One-Pion exchange potential in a strong magnetic field

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Based on work in progress

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Symmetry and Effective Field Theory of Quantum Matter at Niigata Univ. on 2024/11/28

Motivation：Nuclear force in a strong magnetic field

Magnetar

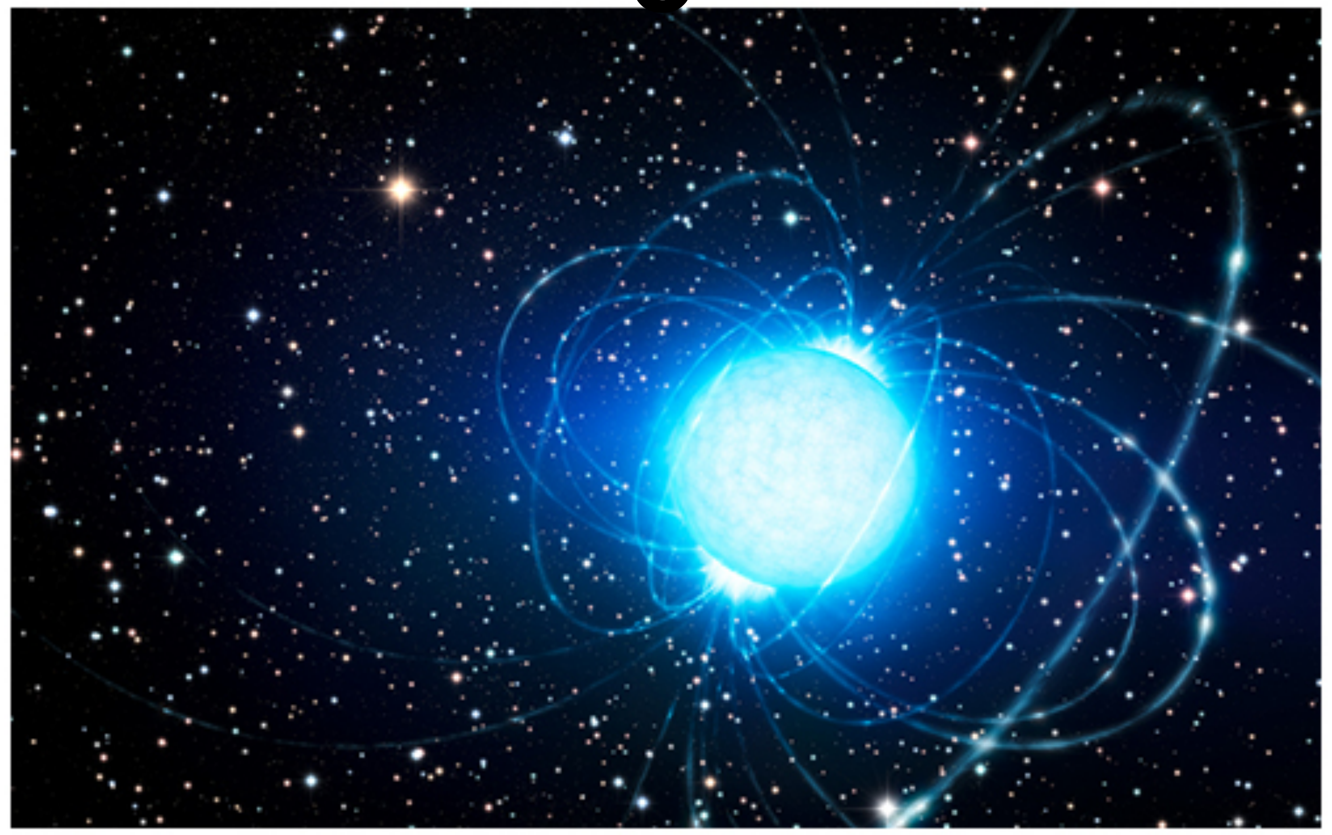


Image credit：ESO/L. Calçada

$\sim 10^{15}$ G McGill Magnetar Catalog

Heavy Ion Collision

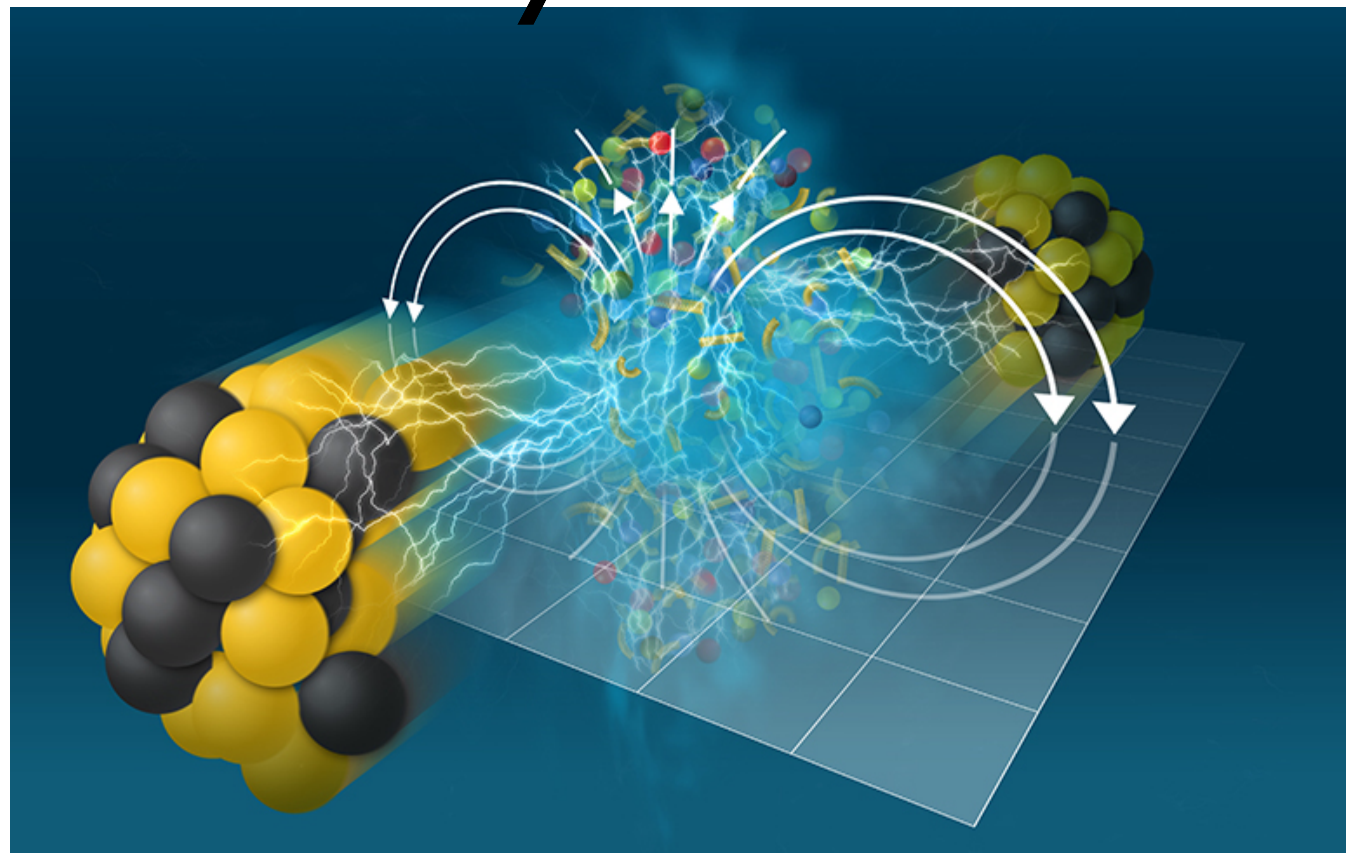


Image credit：T. Bowman and J. Abramowitz/Brookhaven National Laboratory

$\sim 10^{18}$ G STAR Collaboration(2024)

Magnetar : One of the Neutron stars which have a strong magnetic field

On the surface of this star, the magnitude of magnetic field is up to 10^{15}G

Heavy Ion Collision : A magnetic field of up to 10^{18}G is generated

when relativistically accelerated charged particles collide non-centrally

➡ We are inspired and interested in a Nuclear force in a strong magnetic field

Especially, we focus on pion-exchange potential because this is the lightest meson

Motivation : OPEP in a strong magnetic field

The deuteron

- Isospin-singlet $T=0$, Spin-triplet $S=1$, Total angular momentum $J=1$
- Non-zero electric quadrupole moment
- Magnetic moment $0.857\mu_N$ μ_N : Nuclear magneton

$$\Rightarrow \quad |d_M\rangle = C_s\left|{}^3S_1\right\rangle + C_D\left|{}^3D_1\right\rangle \qquad M=0,\pm 1 \qquad |C_S|^2 \simeq 0.96, |C_D|^2 \simeq 0.04$$

This mixture of state is understood to be caused by **tensor operator in a one-pion exchange potential (OPEP)**

OPEP : $\hat{V}_{\mathrm{OPE}} = \#(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left[\left(\frac{1}{r^2}+\frac{m_\pi}{r}+\frac{m_\pi^2}{3}\right)\hat{S}_{12}+\frac{m_\pi^2}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right]\frac{\mathrm{e}^{-m_\pi r}}{r}$

Tensor operator : $\hat{S}_{12} = \frac{3}{r^2}(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{r}) - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$

r : relative distance of two Nucleons

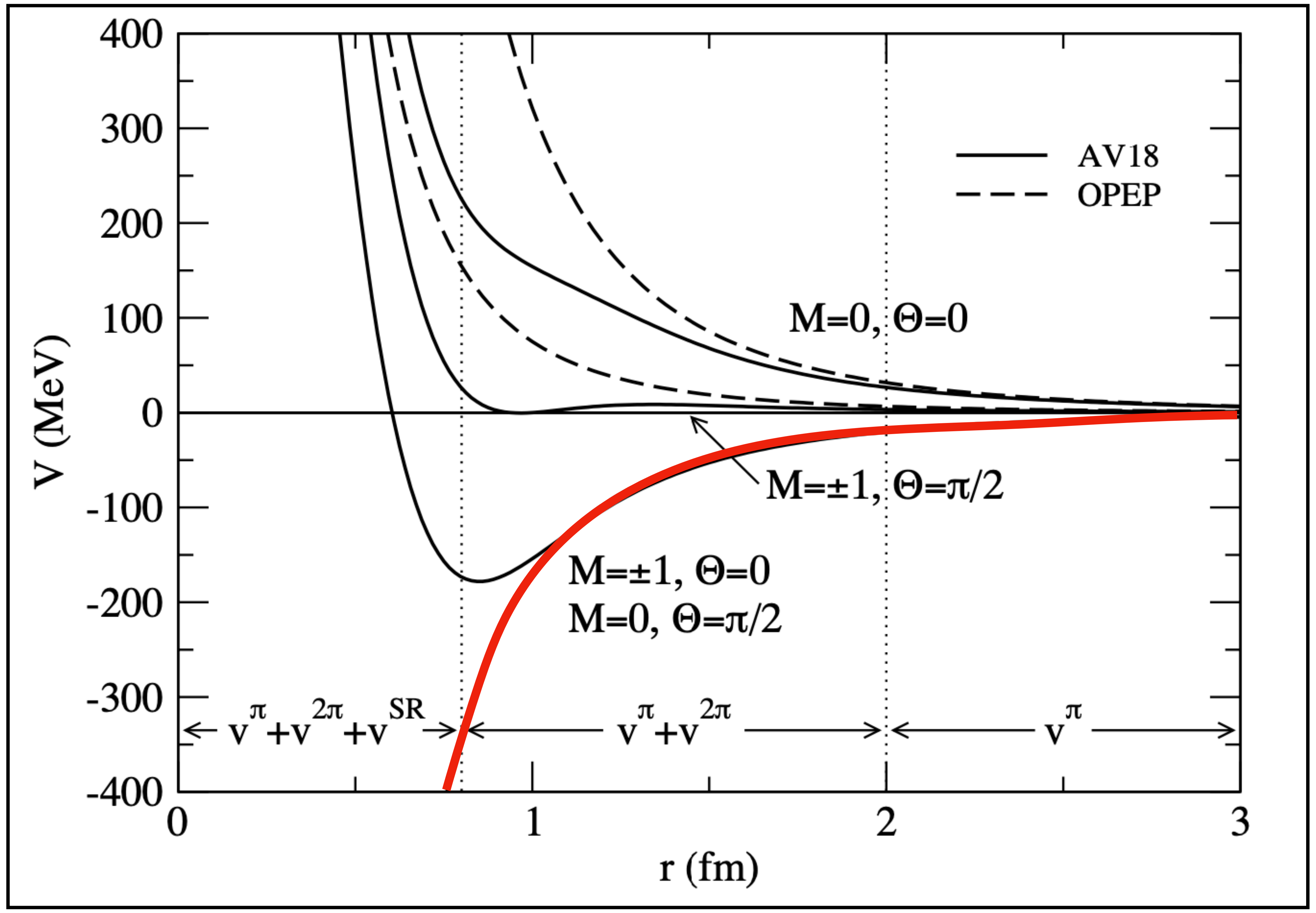
$\tau_{1,2}$: Isospin operator acts Nucleons

$\sigma_{1,2}$: Spin operator

$[\hat{S}_{12}, \hat{\boldsymbol{L}}] \neq 0$ and ${}^3S_1$ and ${}^3D_1$ mix

Motivation : OPEP in a strong magnetic field

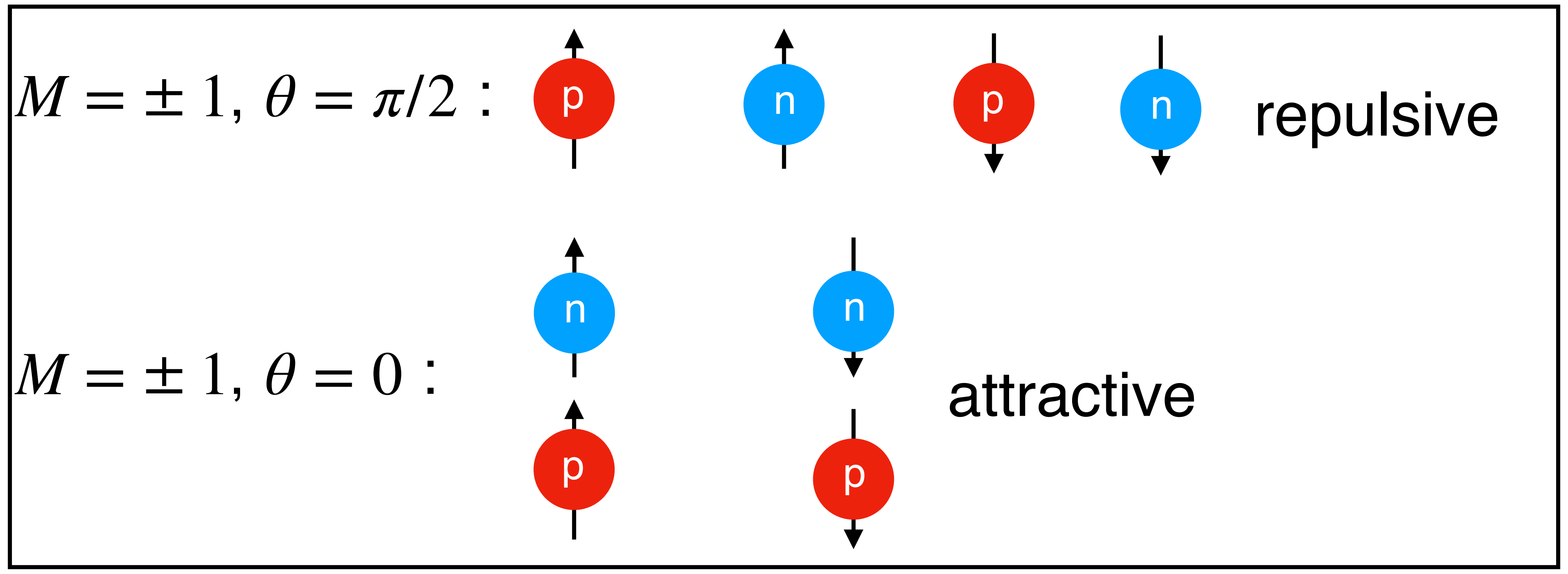
This tensor operator play role for binding proton and neutron

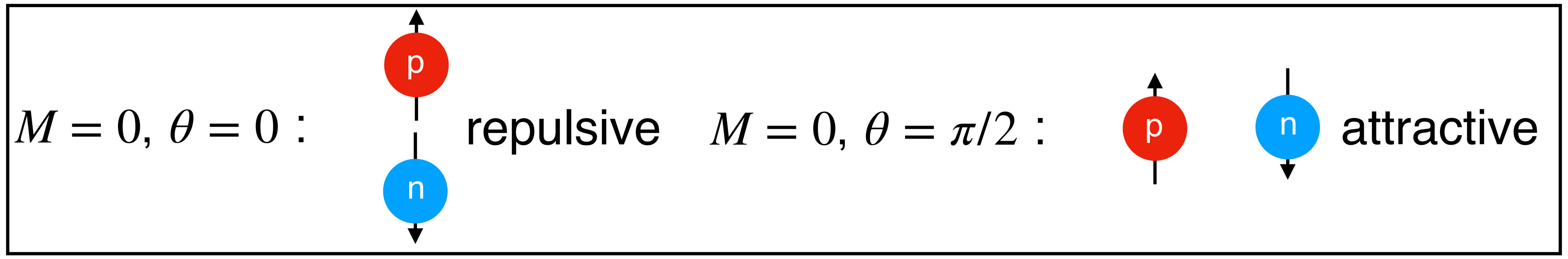


Comparison OPEP with AV 18 potential for $T = 0,\, S = 1$

NuSTEC Class Notes

θ : angle between spin and relative distance





Tensor force from OPEP is very attractive

and this force is the factor that deuterons exist

Based on the above, we derive OPEP in a magnetic field

As application, we examine the deuteron energy shift using OPEP in a magnetic field

One-pion exchange potential in a strong magnetic field

Construction of the interacting Hamiltonian

Chiral Lagrangian with a magnetic field $\overrightarrow{B} = (0,0,B)^t$

$$N = (p, n)^t$$

$$D_\mu^\pm \pi^\pm \equiv \partial_\mu \pi^\pm \pm \mathrm{i} e A_\mu \pi^\pm$$

$$\mathcal{L}_{\text{eff}} = g^{\mu\nu} D_\mu^+ \pi^+ D_\nu^- \pi^- - m_\pi^2 \pi^+ \pi^- + \frac{1}{2}(\partial_\mu \pi^0)^2 - \frac{1}{2} m_\pi^2 (\pi^0)^2$$

$$+ N^\dagger \mathrm{i} D_0 N - \frac{g_A}{2f_\pi} \sum_{a=\pm} D_i^a \pi^a N^\dagger \sigma^i \tau_a N - \frac{g_A}{2f_\pi} \partial_i \pi^0 N^\dagger \sigma^i \tau_0 N$$

Non-rela Nucleon

- Take the heavy baryon limit for nucleons
- A_μ in Covariant derivative
- Magnitude of magnetic field

$$|eB| \sim m_\pi^2 \ll m_N^2 \quad \text{and} \quad |eB| = m_\pi^2 \sim 10^{18}\text{G}$$

- Neglect Zeeman term such as $\dfrac{e}{2m_N} \sigma \cdot \mathbf{B} \ll 1$

Construction of the interacting Hamiltonian

Int. Hamiltonian from Lagrangian

$$H_{\rm int} = \frac{g_A}{2f_\pi}\int \mathrm{d}^3r \sum_{a=\pm} D_i^a \pi^a N^\dagger \sigma^i \tau_a N$$

π field from EoM

$$\pi^a(\boldsymbol{r}) = -\frac{g_A}{2f_\pi}\int \mathrm{d}^3r' N^\dagger_{r'} \sigma^j \tau_{-a} N_{r'} D'^{-}_j \mathrm{i}\Delta^a(\boldsymbol{r},\boldsymbol{r}'|A)$$

This is induced by a Nucleon at position r'

Interacting Hamiltonian

$$H_{\rm int} = -\frac{g_A^2}{4f_\pi^2}\int \mathrm{d}^3r\mathrm{d}^3r' \sum_{a=\pm} N^\dagger_{r'} N^\dagger_r (\sigma^j\tau_{-a})_{r'} (\sigma^i\tau_a)_r D_i^a D_j'^{-a} \mathrm{i}\Delta^a(\boldsymbol{r},\boldsymbol{r}'|A) N_{r'} N_r$$

To get OPEP for the deuteron state

① Charged pion propagator $\mathrm{i}\Delta^a(\boldsymbol{r},\boldsymbol{r}'|A)$

② Acting on the state

Charged Pion propagator in a strong magnetic field

When A_i is translational-invariant such as **Fock-Schwinger gauge**

$$A_j^{\mathrm{FS}}(\boldsymbol{r}-\boldsymbol{r}') \equiv -\frac{1}{2}F_{jk}(r^k - r'^k)$$

propagator satisfy

$$\left[-\delta^{ij}(D_i^{\pm})_{\boldsymbol{r}}(D_j^{\pm})_{\boldsymbol{r}} - m_\pi^2\right] \mathrm{i}\Delta(\boldsymbol{r}-\boldsymbol{r'}|A_{\mathrm{FS}}) = \delta^3(\boldsymbol{r}-\boldsymbol{r'})$$

Applying gauge transformation

$$D_i^{\pm} \to \mathrm{e}^{\mp \mathrm{i}e\alpha(\boldsymbol{r})} D_i^{\pm} \mathrm{e}^{\pm \mathrm{i}e\alpha(\boldsymbol{r})}$$

$$A_i^{\mathrm{FS}} \to A = A_i^{\mathrm{FS}} - \partial_i \alpha(\boldsymbol{r})$$

$$\left[-\delta^{ij}(D_i^{\pm})_{\boldsymbol{r}}(D_j^{\pm})_{\boldsymbol{r}} + m_\pi^2\right] \mathrm{i}\mathrm{e}^{\pm \mathrm{i}e\alpha(\boldsymbol{r}) \mp \mathrm{i}e\alpha(\boldsymbol{r}')}\Delta^{\pm}(\boldsymbol{r},\boldsymbol{r'}|A) = \delta^3(\boldsymbol{r}-\boldsymbol{r'})$$

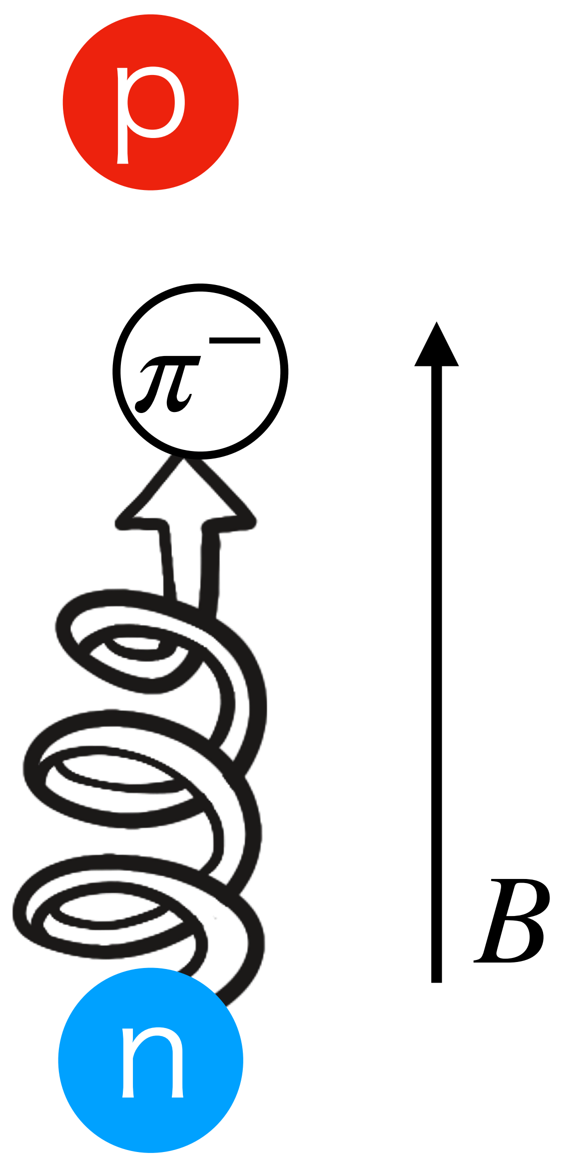
Charged pion propagator in a strong magnetic field

$$\Delta^{\pm}(\boldsymbol{r},\boldsymbol{r'}|A) = \mathrm{e}^{\mp \mathrm{i}e\alpha(\boldsymbol{r}) \pm \mathrm{i}e\alpha(\boldsymbol{r}')}\Delta(\boldsymbol{r}-\boldsymbol{r'}|A_{\mathrm{FS}})$$

Charged Pion propagator in a strong magnetic field

・Propagator in FS gauge

momentum rep.
$$\Delta(\boldsymbol{p}|A_{\mathrm{FS}}) = 2\mathrm{i}e^{-\frac{|\boldsymbol{p}_\perp|^2}{|eB|}} \sum_{n=0}^{\infty} (-1)^n L_n\left(\frac{2|\boldsymbol{p}_\perp|^2}{|eB|}\right) \frac{1}{-p_z^2 - m_\pi^2 - (2n+1)|eB|}$$



F.T.

$$\Delta(\boldsymbol{r}-\boldsymbol{r}'|A_{\mathrm{FS}}) = -\frac{\mathrm{i}}{4\pi}|eB|e^{-\frac{|eB|}{4}|\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp|^2} \sum_{n=0}^{\infty} L_n\left(\frac{|eB|}{2}|\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp|^2\right) \frac{e^{-\sqrt{m_\pi^2+(2n+1)|eB|}|z-z'|}}{\sqrt{m_\pi^2+(2n+1)|eB|}}$$

$\vec{r}_\perp = (x, y)$ $\qquad$ $L_n(x) = L_n^{\alpha=0}(x)$: associated Laguerre polynomials

magnetic field breaks rotational invariance

✔ ① Charged pion propagator $\mathrm{i}\Delta^a(\boldsymbol{r}, \boldsymbol{r}'|A)$ $\qquad$ ② Acting on the state

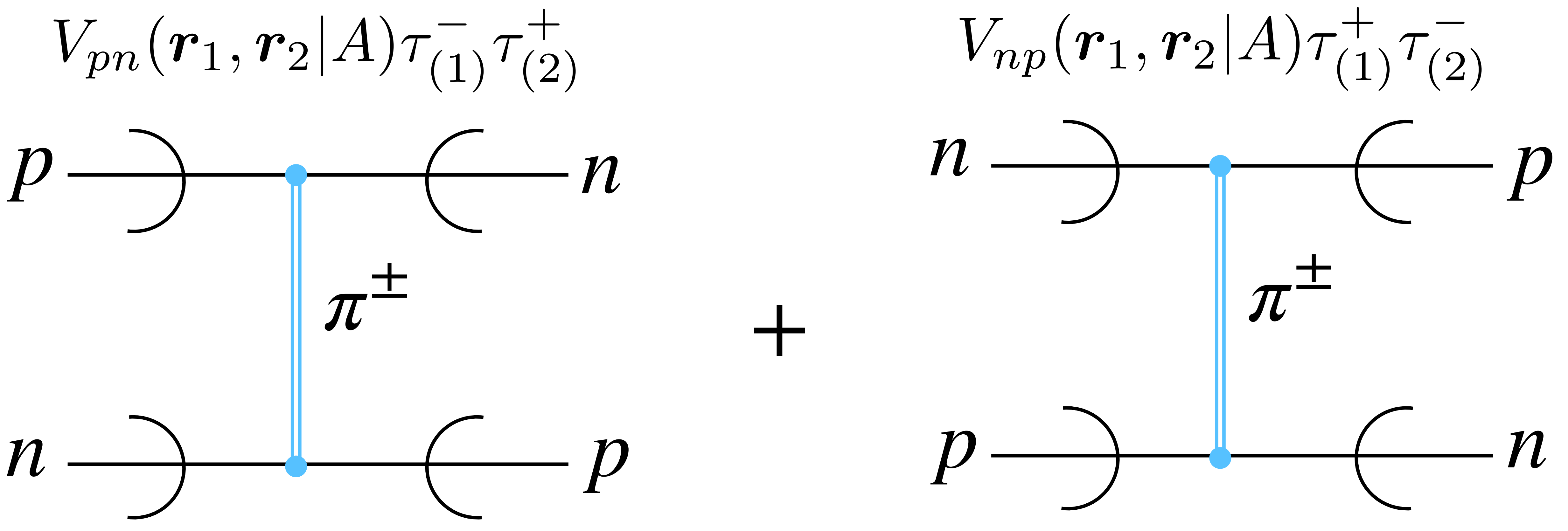
Acting on the state and getting OPEP

- Isospin-singlet $T=0 \longleftrightarrow \dfrac{|pn\rangle - |np\rangle}{\sqrt{2}}$

$$\langle T=0|\, H_{\rm int}\, |T=0\rangle = \langle T=0| \left[V_{pn}(\boldsymbol{r}_1,\boldsymbol{r}_2|A)\tau^-_{(1)}\tau^+_{(2)} + V_{np}(\boldsymbol{r}_1,\boldsymbol{r}_2|A)\tau^+_{(1)}\tau^-_{(2)} \right] |T=0\rangle$$

$$\equiv \hat{V}^{B\neq 0}_{\rm OPE}(\boldsymbol{r}_1,\boldsymbol{r}_2)$$

$$V_{pn}(\boldsymbol{r}_1,\boldsymbol{r}_2|A) = -\frac{g_A^2}{4f_\pi^2}\left[\sigma^i_{(1)}\sigma^j_{(2)} D^-_i D'^+_j \mathrm{i}\Delta^-(\boldsymbol{r},\boldsymbol{r}'|A)\Big|_{\boldsymbol{r}=\boldsymbol{r}_1,\boldsymbol{r}'=\boldsymbol{r}_2} + \sigma^j_{(1)}\sigma^i_{(2)} D^+_i D'^-_j \mathrm{i}\Delta^+(\boldsymbol{r},\boldsymbol{r}'|A)\Big|_{\boldsymbol{r}=\boldsymbol{r}_2,\boldsymbol{r}'=\boldsymbol{r}_1} \right]$$



✔ ① Charged pion propagator ✔ ② Acting on the state

Discussion about gauge transformability

$$V_{pn}(\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{g_A^2}{4f_\pi^2}\left[\sigma^i_{(1)}\sigma^j_{(2)}D_i^- D_j'^{+}\mathrm{i}\Delta^-(\boldsymbol{r},\boldsymbol{r}'|A)\Big|_{\boldsymbol{r}=\boldsymbol{r}_1,\boldsymbol{r}'=\boldsymbol{r}_2} + \sigma^j_{(1)}\sigma^i_{(2)}D_i^+ D_j'^{-}\mathrm{i}\Delta^+(\boldsymbol{r},\boldsymbol{r}'|A)\Big|_{\boldsymbol{r}=\boldsymbol{r}_2,\boldsymbol{r}'=\boldsymbol{r}_1}\right]$$

Applying gauge transformation

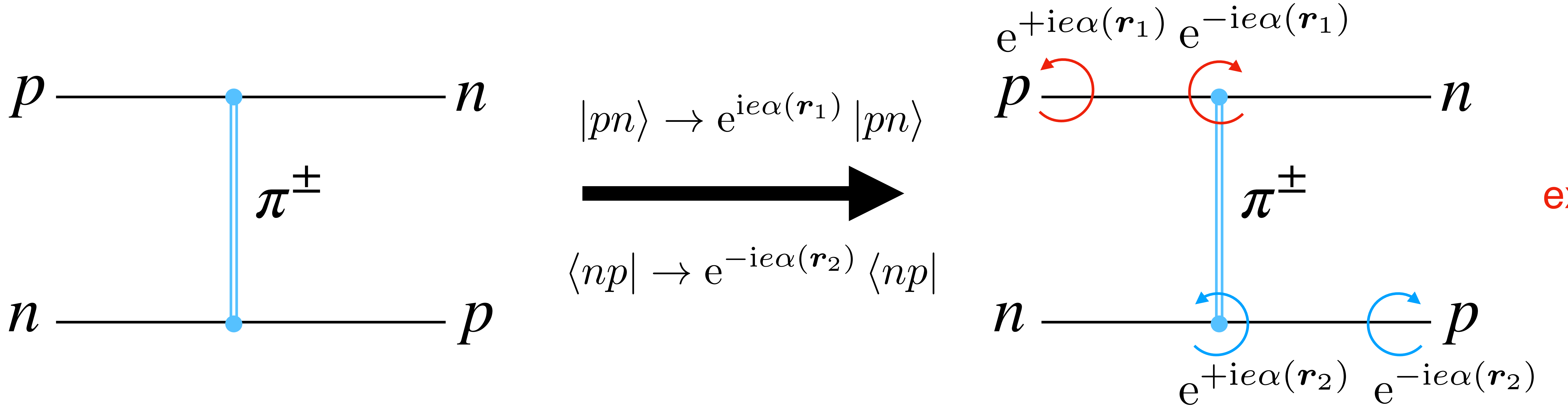
$$\Delta^\pm(\boldsymbol{r},\boldsymbol{r}'|A) \to \Delta^\pm(\boldsymbol{r},\boldsymbol{r}'|A') = \mathrm{e}^{\mp\mathrm{i}e\alpha(\boldsymbol{r})\pm\mathrm{i}e\alpha(\boldsymbol{r}')}\Delta^\pm(\boldsymbol{r}-\boldsymbol{r}'|A)$$

$$D_i^\pm \to \mathrm{e}^{\mp\mathrm{i}e\alpha(\boldsymbol{r})}D_i^\pm\mathrm{e}^{\pm\mathrm{i}e\alpha(\boldsymbol{r})}$$

$$V_{pn}(\boldsymbol{r}_1,\boldsymbol{r}_2|A') = \mathrm{e}^{-\mathrm{i}e\alpha(\boldsymbol{r}_1)+\mathrm{i}e\alpha(\boldsymbol{r}_2)}V_{pn}(\boldsymbol{r}_1,\boldsymbol{r}_2|A)$$

OPEP is not gauge invariant

$$\langle T=0|\, V_{pn}(\boldsymbol{r}_1,\boldsymbol{r}_2|A)\tau^-_{(1)}\tau^+_{(2)}\,|T=0\rangle \qquad \langle T=0|\, V_{pn}(\boldsymbol{r}_1,\boldsymbol{r}_2|A)\tau^-_{(1)}\tau^+_{(2)}\,|T=0\rangle$$

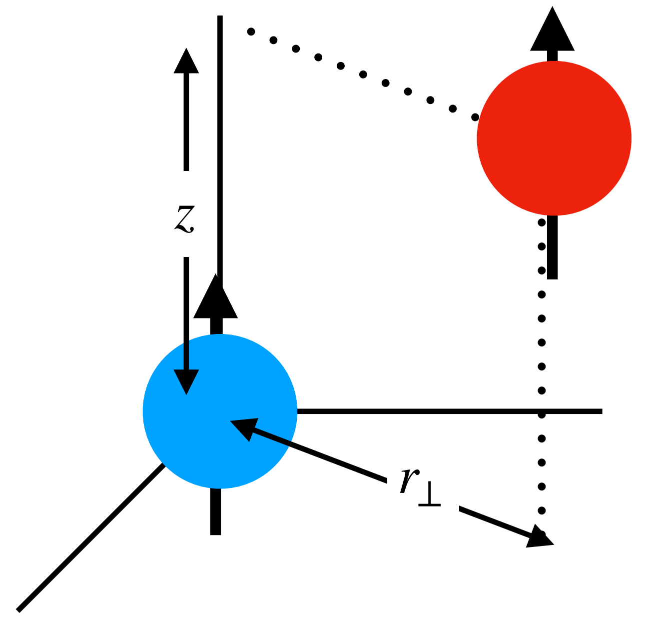


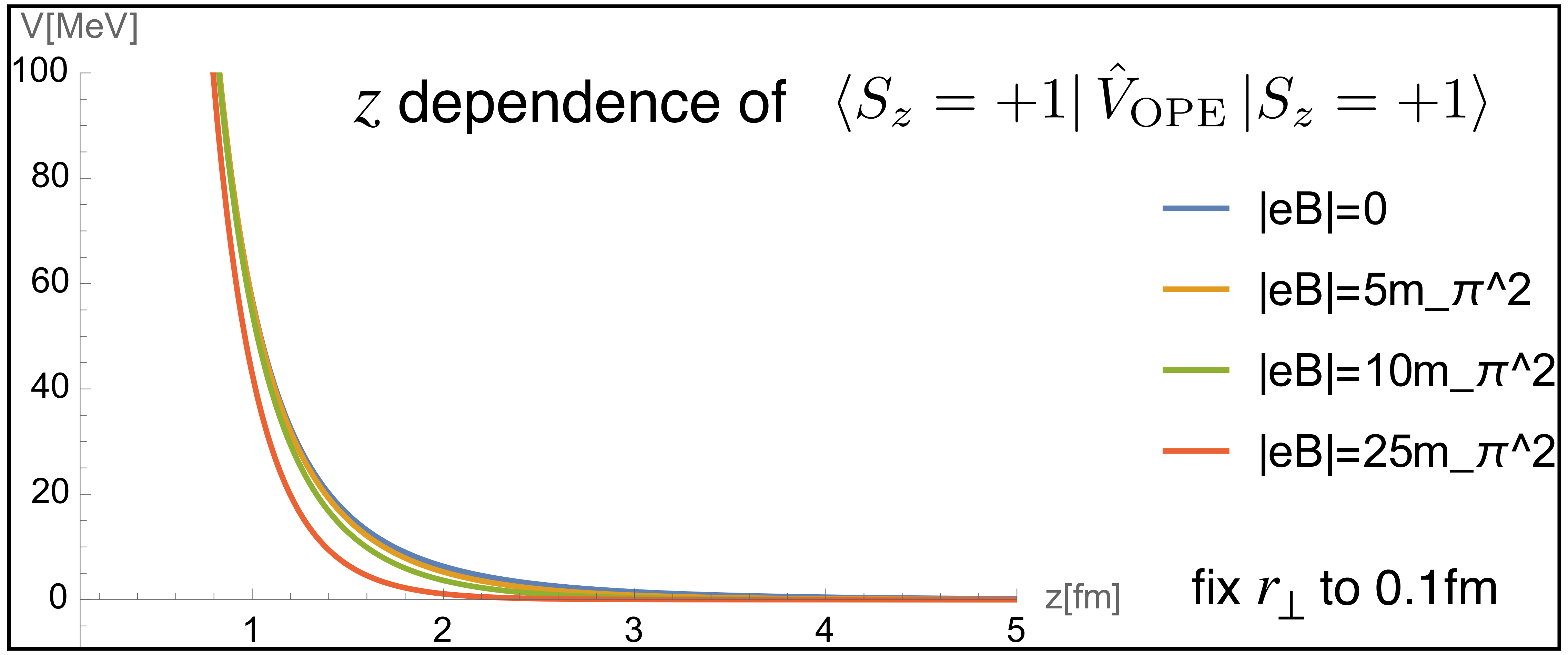
expectation value is gauge invariant

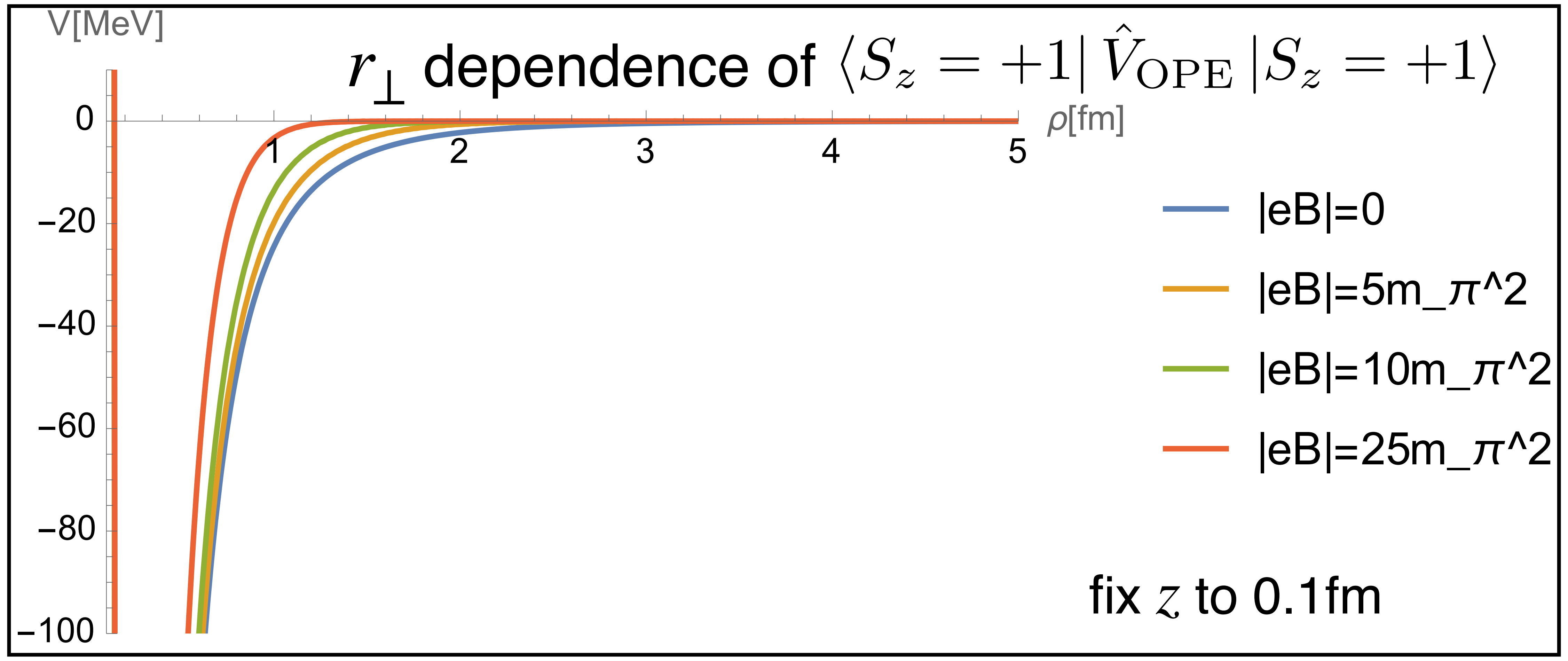
Check behavior of OPEP for deuteron **(Preliminary)**

- $B = 0$ $\langle S_z = +1 | \hat{V}_{\rm OPE}^{B=0} | S_z = +1 \rangle = \frac{g_A^2}{16\pi f_\pi^2} \left[\left(\frac{1}{r^2} + \frac{m_\pi}{r} + \frac{m_\pi^2}{3} \right) \left(\frac{3z^2}{r^2} - 1 \right) + \frac{m_\pi^2}{3} \right] \frac{\mathrm{e}^{-m_\pi r}}{r}$

- $B \neq 0$ $\langle S_z = +1 | \hat{V}_{\rm OPE}^{B\neq 0} | S_z = +1 \rangle = \frac{g_A^2 |eB|}{16\pi f_\pi^2} \mathrm{e}^{-\frac{|eB|}{4}\boldsymbol{r}_\perp^2} \sum_{n=0}^{\infty} \sqrt{m_\pi^2 + (2n+1)|eB|} L_n \left(\frac{|eB|}{2} \boldsymbol{r}_\perp^2 \right) \mathrm{e}^{-\sqrt{m_\pi^2 + (2n+1)|eB|}|z|}$







- These results show that OPEP in a strong magnetic field changes slightly

magnitude of magnetic field is $eB = 5m_\pi^2 \leftrightarrow 10^{19}$G $\gg$ Magnetar surface magnetic field 10^{15}G

Deuteron energy shift

Deuteron energy shift (Preliminary)

deuteron eigenvalue equation for $B=0$: $\left(\hat{V}_{\rm Heavy}+\hat{V}_{\rm OPE}\right)|d_M\rangle=\varepsilon\,|d_M\rangle$ $\quad M=0,\pm1 \quad \varepsilon<\varepsilon_{\rm Bind}=-2.24{\rm MeV}$

Then we put deuteron into a magnetic field

deuteron eigenvalue equation for $B\neq0$: $\left(\hat{V}^{B\neq0}_{\rm Heavy}+\hat{V}^{B\neq0}_{\rm OPE}\right)|\psi\rangle=E\,|\psi\rangle$

evaluate E and examine the deuteron tends to get bound or unbound in a strong magnetic field

$$\hat{V}^{B\neq0}_{\rm Heavy}\simeq\hat{V}_{\rm Heavy}$$

$$\longrightarrow\quad \left(\hat{V}_{\rm Heavy}+\hat{V}_{\rm OPE}+\hat{V}^{B\neq0}_{\rm OPE}-\hat{V}_{\rm OPE}\right)|\psi\rangle=E\,|\psi\rangle$$

Deuteron energy shift (Preliminary)

deuteron eigenvalue equation for $B = 0$: $\left(\hat{V}_{\mathrm{Heavy}} + \hat{V}_{\mathrm{OPE}}\right)|d_M\rangle = \varepsilon\,|d_M\rangle$ $\quad M = 0, \pm 1 \quad \varepsilon < \varepsilon_{\mathrm{Bind}} = -2.24\mathrm{MeV}$

Then we put deuteron into a magnetic field

deuteron eigenvalue equation for $B \neq 0$: $\left(\hat{V}^{B\neq 0}_{\mathrm{Heavy}} + \hat{V}^{B\neq 0}_{\mathrm{OPE}}\right)|\psi\rangle = E\,|\psi\rangle$

evaluate E and examine the deuteron tends to get bound or unbound in a strong magnetic field

$$\hat{V}^{B\neq 0}_{\mathrm{Heavy}} \simeq \hat{V}_{\mathrm{Heavy}}$$

$$\longrightarrow \left(\hat{V}_{\mathrm{Heavy}} + \hat{V}_{\mathrm{OPE}} + \hat{V}^{B\neq 0}_{\mathrm{OPE}} - \hat{V}_{\mathrm{OPE}}\right)|\psi\rangle = E\,|\psi\rangle$$

non-perturbative Hamiltonian — perturbative Hamiltonian

Deuteron energy shift (Preliminary)

Perturbation theory with degeneracies

Energy shift ΔE from the first order of perturbation is given by

$$A\boldsymbol{a} = \Delta E \boldsymbol{a}$$

A is the matrix and matrix elements given by $A_{MM'} = \langle d_M | \hat{V}_{\mathrm{OPE}}^{B\neq 0} - \hat{V}_{\mathrm{OPE}} | d_{M'} \rangle$

Eigenvector $\mathbf{a} = (a_1, a_2, a_3)^t$ tells how to mix eigenstates

$$|\psi\rangle = a_1 |d_{M=1}\rangle + a_2 |d_{M=-1}\rangle + a_3 |d_{M=0}\rangle$$

We calculate this matrix elements and eigenvalue numerically.

Deuteron energy shift **(Preliminary)**

- $\frac{|eB|}{m_\pi^2} = 5 \quad (B \sim 10^{19}\mathrm{G})$

$$\Delta E_1 = 1.32\mathrm{MeV}$$
$$\Delta E_2 = 1.15\mathrm{MeV}$$
$$\Delta E_3 = -0.0142\mathrm{MeV}$$

$$|\psi_{\varepsilon+\Delta E_1}\rangle = 0.707\,|d_{M=+1}\rangle - 0.707\,|d_{M=-1}\rangle$$

$$|\psi_{\varepsilon+\Delta E_2}\rangle = 0.707\,|d_{M=+1}\rangle + 0.707\,|d_{M=-1}\rangle$$

$$|\psi_{\varepsilon+\Delta E_3}\rangle = |d_{M=0}\rangle$$

- Spin un and down states may tend to get unbound in heavy baryon limit

Summary

- We derived the one-pion exchange potential in a strong magnetic field with ChPT

  Effects of a external magnetic field：charged pion propagator and covariant derivative coupling

- There are not much difference between the $\hat{V}_{\mathrm{OPE}}^{B\neq 0}$ and $\hat{V}_{\mathrm{OPE}}$

- We examined deuteron’s energy shift in a magnetic field with perturbation theory

- The energy shift and mixing state are caused by the perturbative Hamiltonian