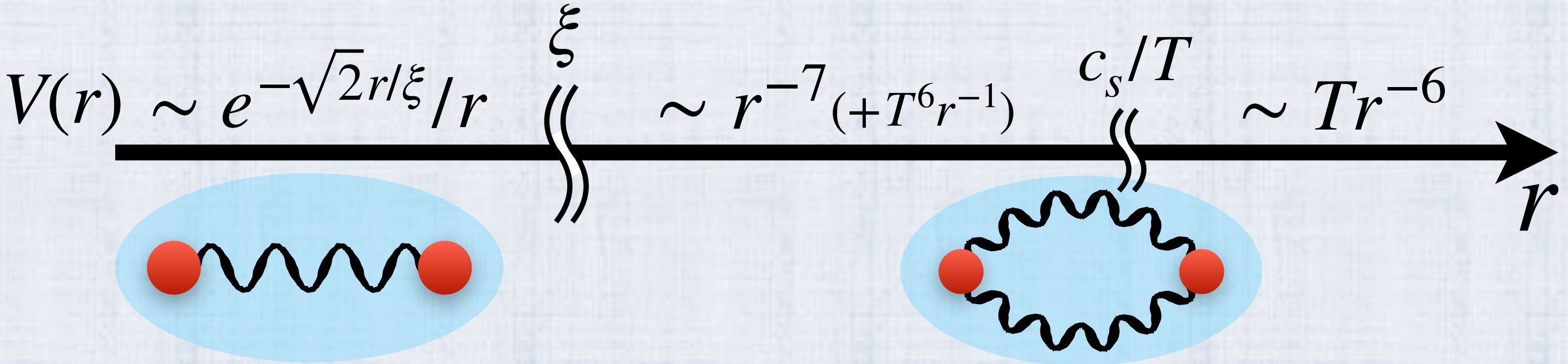
$$V_{T=0}(r) = -g_{IM}^2 \frac{43}{128\pi^3 m^2 c_s^3}\frac{1}{r^7}$$

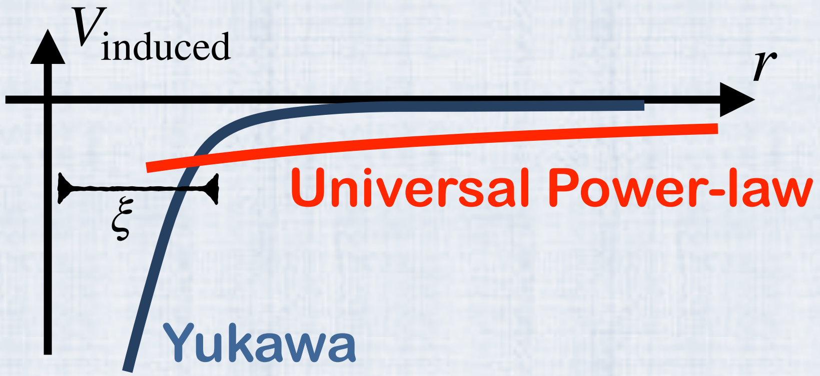
relativistic van der Waals

At finite temperatures (c_s/T : temperature length scale)

$$V(r) = \begin{cases} V_{T=0}(r) - g_{IM}^2 \dfrac{\pi^3 T^6}{135 m^2 c_s^9}\dfrac{1}{r} & (r \ll c_s/T) \\ -g_{IM}^2 \dfrac{3T}{16\pi^2 m^2 c_s^4}\dfrac{1}{r^6} & (r \gg c_s/T) \end{cases}$$

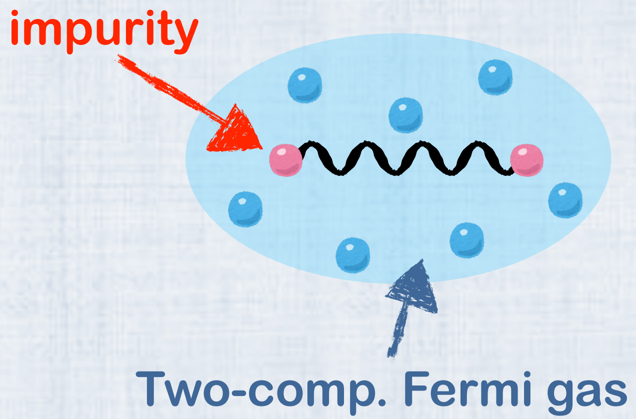
non-relativistic van der Waals





Induced potential in BCS-BEC crossover

Our results are valid in the entire BCS-BEC crossover



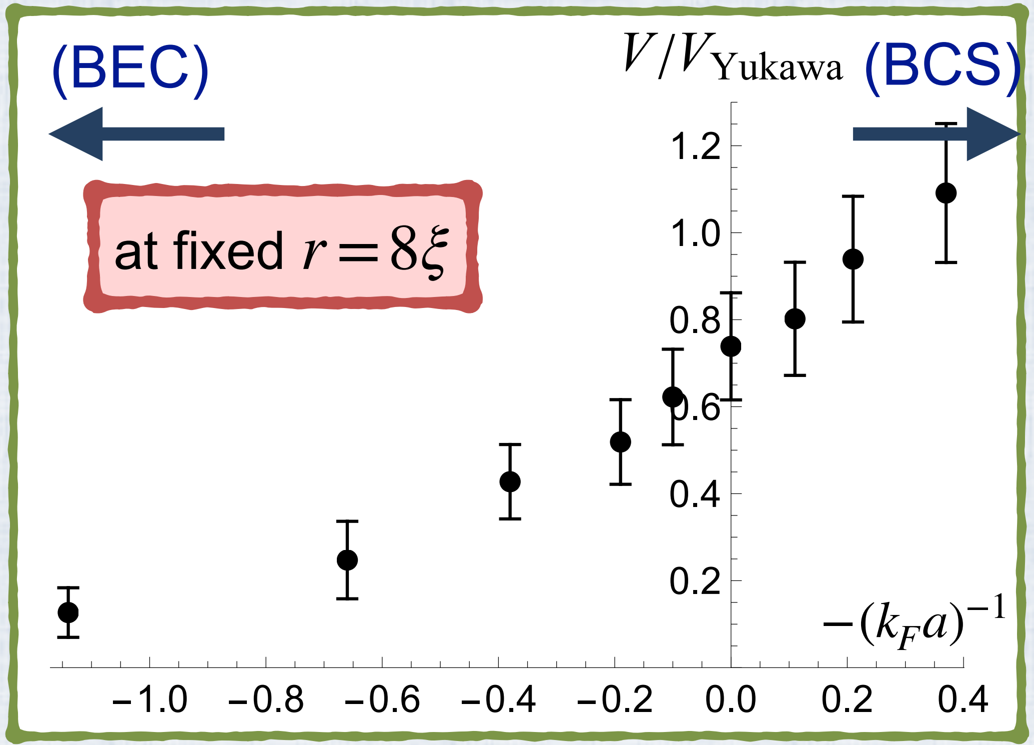
- Our results are based only on two assumptions
  - **Galilean invariant medium**
  - **Contact s-wave impurity-medium coupling**

✓ Plotting the ratio as a function of the scattering length

$$\frac{V_{T=0}}{V_{\rm Yukawa}} = \frac{43}{16\sqrt{2}\pi^2}\frac{1}{n\xi^3}\frac{e^{\sqrt{2}x}}{x^6} \quad \text{with} \quad x = r/\xi$$

with the use of the experimental data

S. Hoinka, et al., Nature Physics **13**, 943 (2017)



- The van der Waals potential is small in the BEC side, but becomes relatively larger when $-(k_F a)^{-1}$ increases
- **At unitarity, the van der Waals potential is dominant in $r \gtrsim 8\xi$.**

At unitarity, our effective theory is robust because of small ξ

Plan of this talk

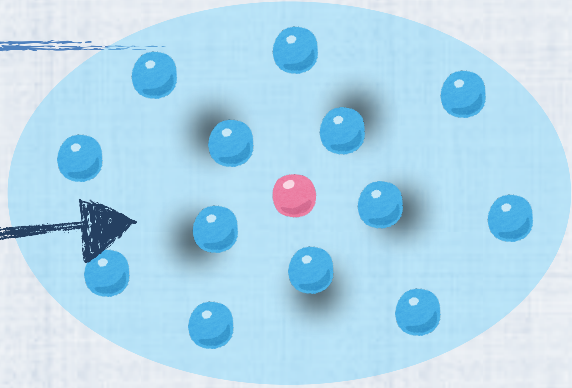
1. Introduction of the polaron

2. EFT approach to induced interactions

3. **Complex-valued induced interaction in finite-temperature media**
   - Imaginary part of $V(r)$
   - Universal low-energy scattering between impurities & the medium

4. Summary

Finite temperature effect



The medium serves as a thermal bath for impurities.

Finite-T medium $\simeq$ thermal bath

- Mediating (quasi-)particles obey the Bose/Fermi distributions.

The induced interaction is smoothed by thermal fluctuations of mediating (quasi)-particles.

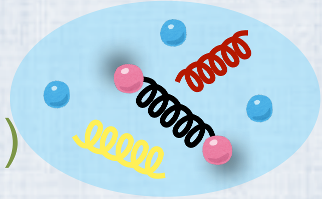
$$V(r)|_{T=0} \sim 1/r^7 \quad \Rightarrow \quad V(r)|_{T>0} \sim 1/r^6$$

Is there any new effect on $V_{\text{induced}}(r)$ specific to finite-T media?

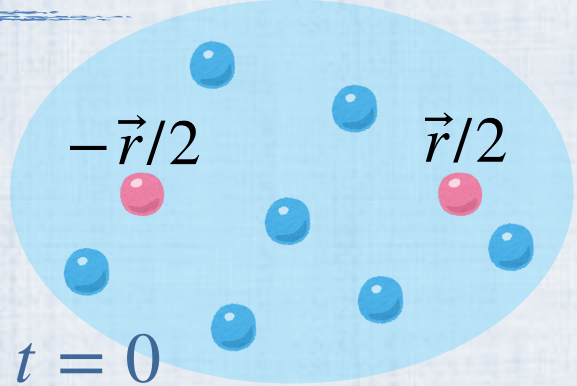
⇒ Yes!! $V_{\text{induced}}(r)$ has **an imaginary-part**
describing the loss of correlation between impurities.

- Possessing **non-Hermitian nature** due to its environmental medium effect

Originally, $V_{\text{Im}}(\vec{r})$ was introduced in subatomic physics.
M. Laine, *et. al.*, JHEP (2007)



Definition of the potential in finite-T media



✓ Real-time correlation function

$$\Psi(\vec{r},t) \sim \langle \hat{\Phi}(\tfrac{\vec{r}}{2},t)\hat{\Phi}(-\tfrac{\vec{r}}{2},t)\hat{\Phi}^\dagger(-\tfrac{\vec{r}}{2},0)\hat{\Phi}^\dagger(\tfrac{\vec{r}}{2},0)\rangle$$

2. **annihilate** two impurities at time t 1. **create** two impurities at $t=0$

$\Psi(\vec{r},t)$ obeys **the Schrödinger equation** at long times as a wave function for relative motion

$$i\frac{\partial}{\partial t}\Psi(\vec{r},t) \simeq \left[-\frac{\nabla^2}{2M} + \Sigma + V(r)\right]\Psi(\vec{r},t)$$

$$= E(\vec{r})$$

The infinitely heavy-mass limit
& subtracting the self-energy part

$$V(\vec{r}) = E(\vec{r}) - \lim_{r\to\infty} E(\vec{r})$$

- The imaginary part describes the decay of the absolute value as $|\Psi(\vec{r},t)| \sim e^{-|\mathrm{Im}E|t}$

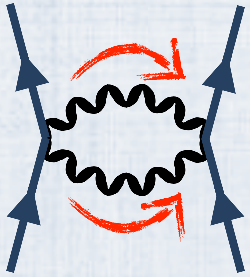
- At weak 

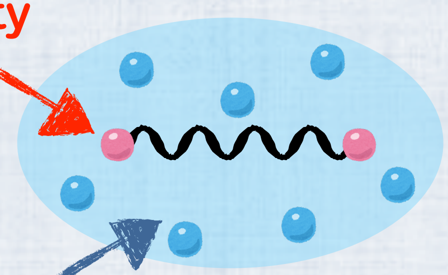
$$V_{\mathrm{Re}}(\vec{r}) \equiv -g^2 \lim_{\omega\to 0}\mathrm{Re}[G^R(\vec{r},\omega)] \qquad V_{\mathrm{Im}}(\vec{r}) \equiv -g^2\frac{2}{\beta}\lim_{\omega\to 0}\frac{\mathrm{Im}[G^R(\vec{r},\omega)]}{\omega}$$

Imaginary part of the induced interaction

$$\mathcal{L}(x) = \mathcal{L}_{\rm imp}(x) - g_{IM} n \Phi^\dagger\Phi + \frac{1}{2}(\partial_t\varphi)^2 - \frac{1}{2}c_s^2(\nabla\varphi)^2 + g_{IM}\left[\sqrt{\chi}\partial_t\varphi + \frac{(\nabla\varphi)^2}{2m}\right]\Phi^\dagger\Phi + \cdots$$

▸ The two-phonon exchange process provides the long-range behavior





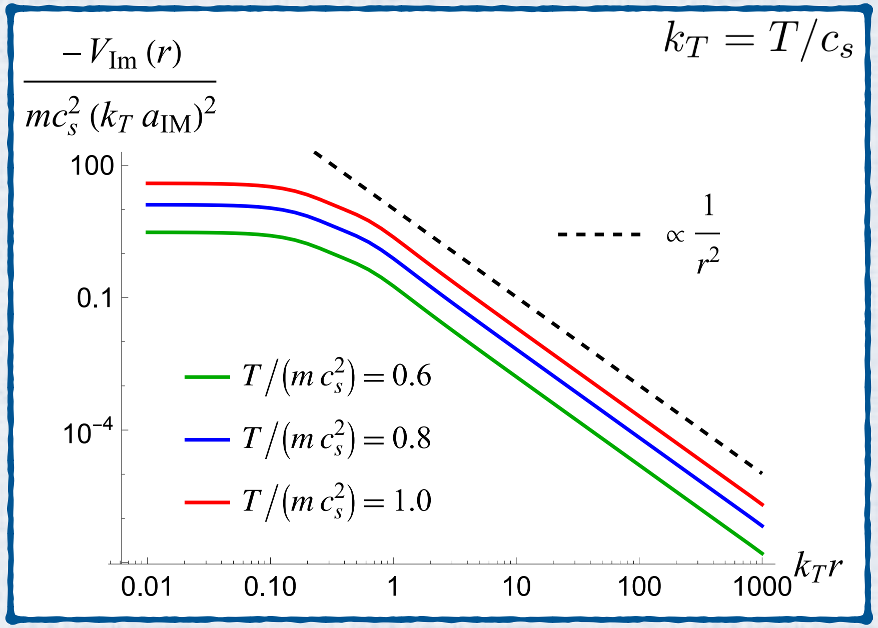
- Real part (c_s/T : temperature length scale)

$$V_{\rm Re}(r) \sim \begin{cases} -g^2 r^{-7} & (r \ll c_s/T) \\ -g^2 T r^{-6} & (r \gg c_s/T) \end{cases}$$

- Imaginary part

$$V_{\rm Im}(r) \sim \begin{cases} -g^2 T^7 & (r \ll c_s/T) \\ -g^2 T^5 r^{-2} & (r \gg c_s/T) \end{cases}$$

Y. Akamatsu, S. Endo, KF, M. Hongo, [arXiv:2312.08241] (2023)



The origin of the power-law decay r^{-2}

- In superfluids $V_{\text{Im}}(r) \sim -g^2 T^5 r^{-2}$
- In non-interacting Fermi gases $\dfrac{V_{\text{Im}}(r \gg k_F^{-1})}{T_F} \simeq -\dfrac{2(k_F a_{IM})^2}{\pi} \dfrac{T/T_F}{1+e^{-T_F/T}} \dfrac{1}{(k_F r)^2}$

Y. Akamatsu, S. Endo, KF, M. Hongo, PRA **110**, 033304 (2024)

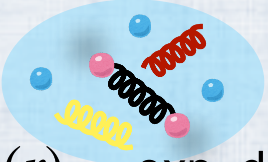
▸ The power law of $V_{\text{Re}}(r)$ at long distance is due to **the gapless nature** of excitations.

What about the power law of $V_{\text{Im}}(r)$?

$$V_{\text{Im}}(\vec{r}) \equiv -g^2 \frac{2}{\beta} \lim_{\omega \to 0} \frac{\text{Im}[G^R(\vec{r},\omega)]}{\omega}$$

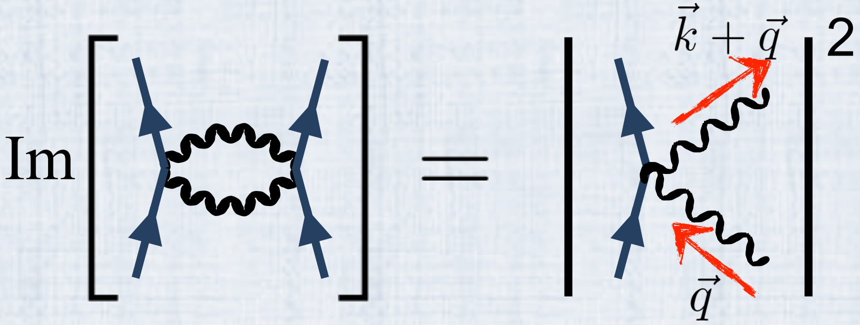
It's NOT due to the gapless nature of excitations.

Counterexample : the induced potential between heavy-quarks in QGP



$V_{\text{Re}}(r) \sim$ exp. damping
$V_{\text{Im}}(r) \sim r^{-2}$

It's due to **the common structure of the low-energy scattering**.



✔ Non-zero scattering cross section in the limit of $k \to 0$

➡ r^{-2} behavior

Summary & Outlooks

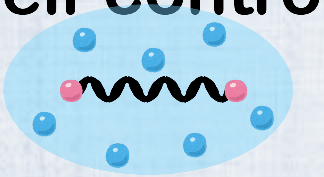
Superfluid EFT can universally predict the long-distance behavior of $V(r)$.

(i) Universal r^{-7} force at zero temperatures

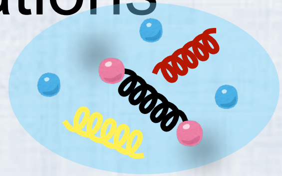
(ii) Universal r^{-2} imaginary part in finite-T superfluids

KF, M. Hongo, & T. Enss, PRL. **129**, 233401 (2022); Y. Akamatsu, S. Endo, KF, M. Hongo, PRA **110**, 033304 (2024)

Insights from **well-controlled ultracold atom** experiments



into uncontrolled experimental situations



Future directions

- Fate of bound states

While $V_{\mathrm{Re}}(r)$ create bound states between impurities, $V_{\mathrm{Im}}(r)$ breaks them.

- Other EFTs with impurities

e.g., Magnon-exchange force from (anti-)ferromagnet EFT, and so on… Let's discuss!!