# Quantum Hall droplets in high-density QCD

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arXiv:2410.07665

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Symmetry and Effective Field Theory of Quantum Matter

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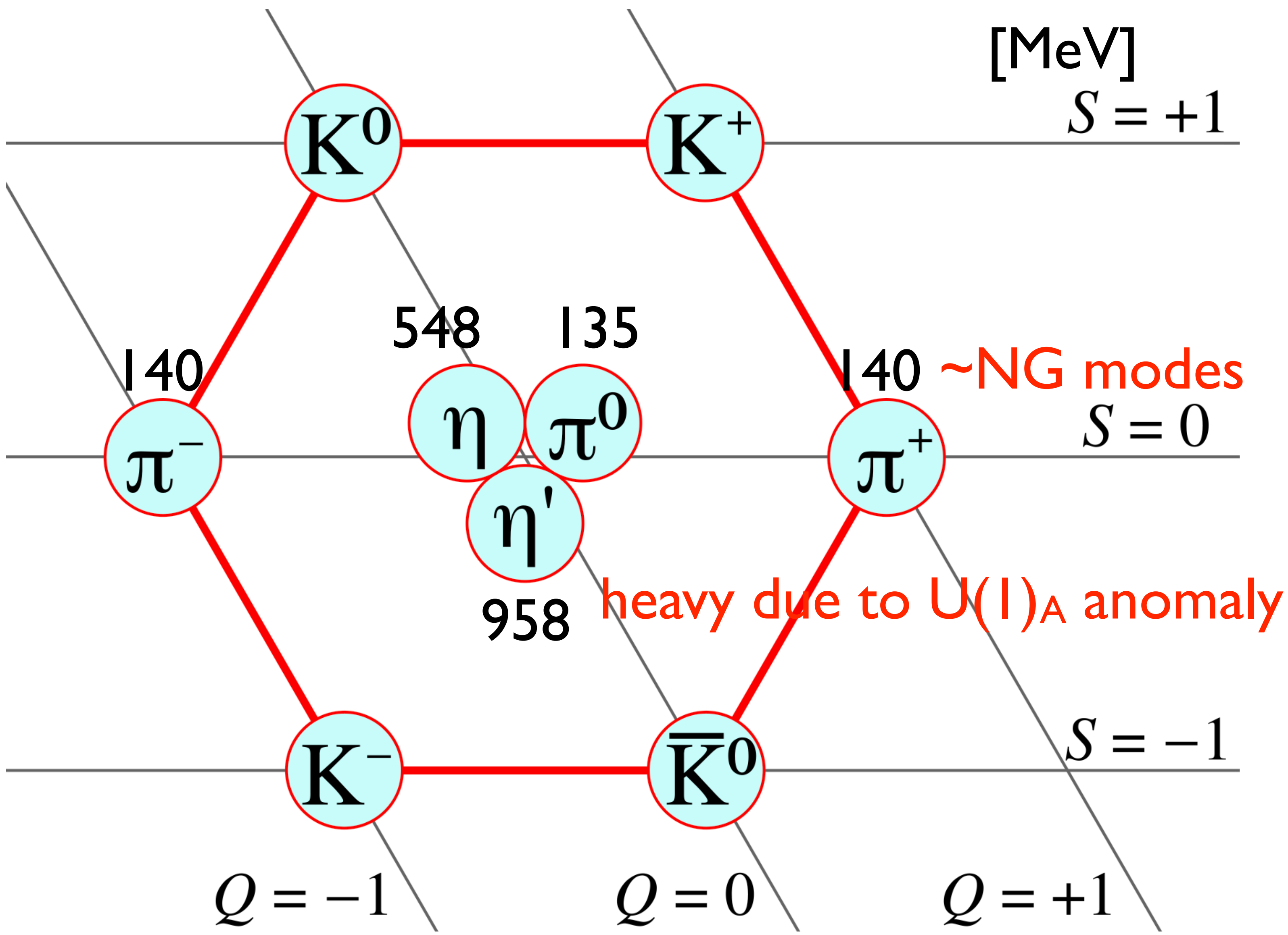
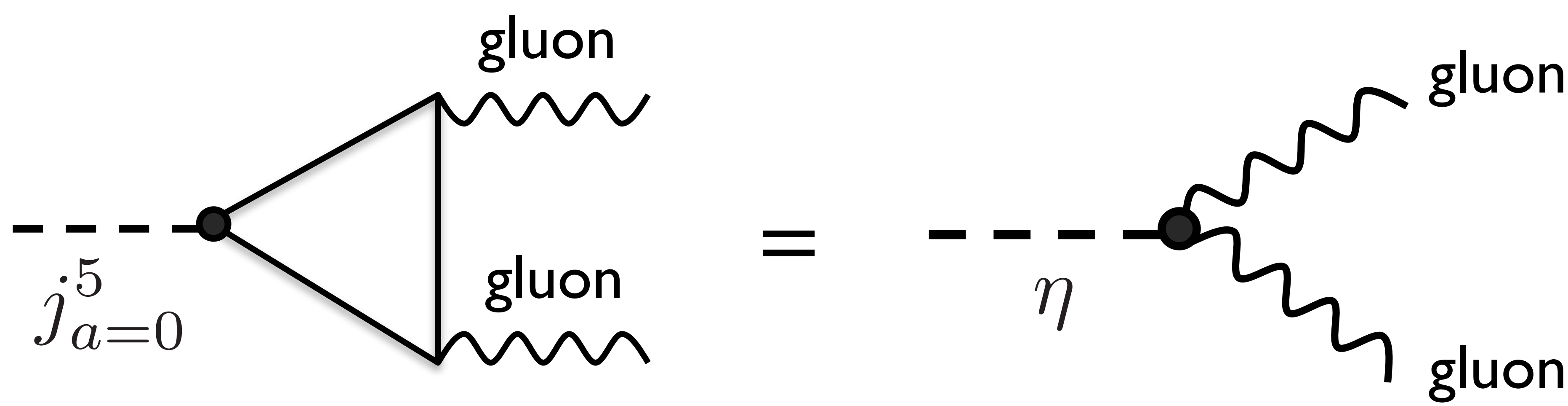
# Mesons ($\bar{q}q$)




Spin 0 pseudoscalar mesons

# Axial anomaly

- Violation of $\mathrm{U(1)_A}$ symmetry ($q_{\mathrm{R}} \to \mathrm{e}^{\mathrm{i}\theta}$, $q_{\mathrm{L}} \to \mathrm{e}^{-\mathrm{i}\theta}$) by quantum effects
- Axial anomaly is nonperturbatively exact




UV (quark)

IR (hadron)

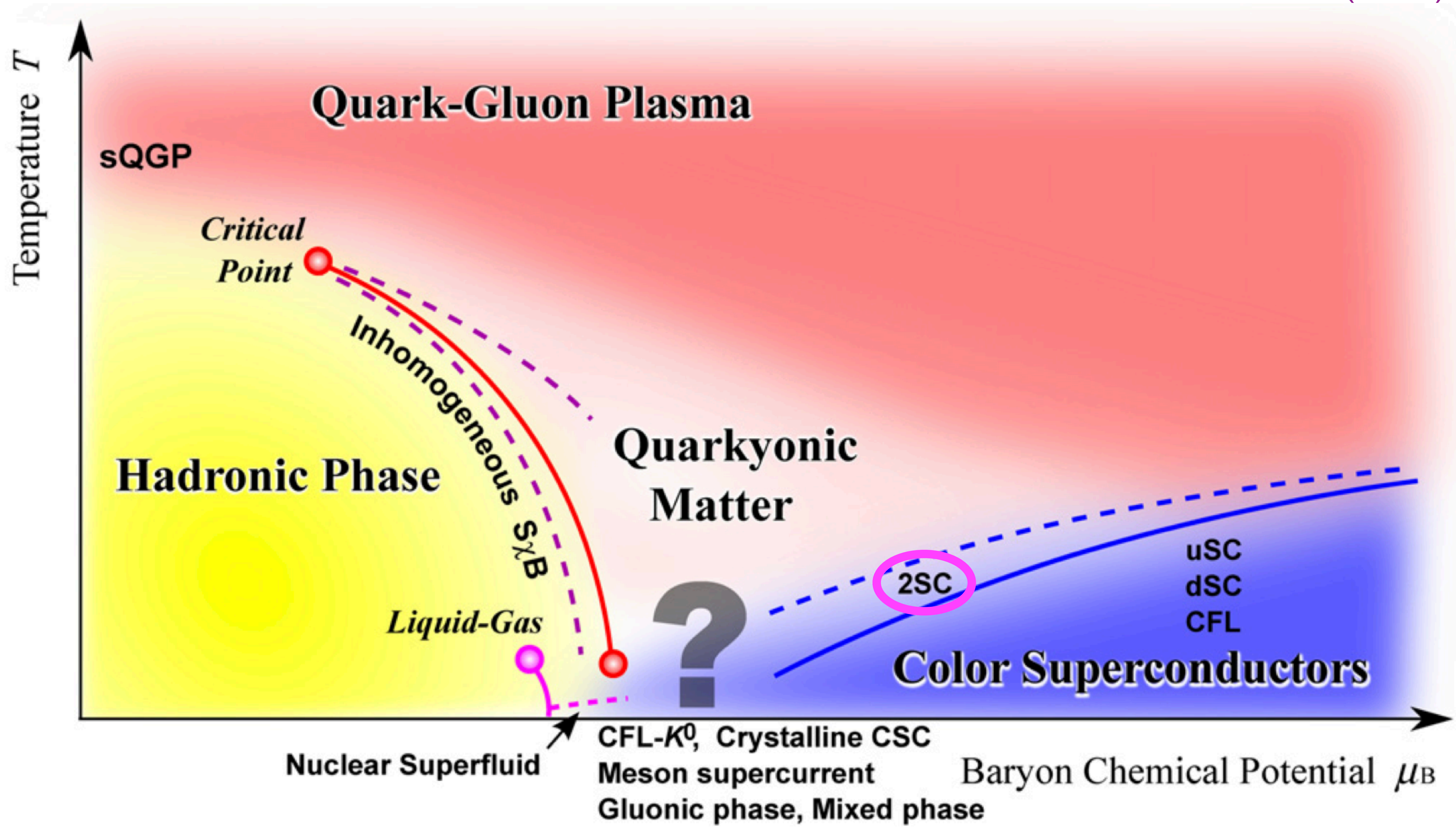
$$\partial_\mu j^{\mu 5} = -\frac{1}{4\pi^2}\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$$

$$S_{\mathrm{anom}} = -\frac{1}{8\pi^2}\int \mathrm{d}^4x \; \eta \; \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$$

$\eta \sim \bar{q}q$ has twice axial charge of a quark

# QCD phase diagram

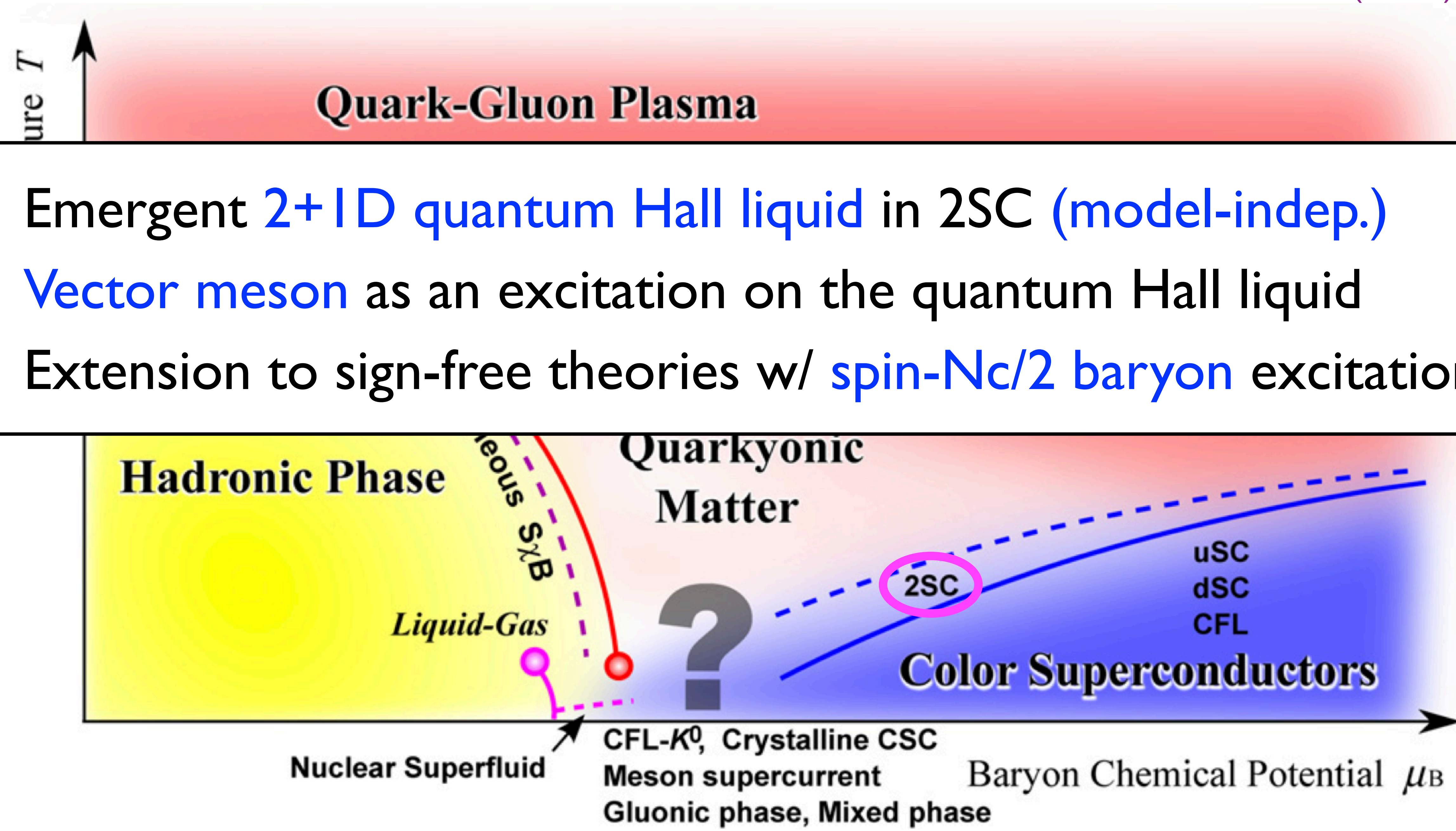
Fukushima-Hatsuda, PPNP (2010)



# QCD phase diagram

Fukushima-Hatsuda, PPNP (2010)

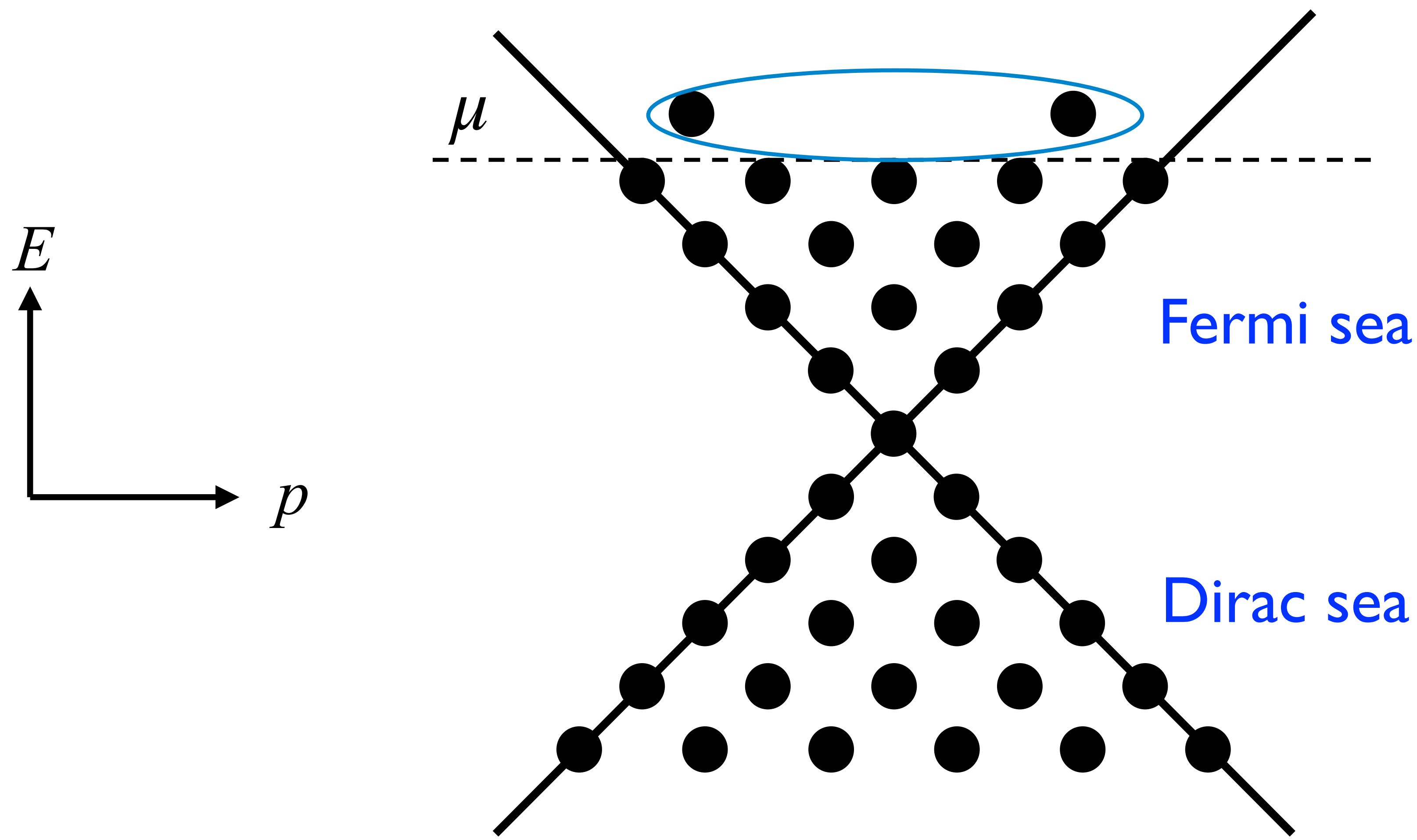
- Emergent 2+1D quantum Hall liquid in 2SC (model-indep.)
- Vector meson as an excitation on the quantum Hall liquid
- Extension to sign-free theories w/ spin-Nc/2 baryon excitation



# Quark matter

- Nuclear matter $\xrightarrow{\text{high density}}$ Quark matter $\xrightarrow{\text{high-density limit}}$ Almost free quark gas

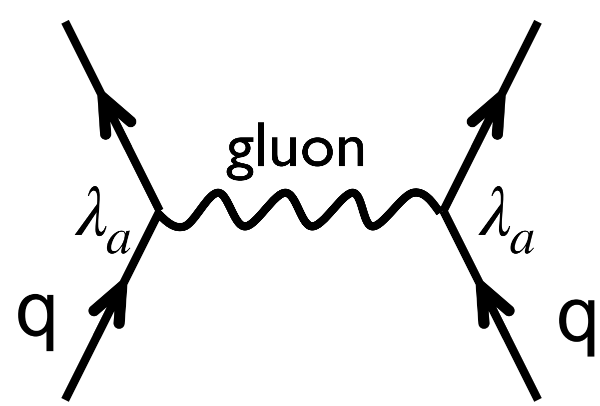
Collins-Perry, PRL (1975)



# Color superconductivity

- High-density limit: one-gluon exchange int. (asymptotic freedom)
- Coulomb interaction can be attractive due to color d.o.f.

cf. quarks are bound to form a nucleon in the vacuum




$$(\lambda_a)_{ij}(\lambda_a)_{kl} = \frac{2}{3}(\lambda_{\rm S})_{ik}(\lambda_{\rm S})_{lj} \underline{- \frac{4}{3}}(\lambda_{\rm A})_{ik}(\lambda_{\rm A})_{lj}$$

cf. metallic superconductivity: Coulomb repulsion between electrons

- Bare interaction leads to quark-quark pairing (BCS mechanism)

# Diquark condensate

- QCD has various quantum numbers:

$$\langle q_{ia}^{\alpha} q_{jb}^{\beta} \rangle = \Delta_P \, P_{ij\,ab}^{\alpha\beta}$$

color a, b = R, G, B
flavor i, j = u, d, …
spin α, β = ↑, ↓

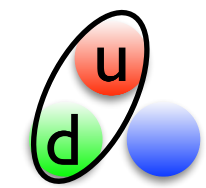
- Color antisymmetric (attractive interaction)
- Spin antisymmetric (isotropy is energetically favorable)

→ Flavor antisymmetric (Pauli principle)

# 2-flavor color superconductivity

color flavor

- 2-flavor limit ($m_\mathrm{u} = m_\mathrm{d} = 0$): $\langle (q_\mathrm{R})_i^a (q_\mathrm{R})_j^b \rangle = \Phi_\mathrm{R} \epsilon^{ab3} \epsilon_{ij}$




Alford-Rajagopal-Wilczek, NPB (1998)

- U(1)$_\mathrm{A}$ is an approximate symmetry due to the screening of instantons

Shuryak, NPB (1982)

- "Symmetry breaking": SU(3)$_\mathrm{C}$ × U(1)$_\mathrm{B}$ × U(1)$_\mathrm{A}$ → SU(2)$_\mathrm{C}$ × U(1)$_{\tilde{\mathrm{B}}}$

$$\tilde{B} = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Baryon number is carried only by unpaired (blue) quarks

# Low-energy excitations in 2SC

- η is a pseudo $U(1)_A$ NG mode ($\bar{q}\bar{q}qq$ state) at high density

  $\Sigma \equiv \Phi_R^\dagger \Phi_L, \quad \Sigma = |\Sigma| e^{-i\eta}$ N.B. η is a $\bar{q}q$ state in the vacuum

- Low-energy d.o.f.s: unpaired quarks, confined SU(2) gluons & η
- SU(2) gluon well below Δ: SU(2) pure Yang-Mills with emergent $\Lambda'_{\mathrm{QCD}}$

  Son-Stephanov-Rischke, PRL (2001)

- Hierarchy at sufficiently high density: $\mu \gg \Delta \gg m_\eta \gg \Lambda'_{\mathrm{QCD}}$

N.B. η does not couple to unpaired quarks at leading order

# Effective theory for η

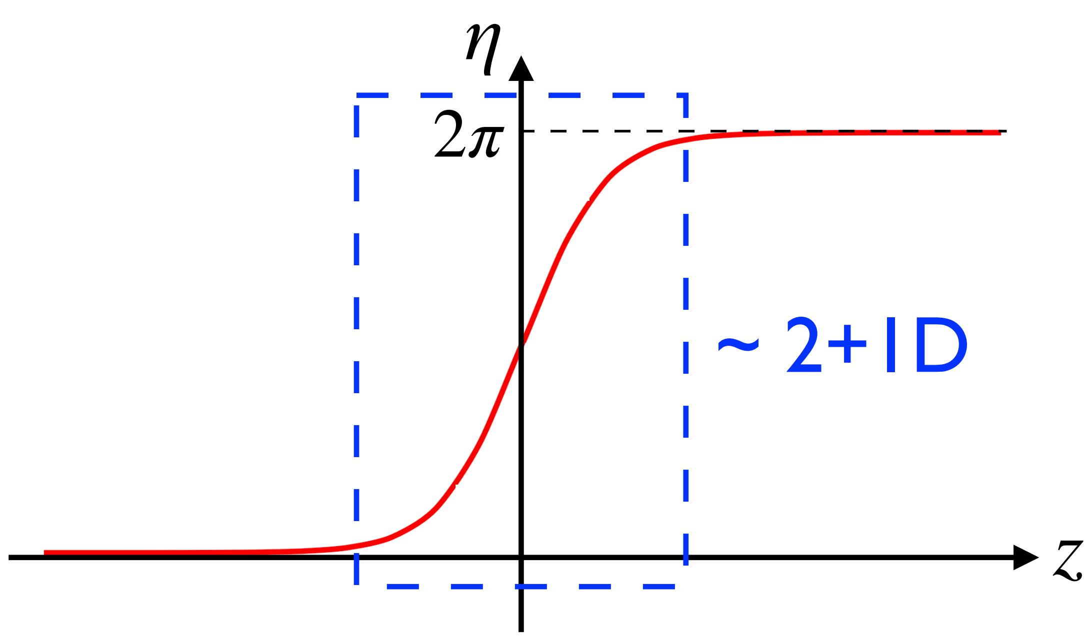
Son-Stephanov-Zhitnitsky, PRL (2001)

- Approximate $U(1)_A$ symmetry: $\eta \to \eta + \text{const}\,.\ (\eta + 2\pi \sim \eta)$

$$\mathcal{L} = f^2[(\partial_t \eta)^2 - v^2(\boldsymbol{\nabla}\eta)^2] + A\cos\eta$$

($f, v, A$ computable by weak-coupling analysis at high density)

- Domain-wall solution from $\eta = 0$ $(z = -\infty)$ to $\eta = 2\pi$ $(z = \infty)$:




Topological charge:

$$Q_{\mathrm{DW}} := \frac{1}{2\pi}\int_{-\infty}^{\infty} \partial_z \eta = 1$$

# Domain Walls of High-Density QCD

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(Received 17 December 2000)

We show that in very dense quark matter there must exist metastable domain walls where the axial U(1) phase of the color-superconducting condensate changes by $2\pi$. The decay rate of the domain walls is exponentially suppressed and we compute it semiclassically. We give an estimate of the critical chemical potential above which our analysis is under theoretical control.

*Discussion.*—It would be interesting to investigate possible astrophysical consequences of the high-density QCD walls. In particular, one would like to know if such walls can be created inside neutron stars. To describe the motion of the wall, one may need more than just the effective Lagrangian (8): the coupling of $\eta$ to ungapped quarks and $SU(2)_c$ gluons could be important. The moving wall may

η-gluon coupling was ignored without any justification: low-energy EFT is incomplete

# Quantum Hall droplet in 2SC

# Topological field theory on the wall

Nishimura, NY, Yokokura, arXiv:2410.07665

- η also couples to confined SU(2) gluons via axial anomaly:

$$S_{\rm anom} = -\frac{1}{16\pi^2}\int \mathrm{d}^4x\ \eta\ \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \qquad \boxed{F_{\mu\nu}\tilde{F}^{\mu\nu} = 2\epsilon^{\mu\nu\rho\sigma}\partial_\mu(A_\nu\partial_\rho A_\sigma + \cdots)}$$

$$\to -\frac{1}{8\pi^2}\underbrace{\int_{-\infty}^{\infty}\mathrm{d}z\partial_z\eta}_{2\pi \text{ for DW}}\int \mathrm{d}^3x\epsilon^{\mu\nu\rho}\mathrm{tr}(A_\mu\partial_\nu A_\rho + \cdots) =: S_{\rm CS}[A]$$

- SU(2)-1 Chern-Simons (CS) theory on the wall:

$$S_{\rm CS}[A] = -\frac{1}{4\pi}\int \mathrm{d}^3x\ \epsilon^{\mu\nu\rho}\mathrm{tr}\left(A_\mu\partial_\nu A_\rho - \frac{2\mathrm{i}}{3}A_\mu A_\nu A_\rho\right)$$

SU(2) gauge field

⇄ U(1)2 CS theory (level-rank duality):

$$S_{\rm CS}[a] = \frac{2}{4\pi}\int \mathrm{d}^3x\ \epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho$$

U(1) gauge field

# Physics at the edge

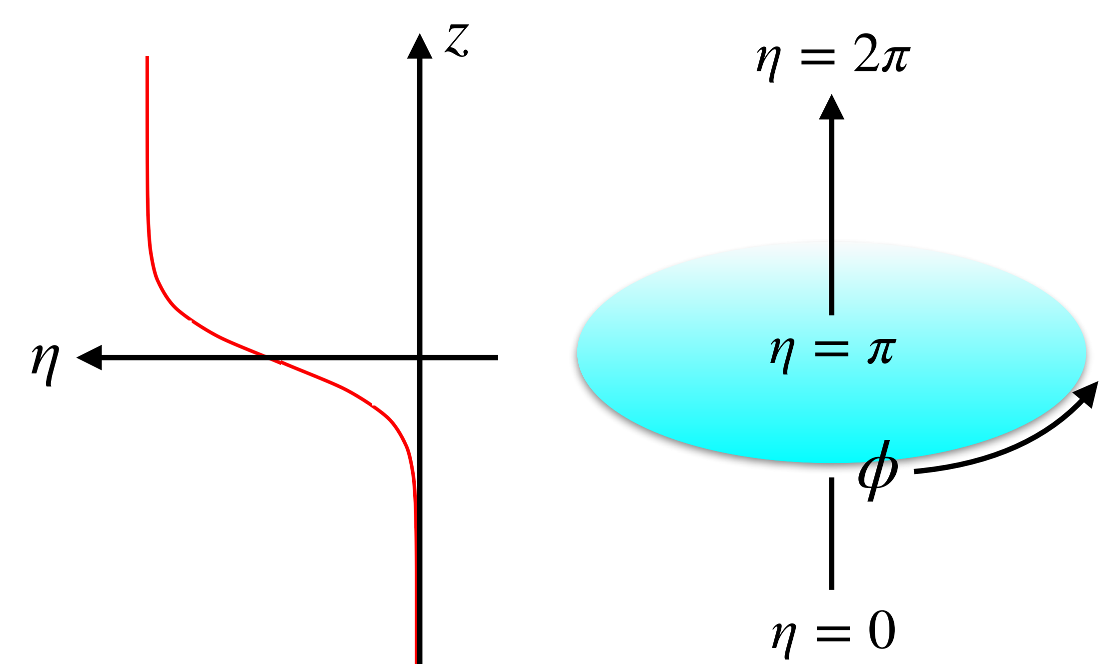
see, e.g., X. G. Wen, "Quantum Field Theory of Many-Body Systems"

- Gauge fixing $a_t + \omega a_\theta = 0$ in $(r, \theta)$ coordinates
- $\tilde{\theta} := \theta - \omega t,\ \tilde{t} := t,\ \tilde{r} := r \rightarrow \tilde{a}_{\tilde{t}} := a_t + \omega a_\theta = 0,\ \tilde{a}_{\tilde{\theta}} = a_\theta,\ \tilde{a}_{\tilde{r}} = a_r$

$$S_{\mathrm{CS}}[a] = \frac{2}{4\pi} \int \mathrm{d}^3 x \epsilon^{\tilde{\mu}\tilde{\nu}\tilde{\rho}} \tilde{a}_{\tilde{\mu}} \partial_{\tilde{\nu}} \tilde{a}_{\tilde{\rho}}$$

- EOM for $\tilde{a}_{\tilde{t}}$: $\tilde{f}_{\tilde{\theta}\tilde{r}} = 0 \rightarrow \tilde{a}_{\tilde{i}} = \partial_{\tilde{i}} \phi\ (\tilde{i} = \tilde{\theta}, \tilde{r})$ as a constraint
- Edge theory = 1+1D CFT (chiral Tomonaga-Luttinger liquid):

$$\begin{aligned} S_{\mathrm{edge}} &= \frac{2}{4\pi} \int \mathrm{d}\tilde{t}\mathrm{d}\tilde{\theta} \partial_{\tilde{t}} \phi \partial_{\tilde{\theta}} \phi \\ &= \frac{2}{4\pi} \int \mathrm{d}t\mathrm{d}\theta (\partial_t + \omega \partial_\theta) \phi \partial_\theta \phi \end{aligned}$$



# 2D CFT analysis

- For generic $O$ under $z \to w(z)$ in complex plane $z = x_1 + \mathrm{i}x_2$,

$$O'(w,\bar{w}) = \left(\frac{\mathrm{d}z}{\mathrm{d}w}\right)^{h} \left(\frac{\mathrm{d}\bar{z}}{\mathrm{d}\bar{w}}\right)^{\bar{h}} O(z,\bar{z})$$

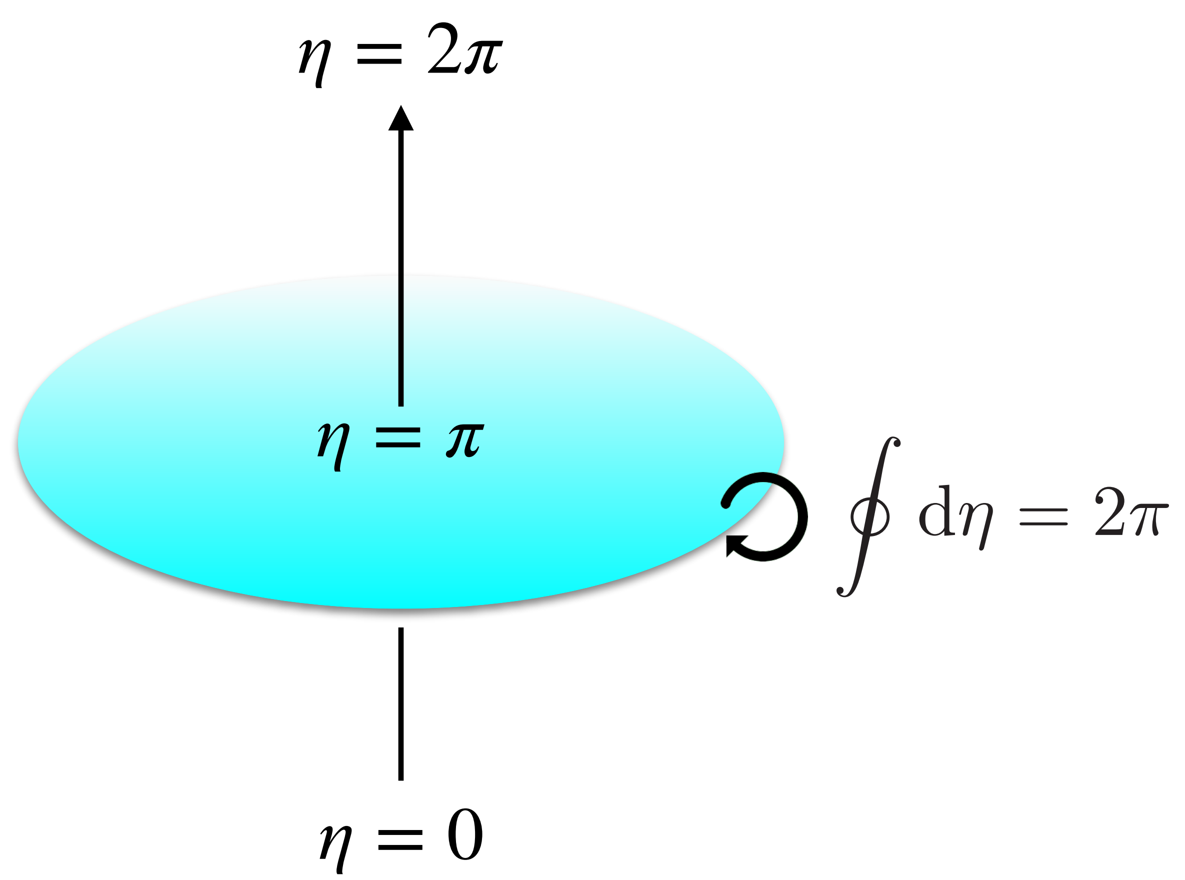
$$\longrightarrow O'(\lambda z, \lambda \bar{z}) = \lambda^{-\underbrace{(h+\bar{h})}_{\text{scaling}}} O(z,\bar{z}), \quad O'(\mathrm{e}^{\mathrm{i}\theta} z, \mathrm{e}^{-\mathrm{i}\theta}\bar{z}) = \mathrm{e}^{-\mathrm{i}\underbrace{(h-\bar{h})}_{\text{spin}}\theta} O(z,\bar{z})$$

- Two-point function for the vertex operator $V_N = \mathrm{e}^{\mathrm{i}N\phi}$ ($N = 2$):

$$\langle V_N(z) V_M(0)\rangle \sim \mathrm{e}^{-NM\langle\phi(z)\phi(0)\rangle} \sim \frac{\delta_{N+M,0}}{z^N} \qquad \because \langle\phi(z)\phi(0)\rangle = -\tfrac{\#}{N}\ln|z|$$

- Scaling dimension & spin $N/2 = 1$, energy ~ $C/R$

# Quantum Hall droplet

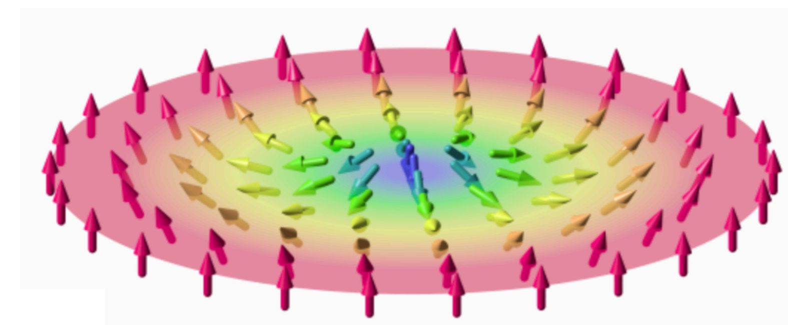



- Total energy of the system: $E(R) = \pi R^2 T_{\mathrm{DW}} + 2\pi R T_{\mathrm{string}} + \dfrac{C}{2\pi R}$
- Droplet stabilized by the edge mode (*S*=1, *B*=0): vector meson

# Historical remarks

- Skyrmion: Multi-flavor baryon in the QCD vacuum
  Skyrme (1961)

  nuclear physics → condensed matter physics



- Quantum Hall droplet: spin N/2 baryon in the large-N vacuum
  Komargodski, arXiv:1812.09253

  condensed matter physics → particle physics

Our finding: quantum Hall droplet as a vector meson in 2SC

# Other theories

- 2-flavor QCD at finite isospin density:
  - $SU(3)_{-2}$ CS theory $\rightleftarrows$ $U(2)_3$ CS theory on the wall
  - Droplet = favored spin 3/2 baryon
- 2-color QCD at finite baryon density:
  - $SU(2)_{-2}$ CS theory $\rightleftarrows$ $U(2)_2$ CS theory on the wall
  - Droplet = favored spin 1 baryon

# Conclusion & Outlook

- Topological field theory emerges in 2SC phase
- Quantum Hall droplet as a vector meson
- Droplets as spinning baryons in other dense QCD-like theories
- Other resonant states in high-density QCD?
- Phenomenological consequences?