Phase Transition of Vortices in Higgs-Confinement Continuity

Yoshimasa Hidaka
(YITP, Kyoto University)

Collaboration with Dan Kondo (Univ. of Tokyo),
Tomoya Hayata (Keio Univ.)

based on arXiv: 2411.03676

Summary

We focus on systems with $U(1)_{\mathrm{global}} \times G_{\mathrm{gauge}}$ symmetry

Pure gauge theory

Confined phase	Deconfined phase

$\longrightarrow \beta_g = 1/g^2$

Strong coupling **Weak coupling**

Adding fundamental charged matter and Higgsing it

and consider SSB of $U(1)_{\mathrm{global}}$

$U(1)$ superfluid ⋮	

$\longrightarrow \beta_g = 1/g^2$

Strong coupling **Weak coupling**

We show something happened between strong and weak coupling.

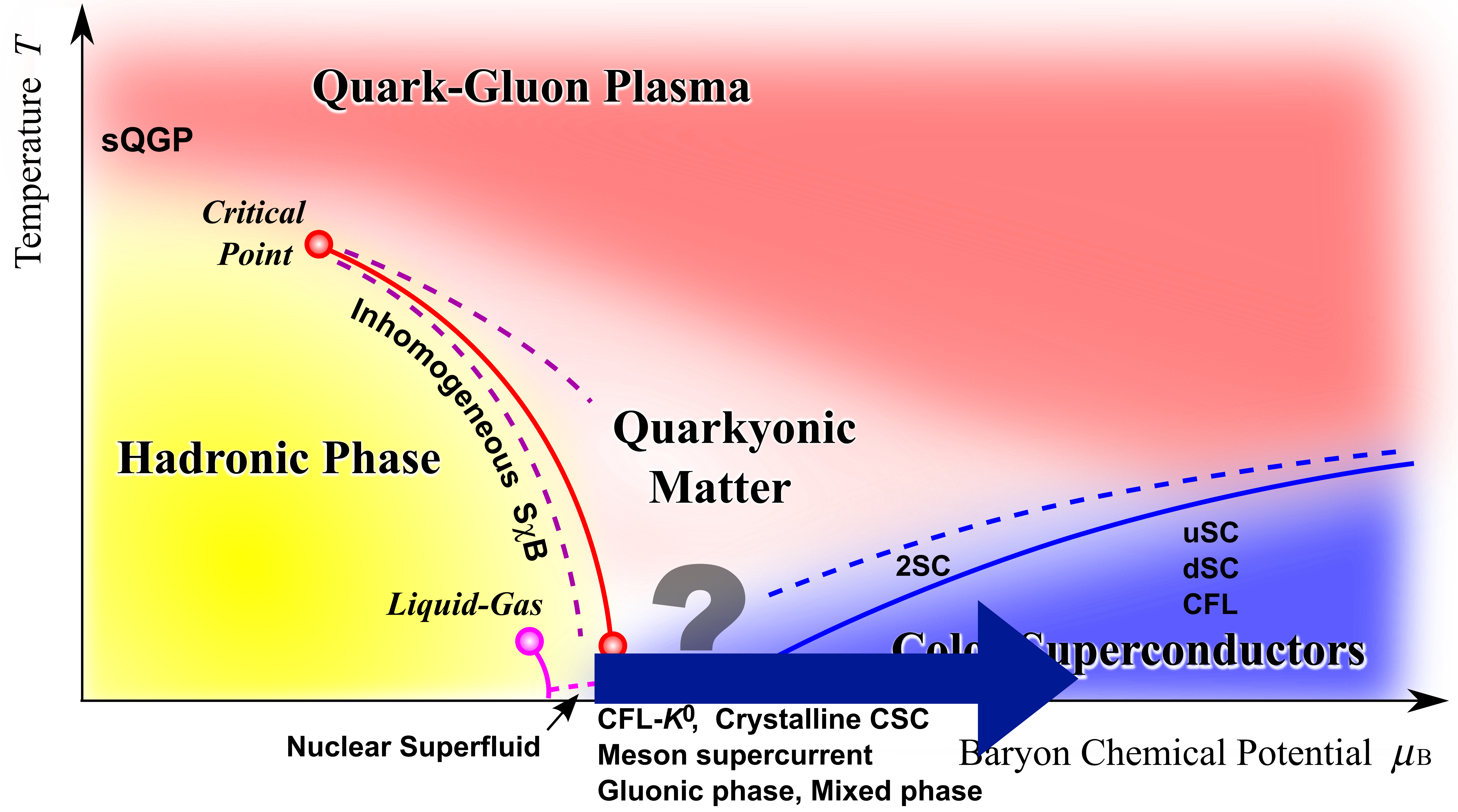
⇒Phase transition on vortex even in bulk has no phase transition.

Outline

- Motivation
- What we know about quark hadron continuity
- Phase transition on a vortices
- Summary and Outlook

Motivation: QCD phase diagram

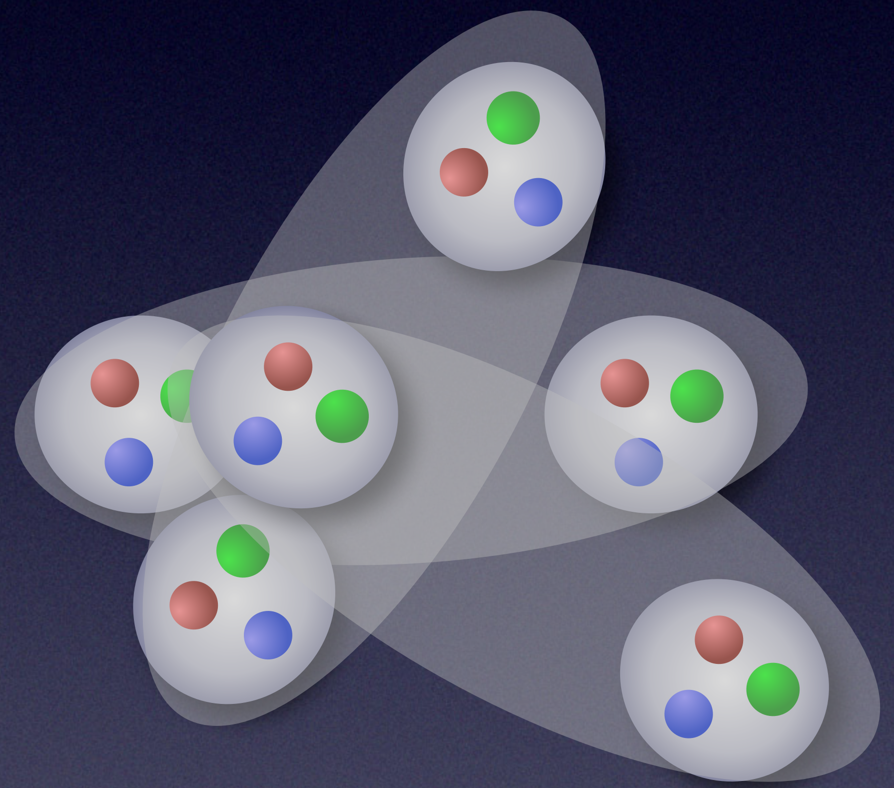
Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011) 014001



What we know

For 3-flavor QCD : $G = SU(3)_f \times U(1)_B$

•Superfluid(dilute phase)

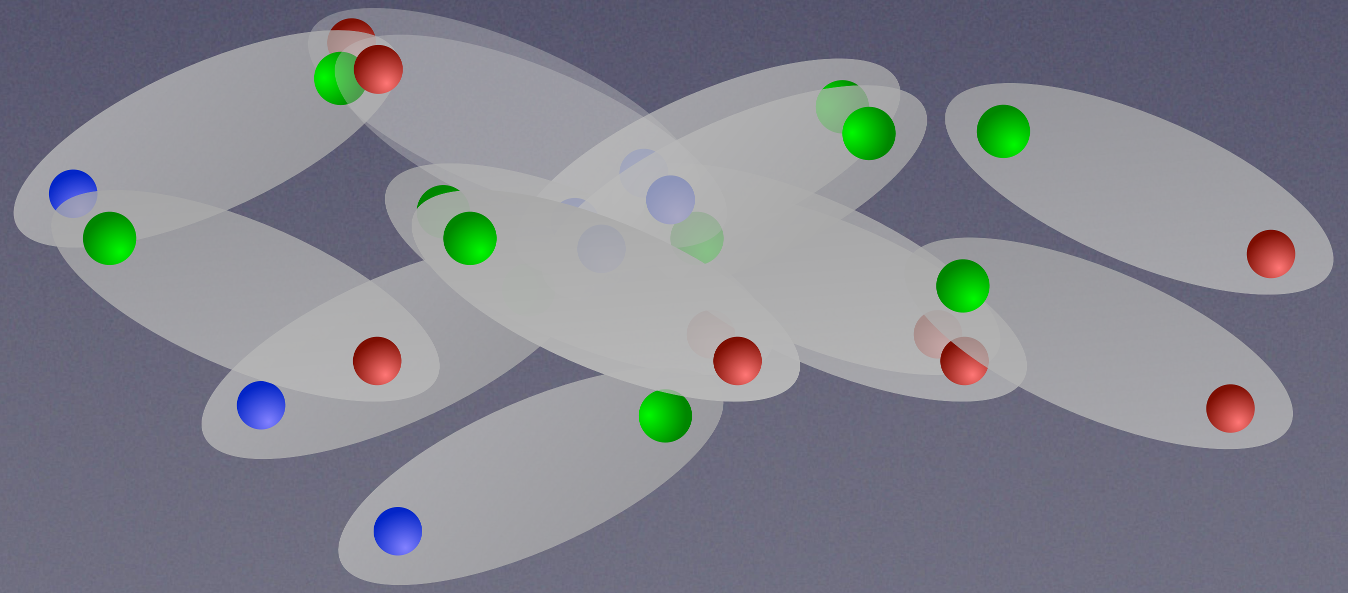


Baryon pair condensation

$$\Delta = \langle \Lambda\Lambda \rangle \neq 0 \qquad \Lambda \sim uds$$

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

•Color super conductor (dense phase)



"quark pair condensate"

$$(\Phi_L)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_L)^b_j (Cq_L)^c_k \rangle = -\epsilon^{ijk}\epsilon_{abc}\langle (q_R)^b_j (Cq_R)^c_k \rangle$$

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

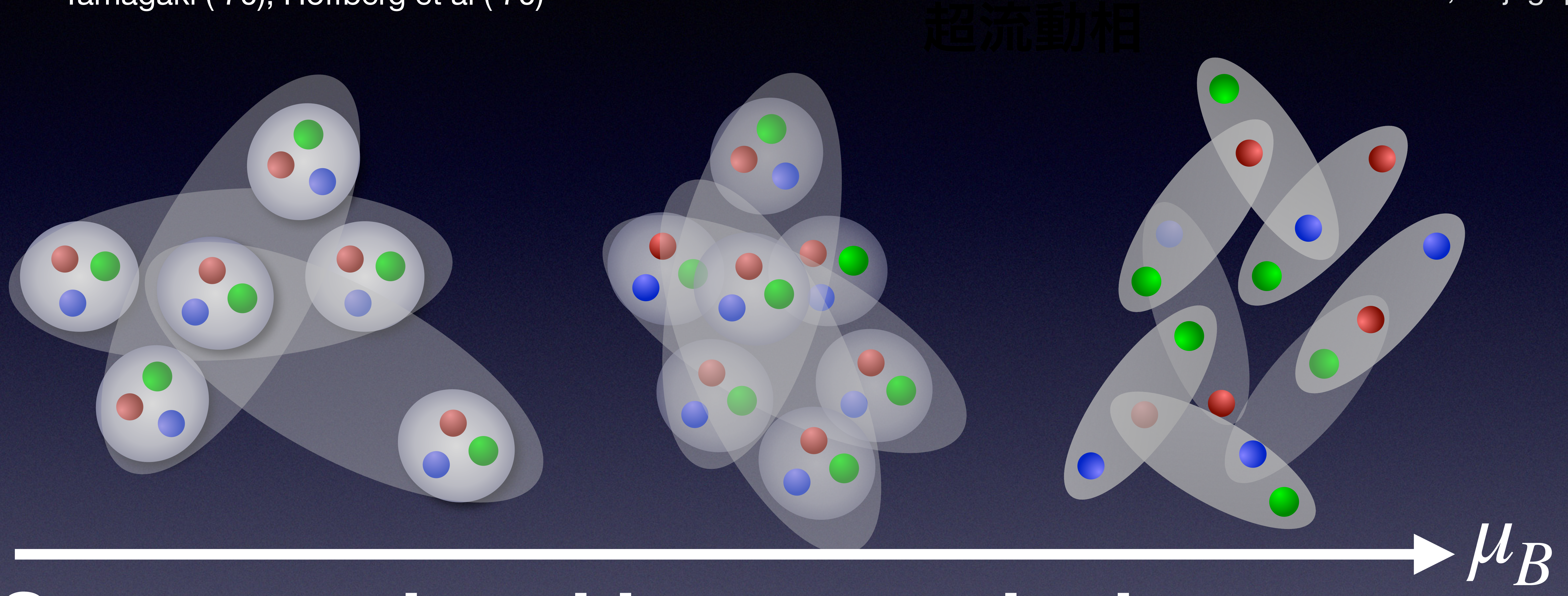
Quark hadron continuity

Hadronic superfluid

Tamagaki ('70), Hoffberg et al ('70)

Color flavor locked phase (CFL phase)

Alford, Rajagopal, Wilczek ('99)



Symmetry breaking patter is the same

⇒Quark hadron continuity

Excitations

Baryons ⇒ Quarks

Vector meson ⇒ Gluons

cf. Hatsuda, Tachibana, Yamamoto, Baym ('06)

Can the two phases be distinguished for topological reasons?

SPT phase, topological ordered phase, …

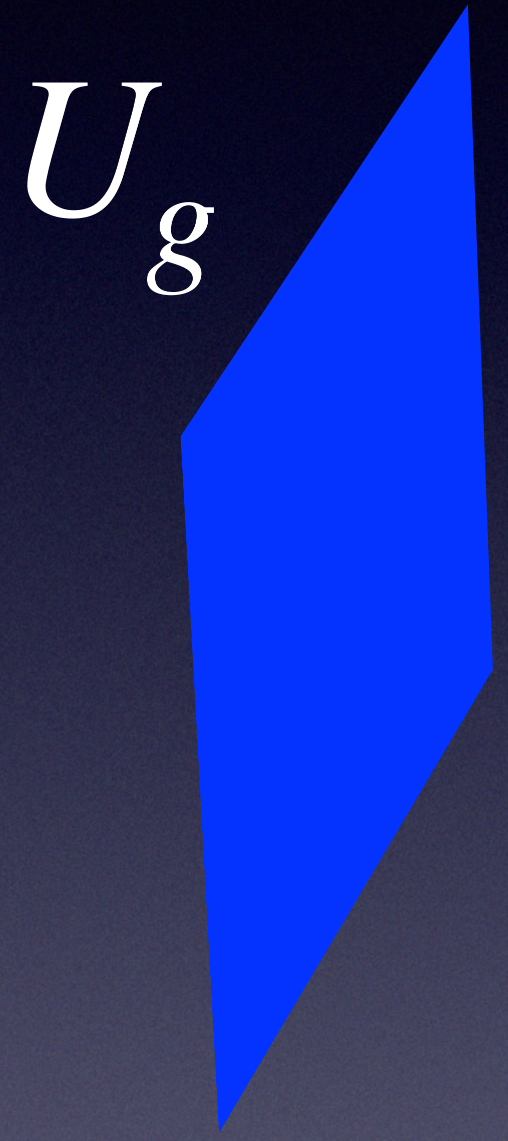
Topological ordered phase

$\approx$ Spontaneously broken generalized (discrete) global symmetries

Generalized global symmetry

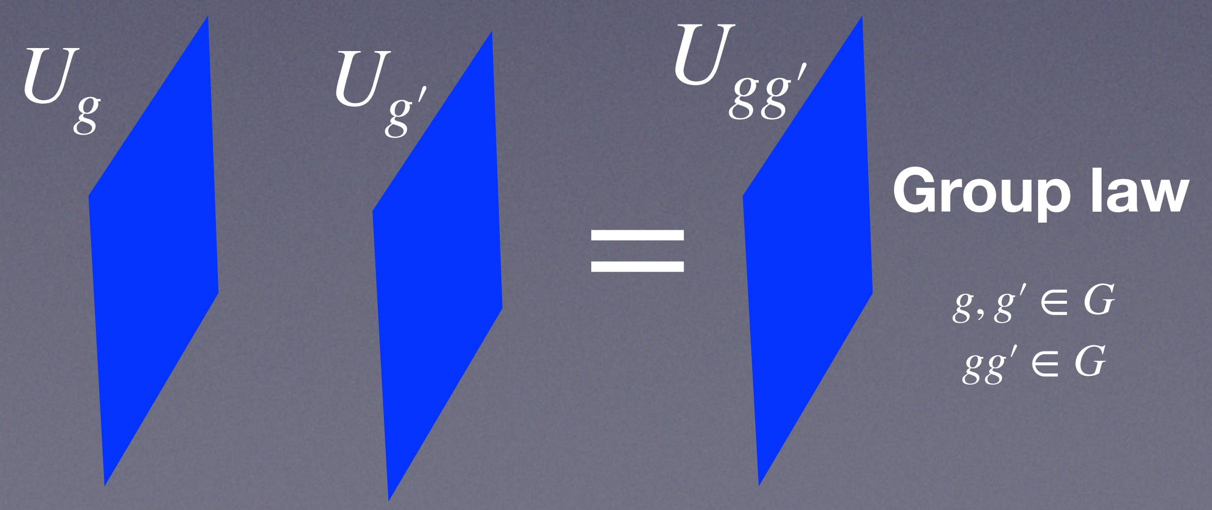
Gaiotto, Kapustin, Seiberg, Willett ('14)

Ordinary global symmetry of Quantum field theory(QFT) in (d+1) dimensions



Symmetry operator

d dimensional topological object labeled by $g \in G$



Charged object

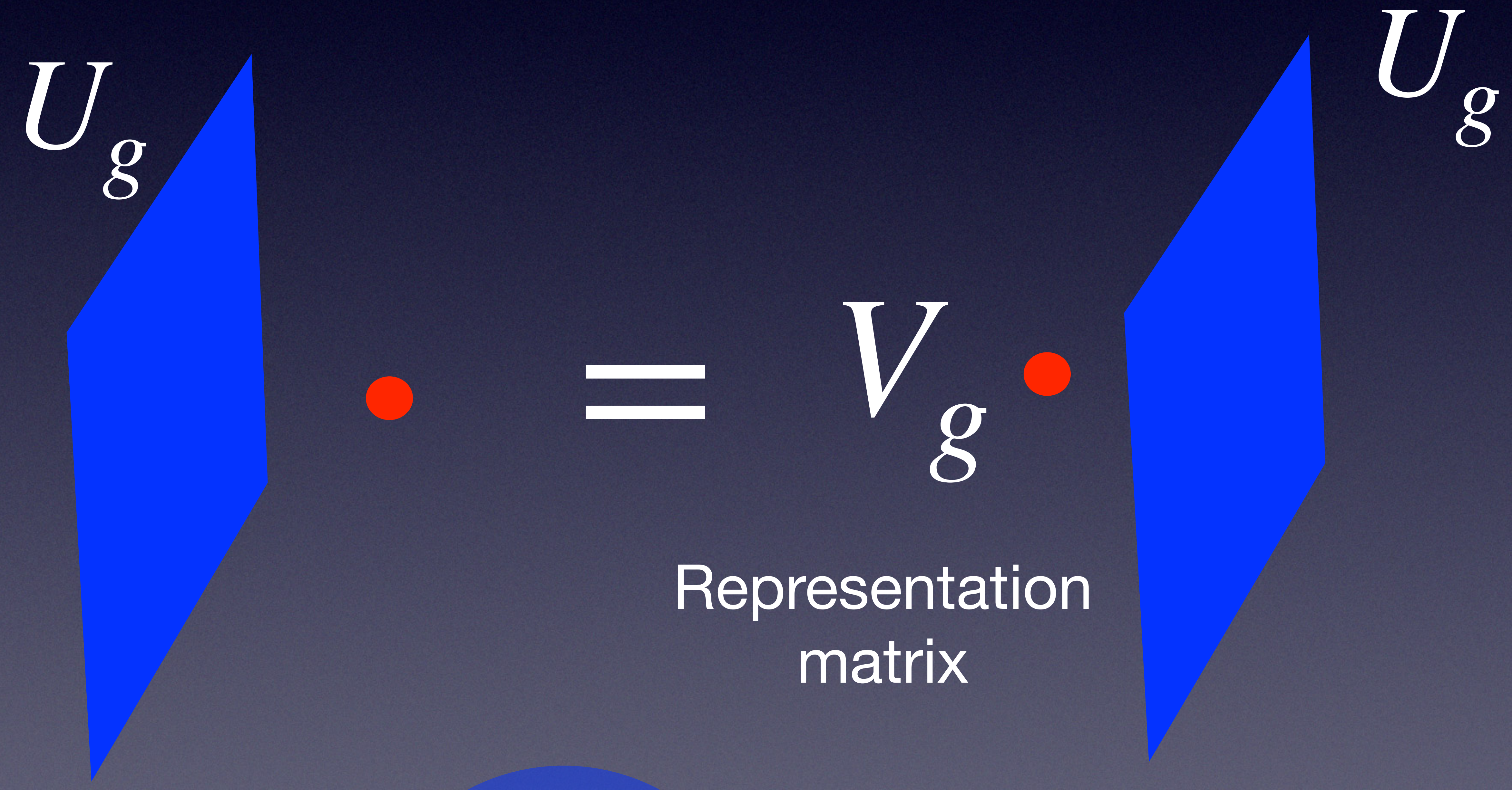
0-dimensional object labeled by representation of G

Generalized global symmetry

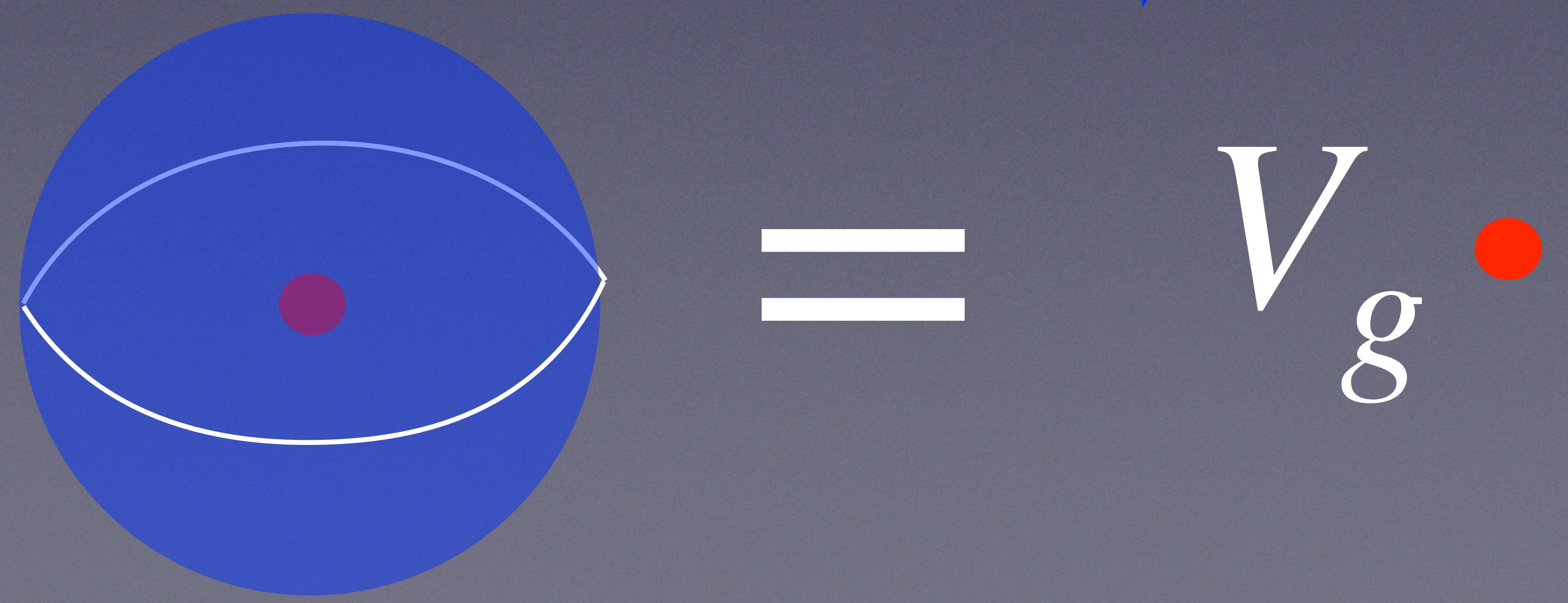
Gaiotto, Kapustin, Seiberg, Willett ('14)

QFT in (d+1) dimension

Transformation: $U_g \phi = V_g \phi U_g$



charged object is surrounded by U_g
⇒symmetry transformation



Generalized global symmetry

Gaiotto, Kapustin, Seiberg, Willett ('14)

QFT in (d+1) dimension

p form symmetry G

U_g

W

Symmetry generator

$d-p$ dimensional topological object labeled by $g \in G$

Charged object

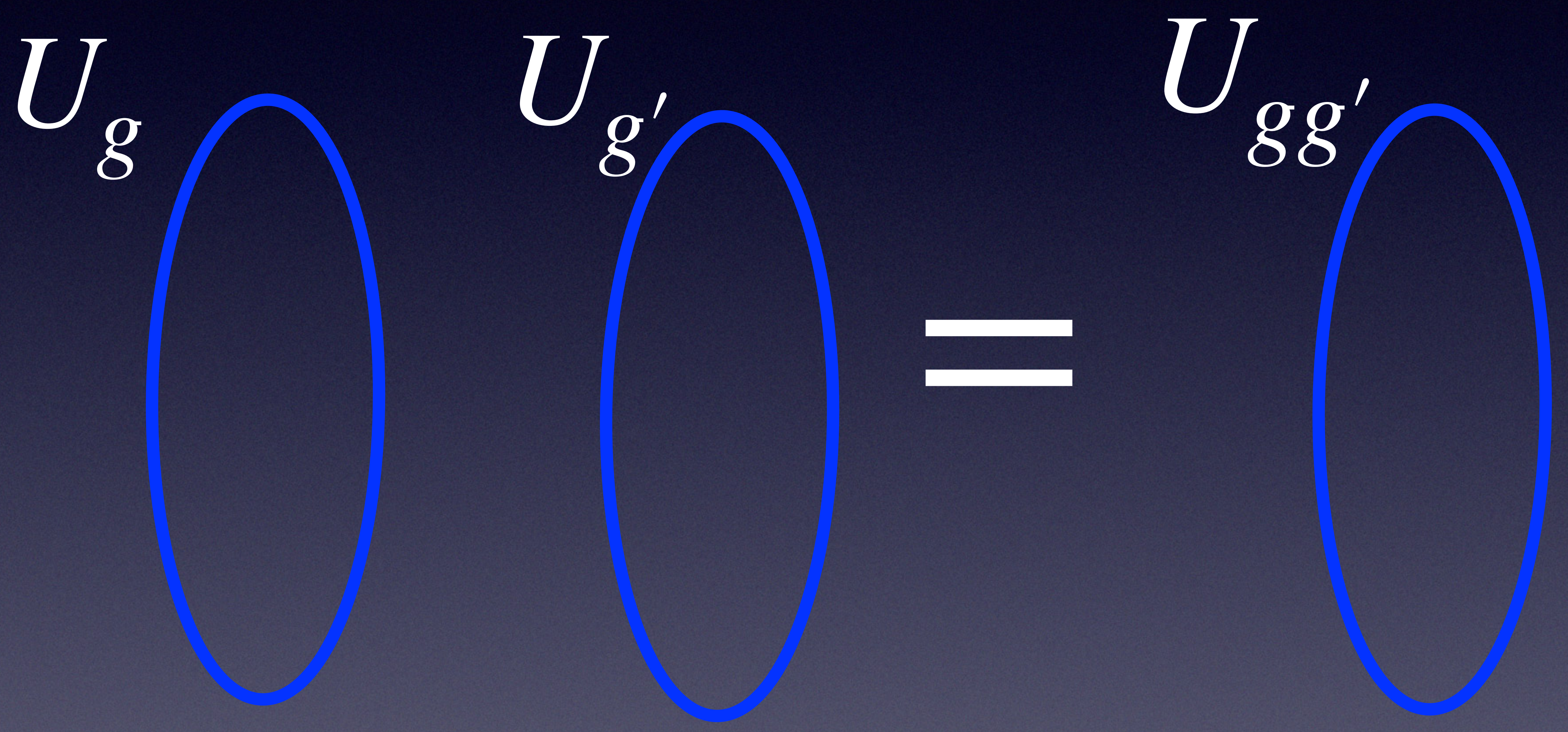
p-dimensional object labeled by representation of G

Generalized global symmetry

Gaiotto, Kapustin, Seiberg, Willett ('14)

QFT in (d+1) dimension

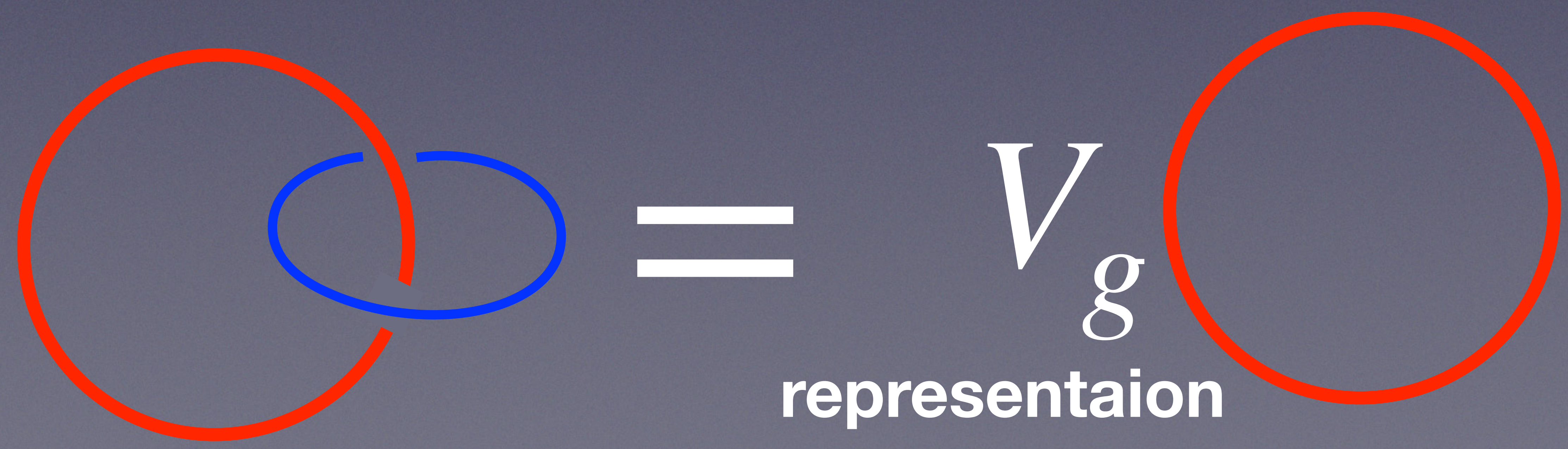
p form symmetry G



group law
$g, g' \in G$
$gg' \in G$

Symmetry generator:
(d-p)-dimensional

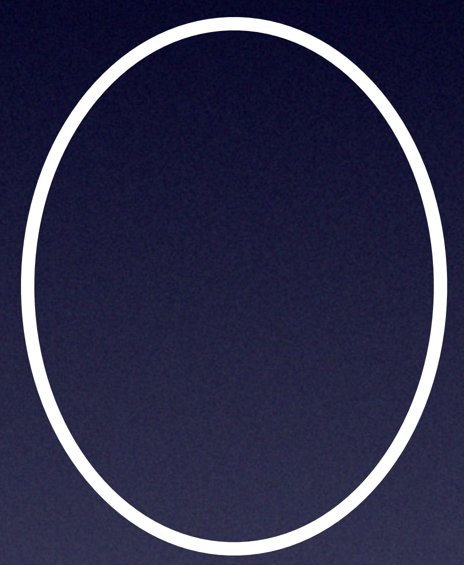
Charged object:
p-dimensional



Example: U(1) gauge theory

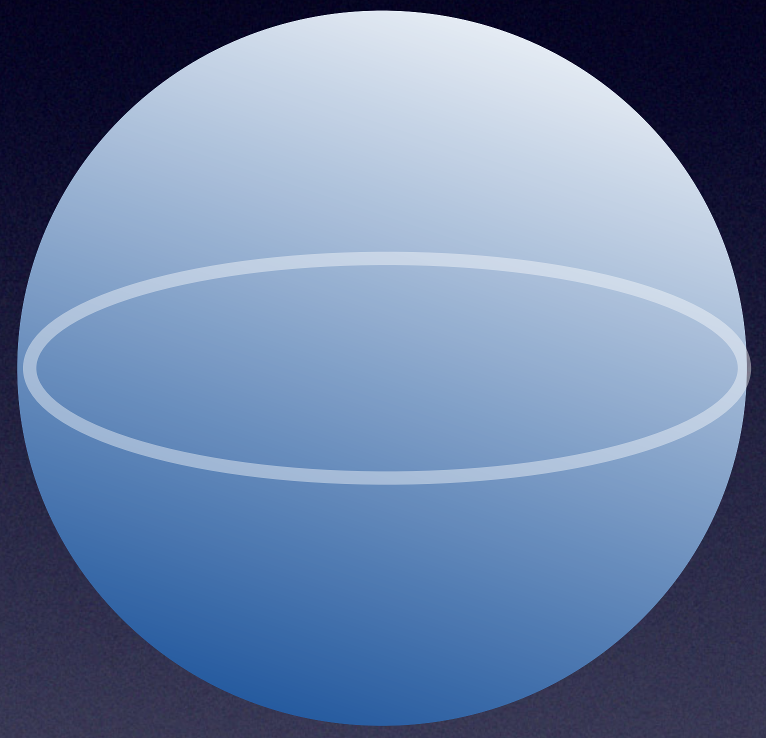
Gaiotto, Kapustin, Seiberg, Willett ('14)

Charged object



Wilson ('t Hooft) loop

Charge



Surface operators

$U(1)^{[1]}_E$ $\quad W = \exp\left[i \oint_{M^{(1)}} A \right]$ $\quad$ **Electric:** $Q_e = \frac{1}{e^2} \int_{M^{(2)}} \star F$

$U(1)^{[1]}_M$ $\quad H = \exp\left[i \oint_{M^{(1)}} \tilde{A} \right]$ $\quad$ **Magnetic:** $Q_m = \frac{1}{2\pi} \int_{M^{(2)}} F$

Quantum electrodynamics

There are $U(1)_M^{[1]}$ magnetic 1-form symmetry

Vacuum

SSB $U(1)_M^{[1]}$

Emergent symmetry $U(1)_E^{[1]}$

Photons are Nambu-Goldstone modes

Superconductor

Unbroken $U(1)_M^{[1]}$

Emergent symmetry (SSB) $\mathbb{Z}_2^{[1]} \times \mathbb{Z}_2^{[2]}$

Topological order

$\mathbb{Z}_2^{[1]}$: cooper pair has charge 2

$\mathbb{Z}_2^{[2]}$:π magnetic flux inside of vortex

Thought experiment : rotating neutron stars

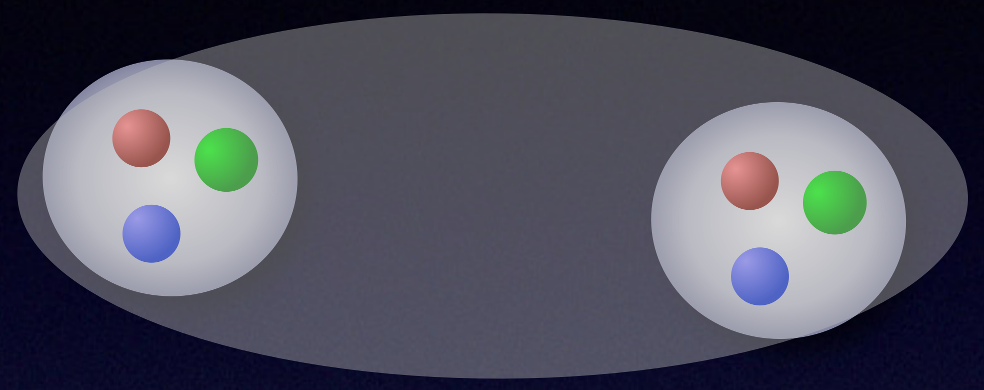
Quantum vortex

Consider continuity of vortices

- Circulation
- Emergent symmetry

Hadronic superfluid phase

di-baryons condense



$$\Delta = \langle \Lambda\Lambda \rangle \neq 0 \quad \Lambda \sim uds$$

Symmetry breaking pattern

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

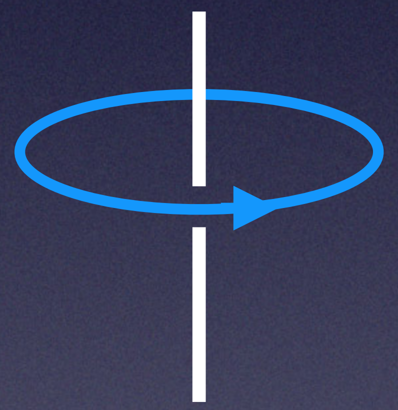
Topological excitation: U(1) vortex $\pi_1(U(1)_B) = \mathbb{Z}$

$$\phi = \Delta f(r) e^{i\theta} \quad \int \frac{d\theta}{2\pi} \in \mathbb{Z} \quad f \xrightarrow{r \to 0} 0 \quad f \xrightarrow{r \to \infty} 1$$

Quantum number in Hadronic superfluid phase

Global $U(1)_B$ symmetry is broken

U(1) vortex: topological defect $\Delta e^{i\theta}$

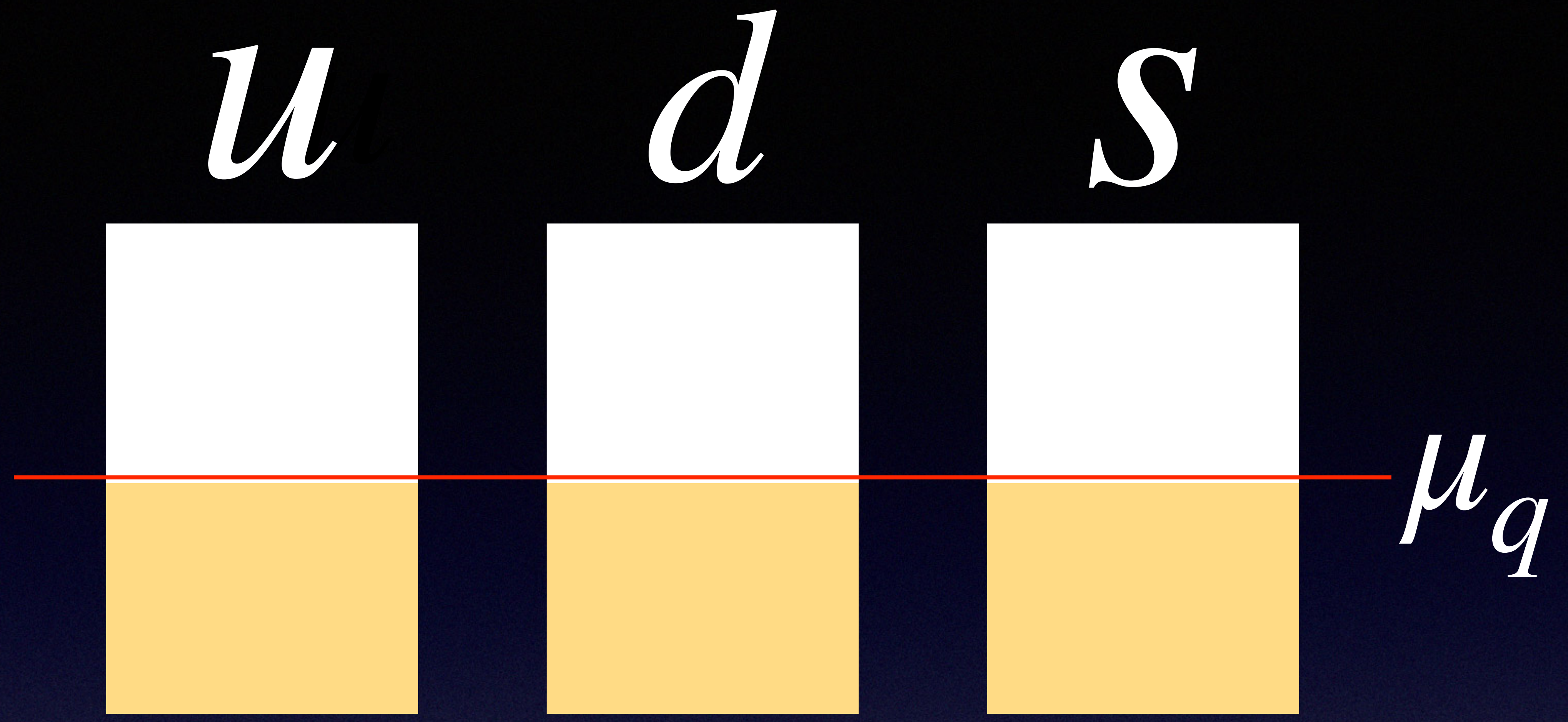


Circulation: $\int v = \int \frac{d\theta}{2\mu_B} = \frac{2\pi\nu_B}{2\mu_B}$

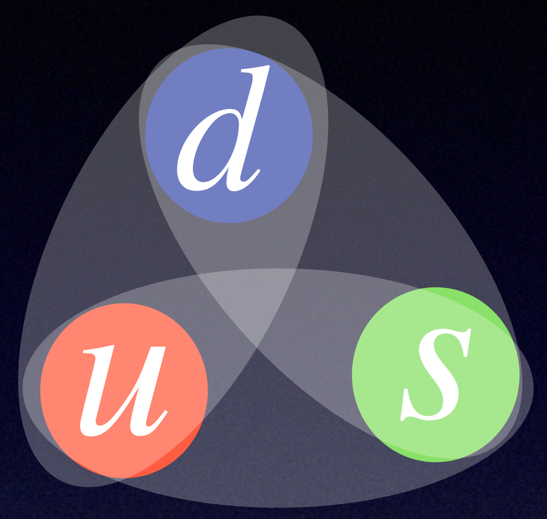
$\nu_B = \int \frac{d\theta}{2\pi}$: Winding number

$2\mu_B$: Baryon chemical potential of order parameter

Color-flavor locking phase



quark pair



$$(\Phi_L)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_L)^b_j (Cq_L)^c_k \rangle \quad (\Phi_R)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_R)^b_j (Cq_R)^c_k \rangle$$

$$\Phi := \Phi_L = -\Phi_R = \begin{pmatrix} \Delta_{\mathrm{CFL}} & 0 & 0 \\ 0 & \Delta_{\mathrm{CFL}} & 0 \\ 0 & 0 & \Delta_{\mathrm{CFL}} \end{pmatrix}$$

Topological excitations

cf. Eto, Hirono, Nitta & Yasui, PTEP 2014, 012D01 (2014)

order parameter space $G/H \simeq \frac{SU(3)_c \times U(1)_B}{\mathbb{Z}_3} \simeq U(3)$

U(1) vortex

$$\Phi := \Delta_{\mathrm{CFL}} \begin{pmatrix} e^{i\theta}f(r) & 0 & 0 \\ 0 & e^{i\theta}f(r) & 0 \\ 0 & 0 & e^{i\theta}f(r) \end{pmatrix}$$

Non-abelian CFL vortex

Balachandran, Digal, Matsuura, PRD73, 074009 (2006)

$$\Phi := \Delta_{\mathrm{CFL}} \begin{pmatrix} e^{i\theta}f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix} = \Delta_{\mathrm{CFL}} e^{i\frac{\theta}{3}} \begin{pmatrix} e^{i\frac{2\theta}{3}}f(r) & 0 & 0 \\ 0 & e^{-i\frac{\theta}{3}}g(r) & 0 \\ 0 & 0 & e^{-i\frac{\theta}{3}}g(r) \end{pmatrix}$$

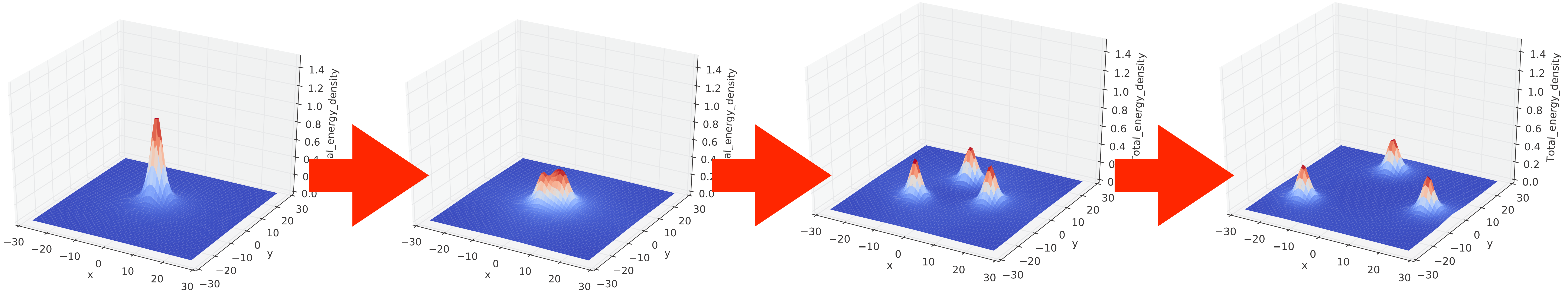
$$A_i = -\frac{\epsilon_{ij}x^j}{g_s^2 r^2}(1 - h(r))\mathrm{diag}\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$$ **both superfluidity and superconductivity**

Numerical Simulation

Alford, Mallavarapu, Vachaspati, Windisch, PRC 93, 045801 (2016)

U(1) vortex

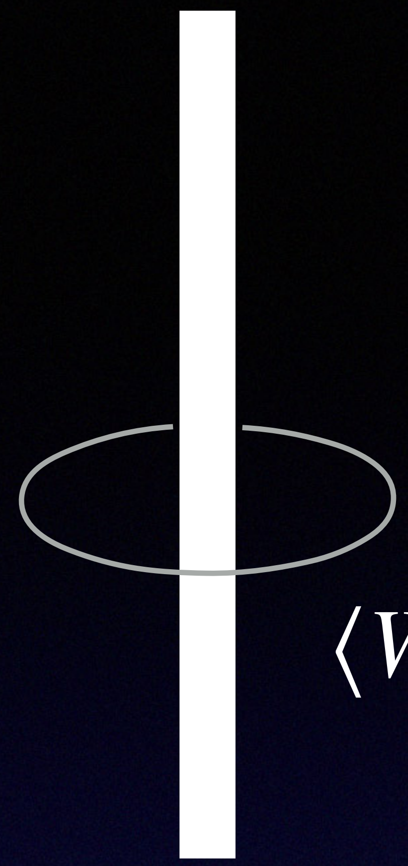
non-abelian vortices



(a) (b) (c) (d)

U(1) vortex decays into three non-abelian vortices

U(1) vortex in Hadronic phase



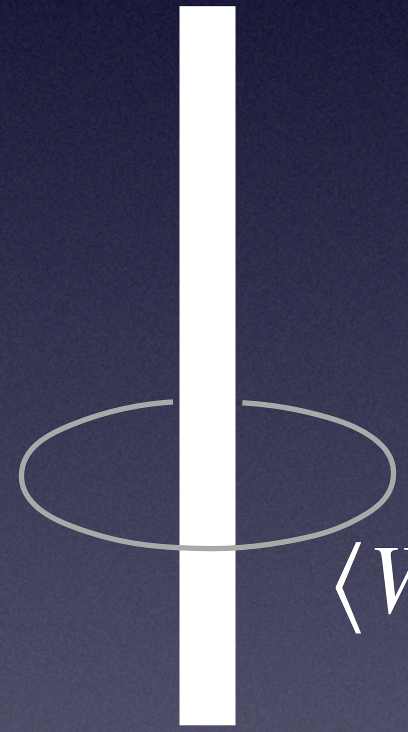
$\langle W \rangle = |\langle W \rangle|$

Circulation $\boxed{2\pi \dfrac{\nu_B}{2\mu_B}}$

Alford, Baym, Fukushima, Hatsuda, Tachibana ('19)

ν_B: **Winding number**

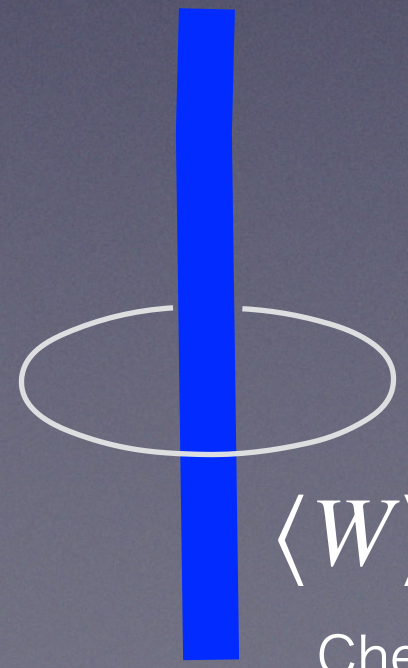
U(1) vortex in CFL



$\langle W \rangle = |\langle W \rangle|$

Circulation $2\pi \dfrac{\nu_A}{2\mu_q} = 2\pi \dfrac{3\nu_A}{2\mu_B}$

Non-abelian vortex in CFL



$\langle W \rangle = e^{i\frac{2\pi\nu_A}{3}} |\langle W \rangle|$

Cherman, Sen, Yaffe, PRD 100, 034015 (2019)

Circulation $\dfrac{2\pi\nu_A/3}{2\mu_q} = \boxed{2\pi \dfrac{\nu_A}{2\mu_B}}$

Topological ordered phase?

CFL vortex: emergent $\mathbb{Z}_3^{[2]}$ symmetry

However, it is not unbroken, i.e. not topological order

Hirono, Tanizaki ('19)

What is the fate of $e^{\frac{2\pi}{3}i}$?

The magnetic flux will not penetrate through the vortices in the hadronic phase

$\Rightarrow$ This allow us to distinguish the phases.

Cherman, Jacobson, Sen, Yaffe ('20), ('24)

Magnetic flux may penetrate through the vortices in the hadronic phase or dissipate during the transition.

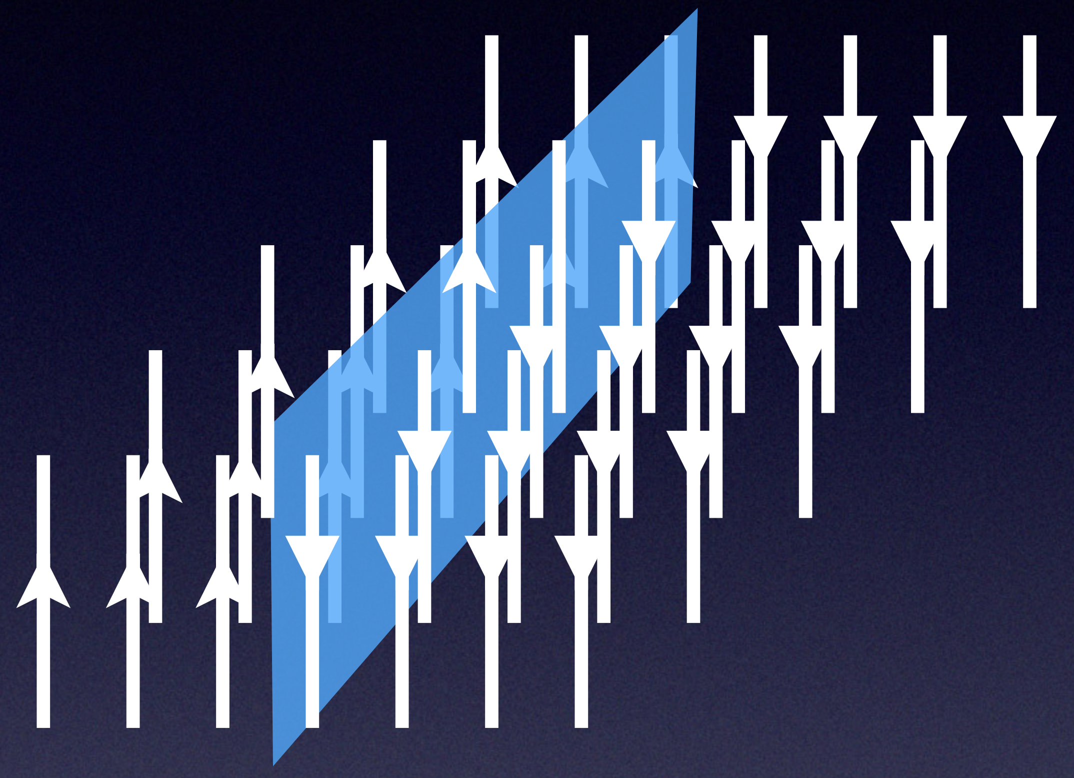
Hayashi ('23)

Outline

- Phase transition on a vortices
- Summary

Phase transition on a topological defect, while the bulk remains continuous?

Domain wall



vortex

Our answer is YES!

Effective theory on a topological defect=
a lower-dimensional field theory may exhibit phase transition

Phase transitions may occur in quantum vortices.

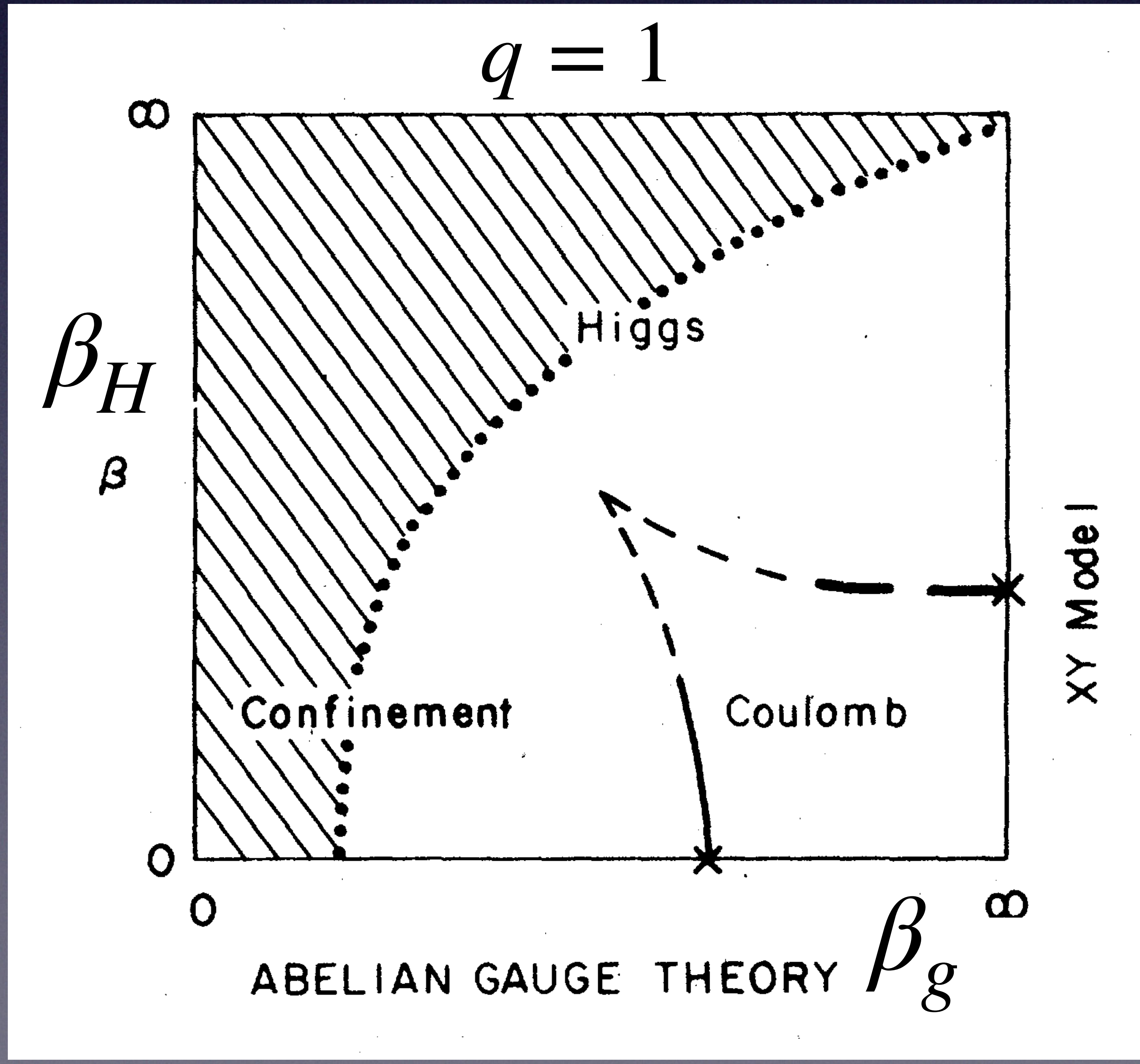
Abelian Higgs model in (3+1) dimensions

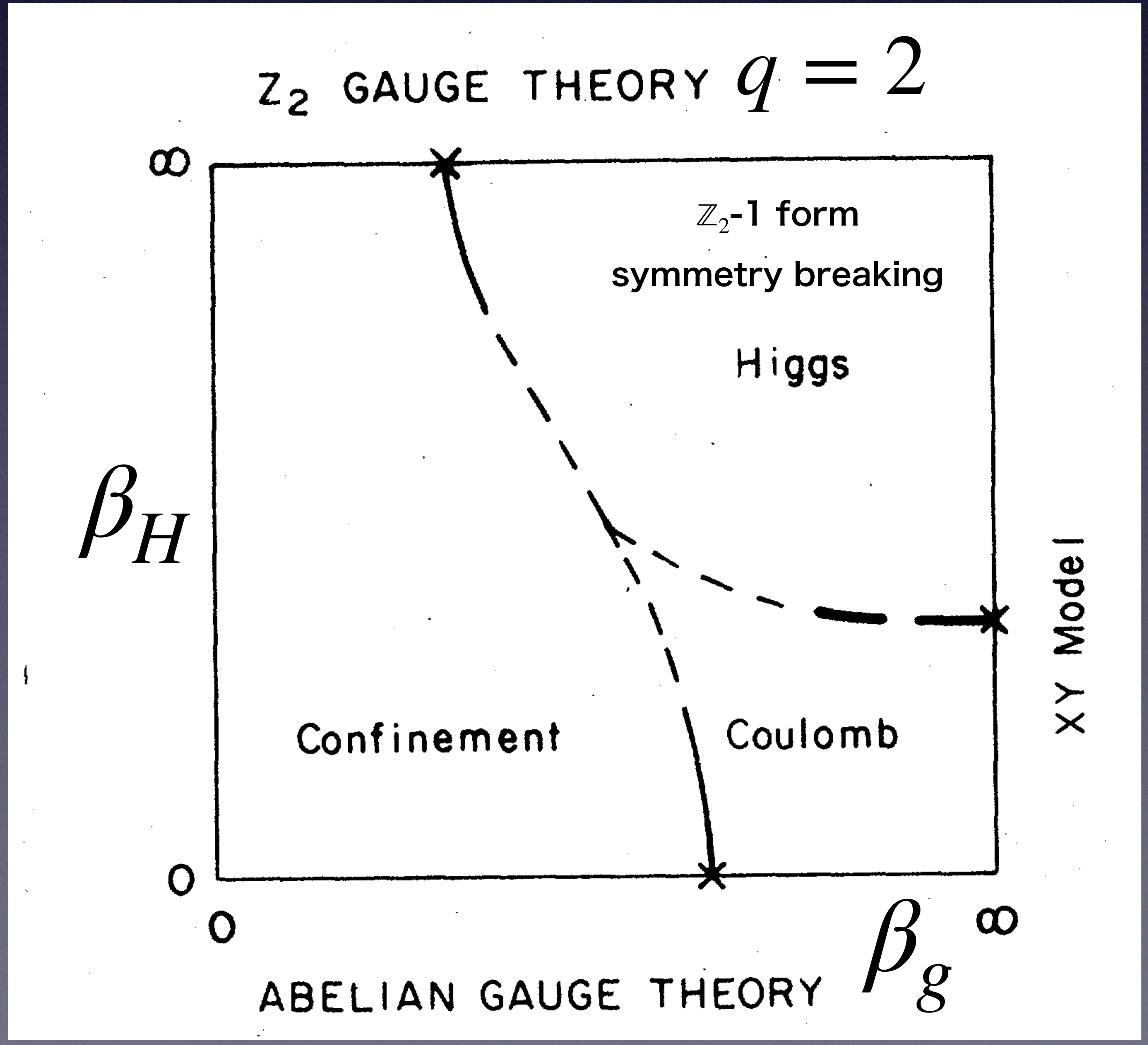
$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \cos\left(\Delta_\mu \varphi(x) - qA_\mu(x)\right)$$

Field strength **Scalar field (phase dof)** **Gauge field**

$$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$$

Fradkin-Schenker Phys. Rev. D 19, 3682 ('79)





$\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ lattice model

cf. Motrunich, Senthil ('05)

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Field strength **Scalar field** (phase dof) **Gauge field**

$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$

Symmetry

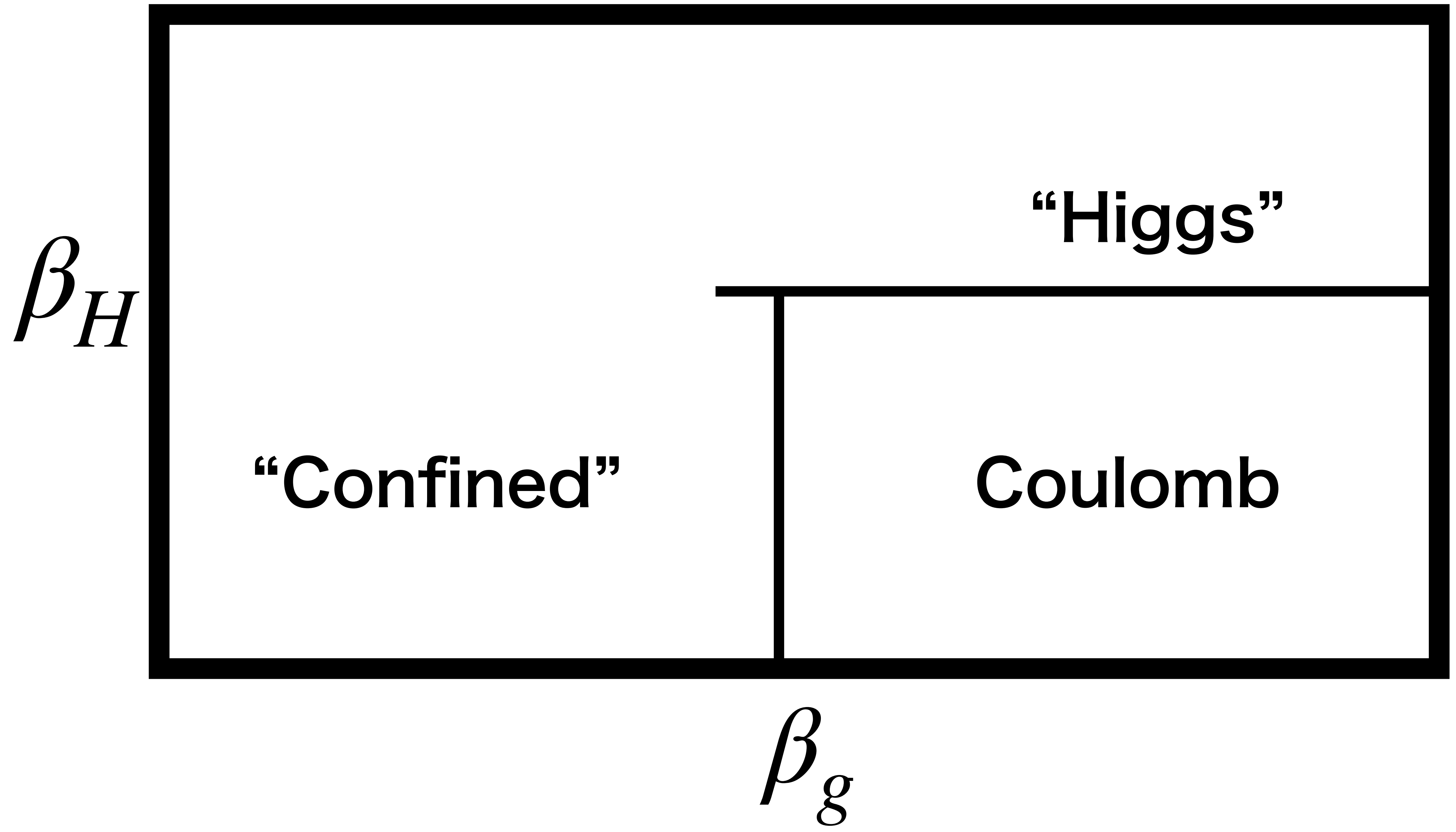
$$\mathrm{U}(1)_{\mathrm{gauge}} : \begin{array}{l} \varphi_1 \to \varphi_1 - \lambda \\ \varphi_2 \to \varphi_2 - \lambda \\ A_\mu \to A_\mu + \Delta_\mu \lambda \end{array}$$

$$\mathrm{U}(1)_{\mathrm{global}} : \begin{array}{l} \varphi_1 \to \varphi_1 + \theta \\ \varphi_2 \to \varphi_2 - \theta \end{array}$$

$$\mathbb{Z}_{2F} : \begin{array}{l} \varphi_1 \to \varphi_2 \\ \varphi_2 \to \varphi_1 \end{array}$$

Phase diagram

Fradkin-Schenker



$\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ lattice model

cf. Motrunich, Senthil ('05)

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Field strength

Scalar field **(phase dof)**

Gauge field

$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$

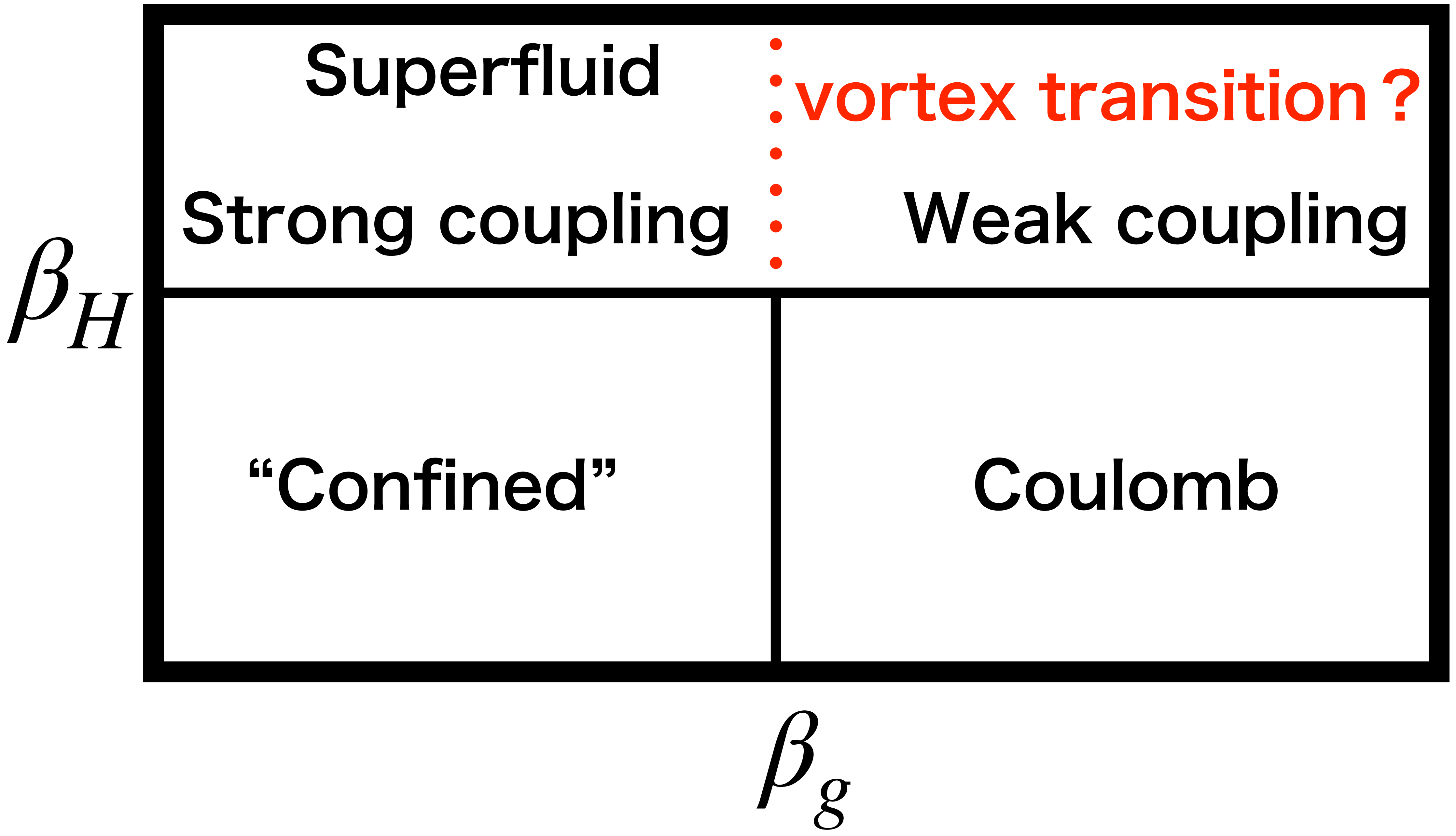
Symmetry

$$\mathrm{U}(1)_{\mathrm{gauge}} : \begin{array}{l} \varphi_1 \to \varphi_1 - \lambda \\ \varphi_2 \to \varphi_2 - \lambda \\ A_\mu \to A_\mu + \Delta_\mu \lambda \end{array}$$

$$\mathrm{U}(1)_{\mathrm{global}} : \begin{array}{l} \varphi_1 \to \varphi_1 + \theta \\ \varphi_2 \to \varphi_2 - \theta \end{array}$$

$$\mathbb{Z}_{2F} : \begin{array}{l} \varphi_1 \to \varphi_2 \\ \varphi_2 \to \varphi_1 \end{array}$$

Phase diagram



$\mathrm{U(1)_{gauge}} \times \mathrm{U(1)_{global}}$ lattice model

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Field strength

Scalar field (phase dof)

Gauge field

$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$

Emergent symmetry at large β_H (SSB of $U(1)_{\text{global}}$)

YH, Kondo ('22)

Emergent	$U(1)^{[2]}$	$\mathbb{Z}_2^{[2]}$
Symmetry operator	$e^{i\frac{\theta}{2\pi}\int_C (d\varphi_1 - d\varphi_2)}$	$e^{i\frac{1}{2}\int_C (d\varphi_1 + d\varphi_2)}$

Example: $\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ lattice model

Strong coupling $\beta_g \ll 1$

Integrating over gauge fields

$$S_{\mathrm{eff}} = -\sum_{x,\mu} \ln I_0 \left[2\beta_H \cos\left(\frac{\Delta_\mu \varphi_1(x) - \Delta_\mu \varphi_2(x)}{2} \right) \right]$$

$I_0(z)$:Modified Bessel

Essential d.o.f. is $\varphi_1 - \varphi_2$

i.e., one d.o.f.

Weak coupling $\beta_g \gg 1$

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right)$$
$$-\beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Distinguishable φ_1 and φ_2

$\mathbb{Z}_{2F}$ is spontaneously broken on the vortices

Criterion of symmetry breaking:

When discrete symmetry is broken:
twisting the boundary conditions by the symmetry causes the formation of domain walls

Example: Ising model

$\mathbb{Z}_2$ broken phase

↑↑↑↑↑↑|↓↓↓↓↓↓

domain wall

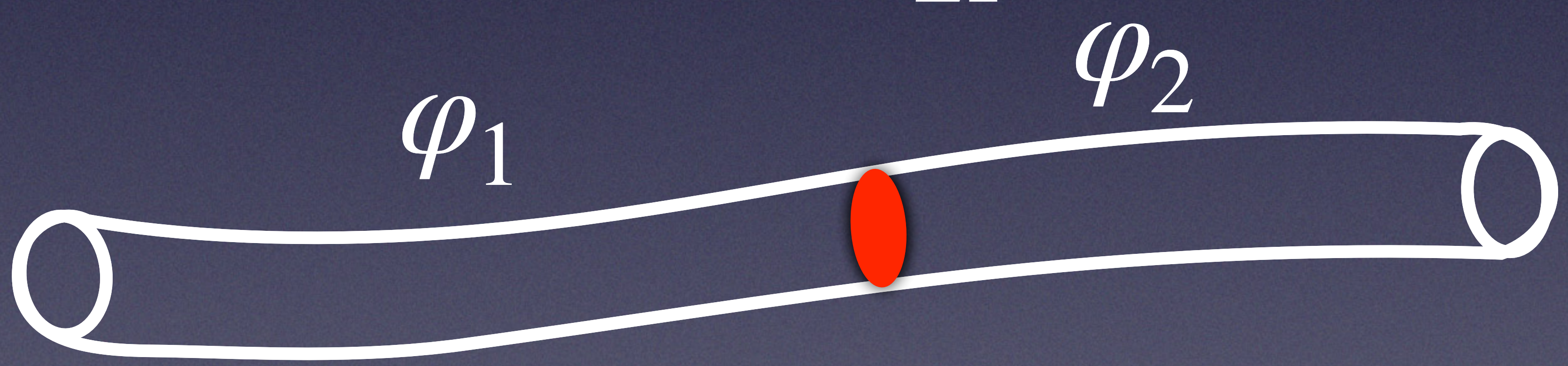
$\mathbb{Z}_2$ unbroken phase

random configuration

↑↓↑↑↓↑|↑↓↑↓↓↓

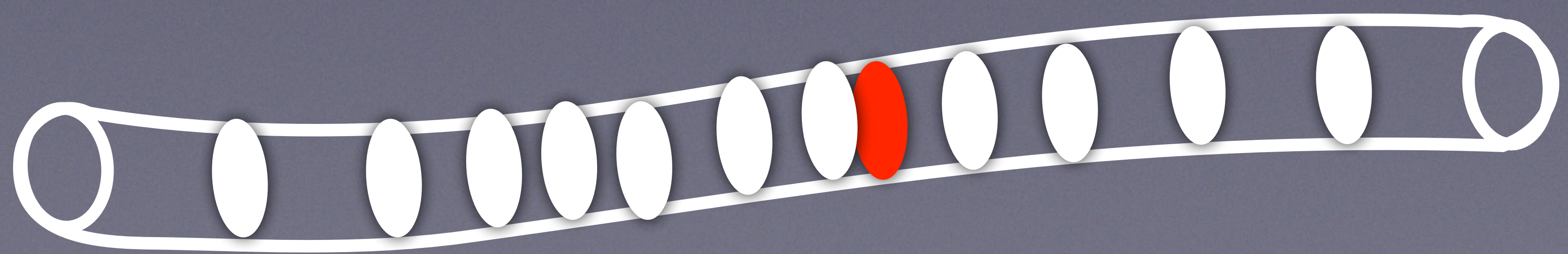
$\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ model

Weak coupling ($\mathbb{Z}_{2F}$ broken)



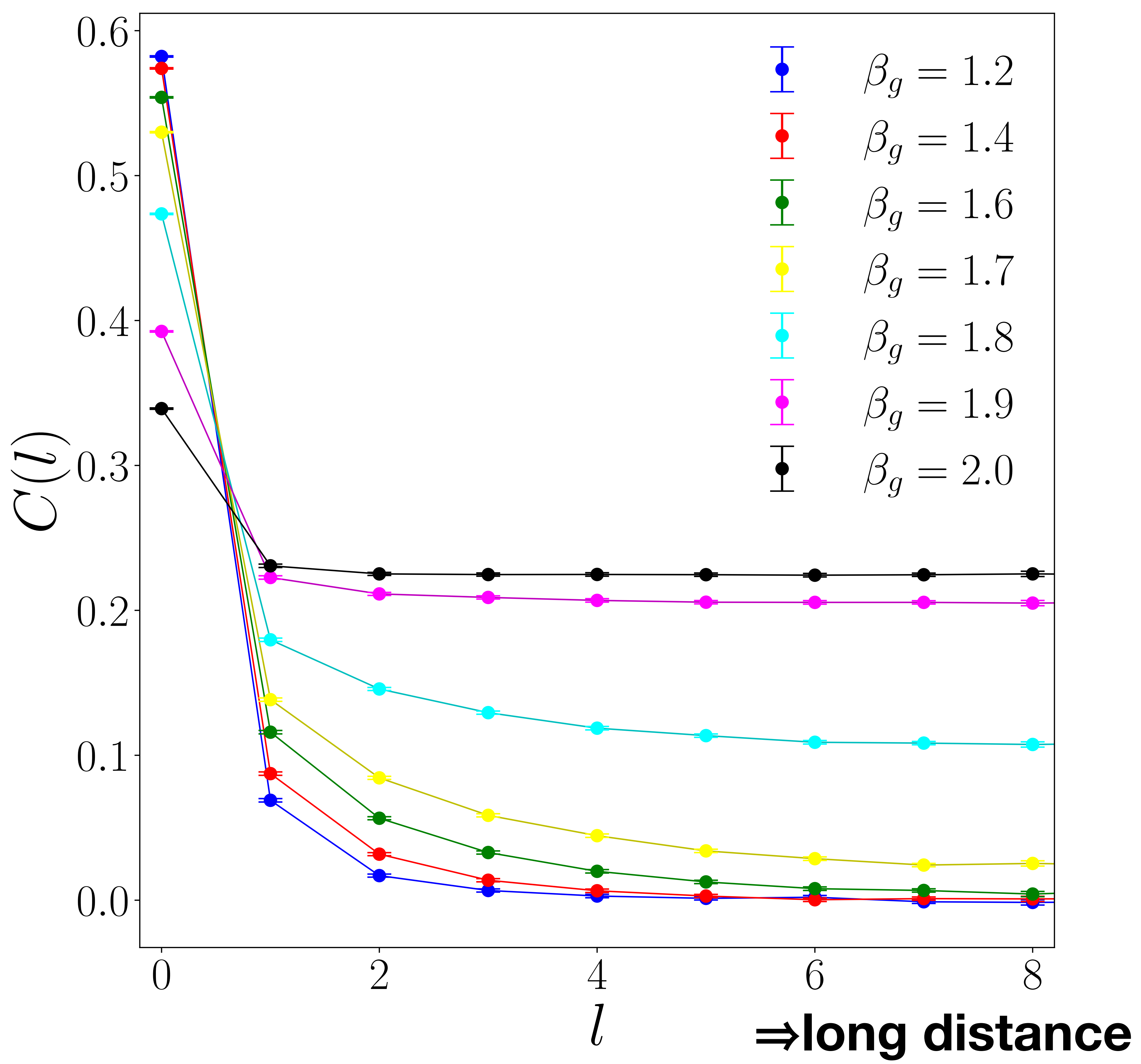
Strong coupling ($\mathbb{Z}_{2F}$ unbroken)

randomized junctions



Numerical simulation

Correlation function of magnetic flux



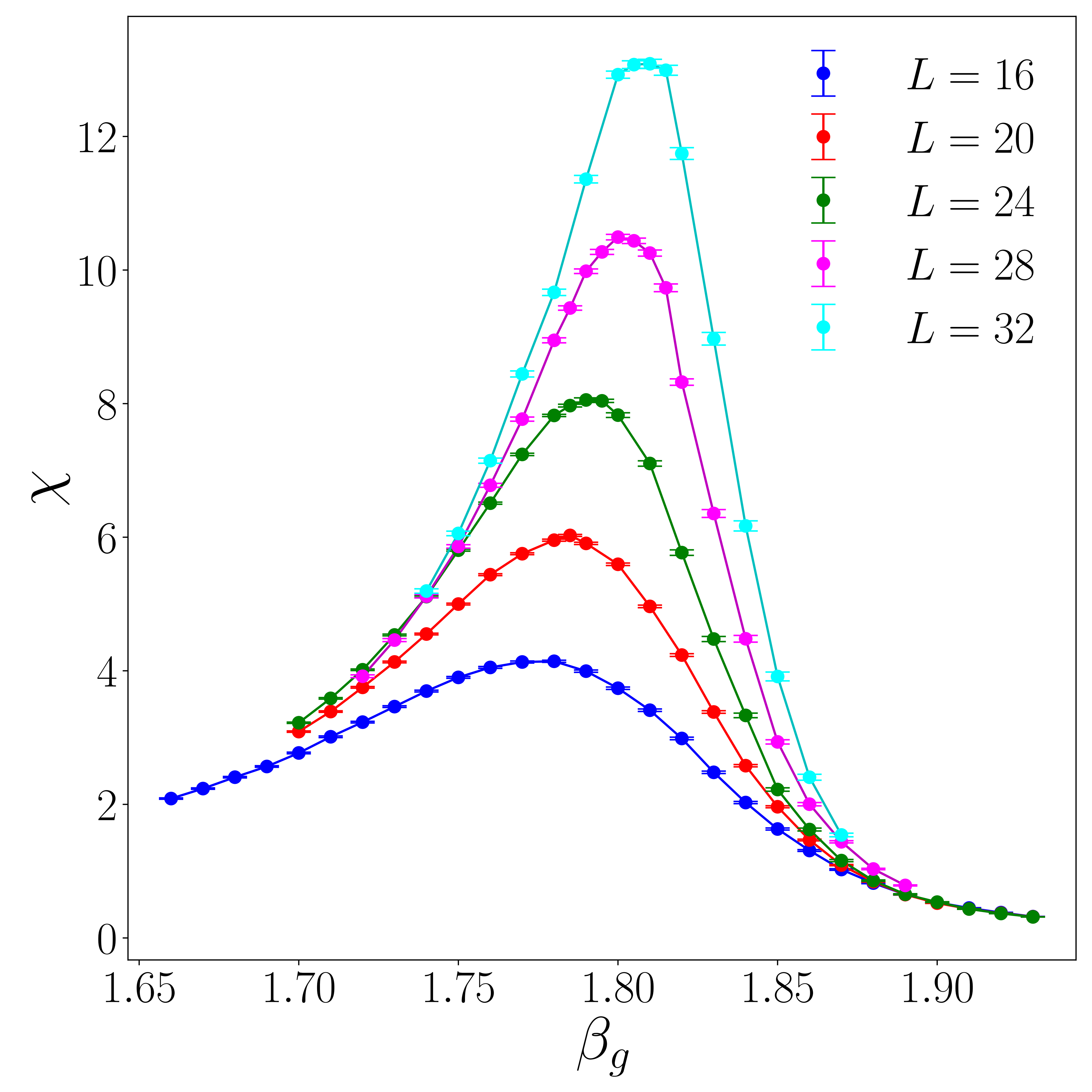
vortex

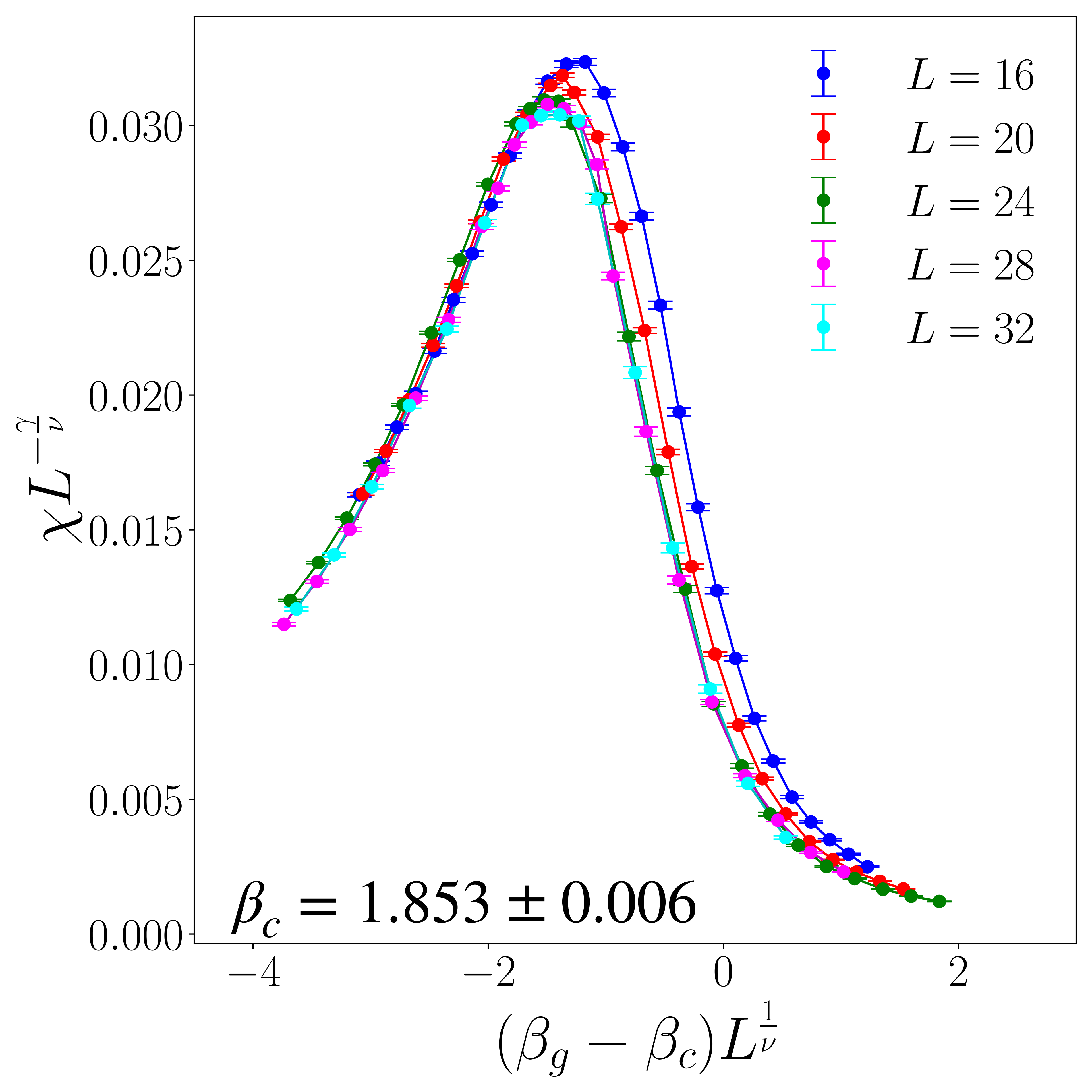
At weak coupling long-range correlation

Spontaneous symmetry breaking

⇒ **Phase transition on a vortex**

Critical point





Ising universality class $\nu = 1, \gamma = 7/4$

predicted in Motrunich, Senthil ('05)

Summary

We found the phase transition on a vortex between strong and weak gauge couplings in superfluid phase

More generally, there can be phase transitions of various phase defects

Codimension 1: transition on a domain wall
Codimension 2: transition on a vortex
Codimension 3: Level crossing

Phase transitions on domain wall junctions are also possible

Outlook

EFT on $U(1) \times U(1)$ model〜Ising model

EFT of CFL phase $\sim CP(2)$ model

Ground state of $CP(2)$ model
Gapped phase, no flavor breaking
⇒ continuously connects to the hadronic phase？

What happens if fermion d.o.f. is included？