Generalized chiral instabilities, linking numbers, and non-invertible symmetries

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Symmetry and Effective Field Theory of Quantum Matter

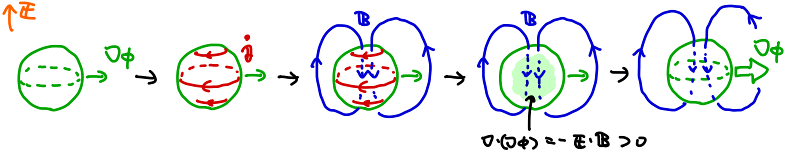
@ Niigata University

Based on N. Yamamoto & RY, JHEP **07** (2023) 045 [2305.01234]

Message

Non-invertible symmetries can be applicable to dynamics.

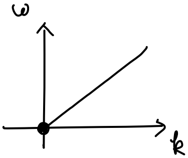
Overview



- Axion electrodynamics in $(3+1)$ dimensions exhibits instability
  in the presence of background time dependent axion $\partial_t \phi$ or electric field.
- Generalized chiral instabilities: universal mechanism of these instabilities
  - Instabilities tend to be weakened.
  - $\boldsymbol{B}$ & $\nabla\phi$ with linking number are generated.
  - Stability of generated fields can be stable due to non-invertible symmetries.

Contents

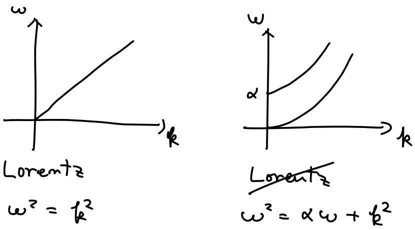
Gapless modes = modes without energy (mass) gap



- Dispersion relation: $\omega = 0$ for $\boldsymbol{k} = \mathbf{0}$.
- Long wave excitation by infinitesimal energy $\rightarrow$ Dominating infrared (IR) physics
- Characterizing phase of matter: gapless phase
- Ubiquitous in physics: photon, phonon, Nambu-Goldstone bosons

The Lorentz symmetry is important for gapless modes.

Gapless modes and Lorentz symmetry



With Lorentz symmetry:

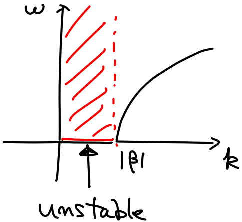
- Linear dispersion $\omega^2 = k^2$ (I neglect higher order terms in this talk)

Without Lorentz symmetry (e.g., explicit breaking by background fields)

$\rightarrow$ possibility of corrections in IR

- 1st order of ω: $\omega^2 = \alpha\omega + k^2 \rightarrow$ gapped mode $\omega = \alpha + \frac{1}{\alpha}k^2$
- 1st order of k: $\omega^2 = \beta k + k^2 \rightarrow$ unstable mode

Unstable mode



- Dispersion relation $\omega = \sqrt{k^2 + \beta k}$
- For $\beta < 0$, there is instability $\omega = i\sqrt{|\beta k| - k^2}$ in finite IR region $0 < |k| < |\beta|$

(Tachyonic mode $e^{-i\omega t + ikx} \propto e^{\sqrt{|\beta k| - k^2}\, t}$)

Such an instability arises in realistic systems!

Axion electrodynamics = axion ϕ + photon a_μ + topological coupling [Wilczek '87]

Action (massless axion & photon)

$$S = -\int d^4x \left(\frac{v^2}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{16\pi^2}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \right)$$

v: decay constant, e: coupling constant (I sometimes omit them)

- Axion ϕ: pseudo-scalar field, photon A_μ: $U(1)$ gauge field with Dirac quantization condition

Features

1. Simple and ubiquitous in modern physics

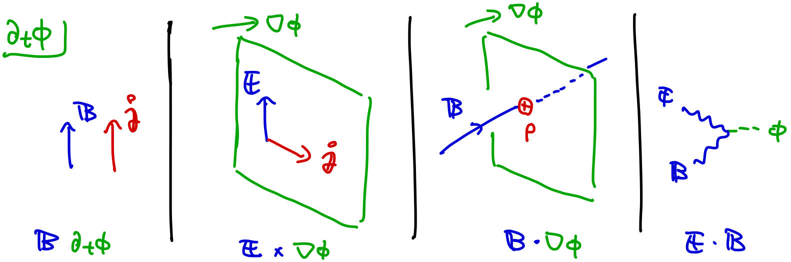
   QCD axion, inflaton, moduli from string theory, π^0 meson, quasi-particle excitation,...

2. Cubic topological coupling $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$: determined by chiral anomaly in UV

   Toy model of 10d, 11d supergravities $\sim C_3 \wedge F_4 \wedge F_4$ [Townsend '93; Harvey & Ruchayskiy '00]

Cubic topological coupling $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$ leads to non-trivial effects

Four effects due to topological coupling



- Induced current: $\nabla \times \boldsymbol{B} - \partial_t \boldsymbol{E} = \frac{1}{4\pi^2}(\boldsymbol{B}\partial_t \phi - \boldsymbol{E} \times \nabla\phi)$

  Chiral magnetic effect [Fukushima, et al. '08]; anomalous Hall effect [Sikivie '84]

- Induced charge: $\nabla \cdot \boldsymbol{E} = -\frac{1}{4\pi^2}\boldsymbol{B} \cdot \nabla\phi$ [Sikivie '84]

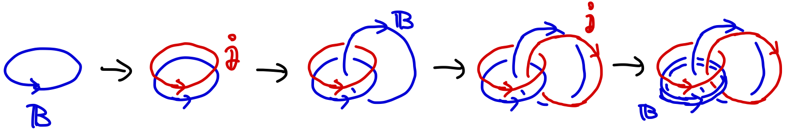
- Photon to axion: $(\partial_t^2 - \nabla^2)\phi = \frac{1}{4\pi^2}\boldsymbol{E} \cdot \boldsymbol{B}$

Background axion velocity $\partial_t \phi = \mathrm{const} \to$ instability of photon

Chiral instability

Review based on Akamatsu & Yamamoto '13 and so on

Chiral instability [Carroll, et al. '89; Joyce & Shaposhnikov '97; Anber & Sorbo '07; Akamatsu & Yamamoto '13]



- Ampère law $\nabla \times \boldsymbol{B} = \frac{1}{4\pi^2} \boldsymbol{B} \partial_t \phi$
- Background $\partial_t \phi \neq 0 \to \boldsymbol{j} \propto \boldsymbol{B}$ amplifies magnetic field

Dispersion relation?

Dispersion relation

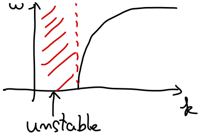
$C = \partial_t \phi$

For $\boldsymbol{k} = (k, 0, 0)$, EOM is

$$(\omega^2 - k^2) \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = iC \begin{pmatrix} 0 & & \\ & & -k \\ & k & \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

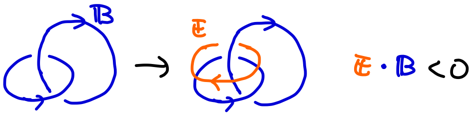
Instability in IR region $k < C$

- Tachyonic mode $\omega = i\sqrt{Ck - k^2}$



Is the instability pathological?

Instability tends to be weakened (linear analysis)

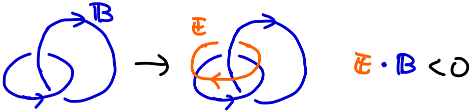


$\partial_t \phi$ decreases (linear analysis)

- Faraday law: $\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$
- EOM of axion: $\partial_t^2 \phi = \frac{1}{4\pi^2} \boldsymbol{E} \cdot \boldsymbol{B} < 0$

Generated magnetic field is stable

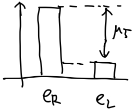
Generation of stable magnetic field

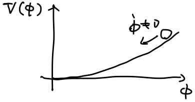


- EOM of axion $\partial_\mu(\partial^\mu \phi + \frac{1}{8\pi^2} A_\nu \tilde{F}^{\mu\nu}) = 0 \quad \rightarrow \quad \int d^3\boldsymbol{x} (\partial_t \phi + \frac{1}{8\pi^2} \boldsymbol{A} \cdot \boldsymbol{B})$ is conserved
- Decrease of $\partial_t \phi \rightarrow$ increase of $\boldsymbol{B}$ with magnetic helicity $\int d^3\boldsymbol{x}\, \boldsymbol{A} \cdot \boldsymbol{B}$
- Stability of $\boldsymbol{B}$ = stability of magnetic helicity
- Applications: generation of magnetic fields in cosmology and neutron stars

  [Joyce & Shaposhnikov '97; Anber & Sorbo '07; Akamatsu & Yamamoto '13]

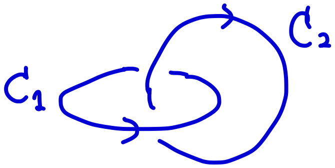
  $\partial_t \phi$: chiral chemical potential or time deriv. of inflaton





Physical meaning of magnetic helicity?

Magnetic helicity = linking number of magnetic flux [Demoulin, et al., '06]



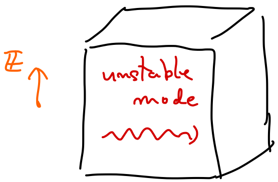
Consider magnetic flux tubes for simplicity.

$$\int d^3\boldsymbol{x}\, \boldsymbol{A} \cdot \boldsymbol{B} = 2\Phi_1\Phi_2\, \mathrm{Link}\,(C_1, C_2)$$

- Φ_1, Φ_2 magnetic flux of flux tubes C_1, C_2
- $\mathrm{Link}\,(C_1, C_2)$: linking number between C_1 & C_2

  Derivation: use Biot-Savart law $\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{4\pi} \int d^3\boldsymbol{x}' \frac{\boldsymbol{B}(\boldsymbol{x}') \times (\boldsymbol{x}-\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|^3}$

Q. How universal is the chiral instability?



Similar instabilities have been found in the context of holography

- Axion ED in background elec. field [Bergman et al., '11; Ooguri & Oshikawa '11](massive axion)
- $(4+1)$ dim. Maxwell-Chern-Simons thy in background elec. field [Nakamura et al., '09]

Electric fields decrease? Magnetic fields with topological quantities increase?

Result [Yamamoto & RY, '23]

- Decrease of bg. elec. fields & increase of mag. fields with topological quantities hold for them.
- Further generalization is possible

Generalized chiral instabilities

- Setup: massless Abelian p-form gauge theories with cubic topological couplings in flat spacetime
- IR instabilities in background elec. fields
- Decrease of bg. elec. fields & increase of mag. fields (linear analysis)
- Mag. fields are protected by non-invertible symmetries

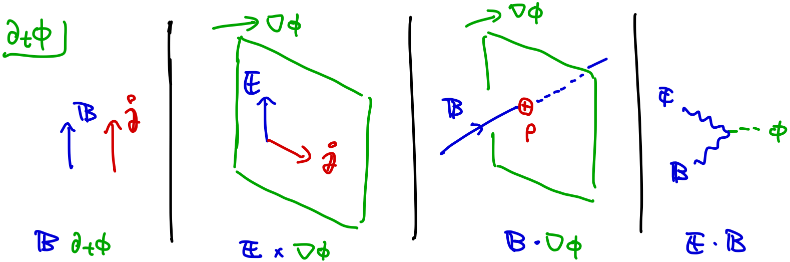
In this talk, I consider axion ED in elec. field for concreteness.

Instability of axion electrodynamics in background electric field

as an example of generalized chiral instabilities

Yamamoto & RY, 2305.01234

Four effects due to topological coupling



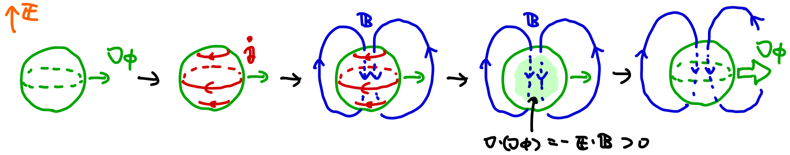
- Induced current: $\nabla \times \boldsymbol{B} - \partial_t \boldsymbol{E} = \frac{1}{4\pi^2}(-\boldsymbol{E} \times \nabla\phi + \boldsymbol{B}\partial_t\phi)$

  Anomalous Hall effect [Sikivie '84]

- Photon to axion: $(\partial_t^2 - \nabla^2)\phi = \frac{1}{4\pi^2 v^2}\boldsymbol{E} \cdot \boldsymbol{B}$

Background $\boldsymbol{E} \to$ instability of $\nabla\phi$ & $\boldsymbol{B}$

Instability of axion ED in bg. elec. field [Yamamoto & RY, '23]



Amplification of $\nabla\phi$ & $\boldsymbol{B}$ due to

- Ampère law $\nabla \times \boldsymbol{B} = -\frac{1}{4\pi^2}\boldsymbol{E} \times \nabla\phi$
- EOM of axion $\nabla^2\phi = -\frac{1}{4\pi^2}\boldsymbol{E} \cdot \boldsymbol{B}$

Dispersion relation?

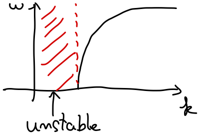
Dispersion relation [Bergman et al., '11; Ooguri & Oshikawa '11]

For $\boldsymbol{k} = (k, 0, 0)$, $\boldsymbol{E} = (0, E, 0)$ EOM is

$$(\omega^2 - k^2) \begin{pmatrix} v\phi \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = i \frac{E}{v} \begin{pmatrix} 0 & & & k \\ & 0 & & \\ & & 0 & \\ -k & & & 0 \end{pmatrix} \begin{pmatrix} v\phi \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

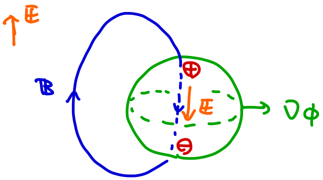
Instability in IR region $k < \frac{E}{v}$

- Tachyonic mode $\omega = i\sqrt{\frac{E}{v}k - k^2}$



Amplification of $\nabla\phi$ & $\boldsymbol{B} \to$ decrease of $\boldsymbol{E}$

Decrease of $\boldsymbol{E}$ [Yamamoto & RY, '23]

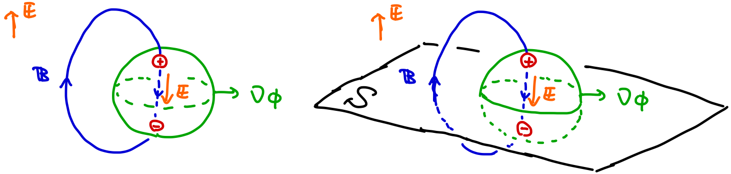


Induced charge screens elec. field

- Elec. Gauss law $\nabla \cdot \boldsymbol{E} = -\frac{1}{4\pi^2}\boldsymbol{B} \cdot \nabla\phi$
- Direction of induced elec. field is opposite to $\boldsymbol{E}$

Generated ϕ and $\boldsymbol{B}$ are stable due to dielectric polarization

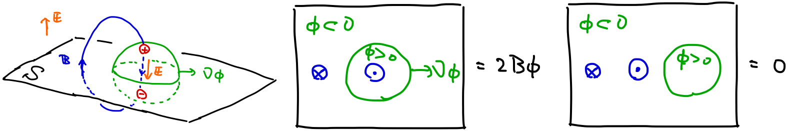
Increase of dielectric polarization [Yamamoto & RY, '23]



- Gauss law $\nabla \cdot \boldsymbol{E} = -\frac{1}{4\pi^2}\boldsymbol{B} \cdot \nabla\phi$ → conservation of elec. flux $\int_S d\boldsymbol{S} \cdot (\boldsymbol{E} + \frac{1}{4\pi^2}\phi\boldsymbol{B})$
- $\boldsymbol{E}$ decreases → dielectric polarization $\int_S d\boldsymbol{S} \cdot \phi\boldsymbol{B}$ increases
- Stability of $\nabla\phi$ and $\boldsymbol{B}$ = stability of dielectric polarization

Topological meaning of $\int_S d\boldsymbol{S} \cdot \phi\boldsymbol{B}$? (cf. magnetic helicity & linking number)

$\int_S d\boldsymbol{S} \cdot \phi \boldsymbol{B}$: linking number of $\boldsymbol{B}$ & $\nabla\phi$ on S [Yamamoto & RY, '23]



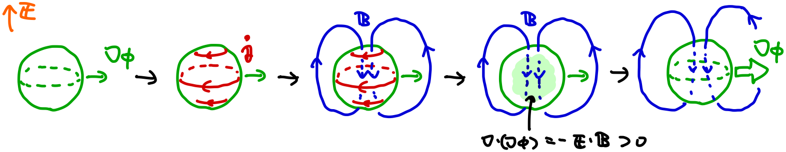
Consider flux tube of $\boldsymbol{B}$ & thin wall of $\nabla\phi$

- $\boldsymbol{B}$: two points with signs, $\nabla\phi$: circle on integral surface S
- Sign of ϕ changes between outside and inside the circle.
- If circle surrounds either point, surface integral is non-zero, otherwise it is zero.

Generated $\boldsymbol{B}$ and $\nabla\phi$ are topologically stable.

I will call the integral "generalized magnetic helicity"

Summary of instability of axion ED in $\boldsymbol{E}$



- Background $\boldsymbol{E} \to$ instability
- Tachyonic generation of $\boldsymbol{B}$ & $\nabla\phi$
- Decrease of $\boldsymbol{E}$
- Stable $\nabla\phi$ and $\boldsymbol{B}$ due to generalized magnetic helicity $\int_S d\boldsymbol{S}\cdot\phi\boldsymbol{B}$

For further generalization, please see our paper [Yamamoto & RY '23].

Contents

Magnetic helicity and non-invertible symmetry

Conserved charges ⇒ symmetries? (converse of Noether theorem)

For the stable magnetic fields, conserved charges e.g., $\int d^3\boldsymbol{x}(\partial_0\phi + \frac{1}{8\pi^2}\boldsymbol{A}\cdot\boldsymbol{B})$ are important.

Q. Does a symmetry exist for this charge?

A. Yes, but it cannot be an ordinary symmetry.

Q. What is the problem with the conserved charge or symmetry generator, e.g.,

$$U = \exp\left(i\alpha \int_V d^3\boldsymbol{x}(\partial_0\phi + \frac{1}{8\pi^2}\boldsymbol{A}\cdot\boldsymbol{B})\right) \quad \text{for} \quad \alpha\in\mathbb{R},\ V\text{: closed 3d space}$$

acting on axion $Ue^{i\phi}U^\dagger = e^{i\alpha}e^{i\phi}$

A1. Just a consequence of chiral anomaly (assuming a UV model with Dirac fermions)

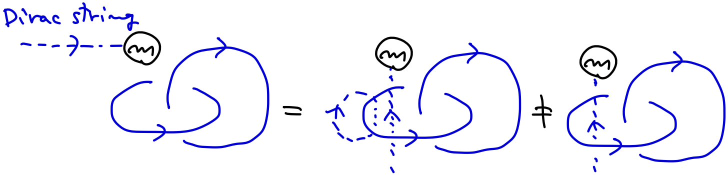
A2. Exp. of magnetic helicity $\exp\left(i\alpha\int d^3\boldsymbol{x}\frac{1}{8\pi^2}\boldsymbol{A}\cdot\boldsymbol{B}\right)$ is not large gauge invariant, so U is not physical

Why does the magnetic helicity $\int d^3\boldsymbol{x}\frac{1}{8\pi^2}\boldsymbol{A}\cdot\boldsymbol{B}$ violate the large gauge invariance?

On large gauge invariance of magnetic helicity (1/3)

Large gauge invariance = Dirac string should be invisible

- Magnetic monopole $\int_S \boldsymbol{B} \cdot d\boldsymbol{S} = 2\pi m$
- Dirac string = unphysical magnetic flux tube to have single-valued $\boldsymbol{A}$
- Invisibility of Dirac string: independence of the choice of Dirac strings
- Magnetic helicity depends on the choice of Dirac strings



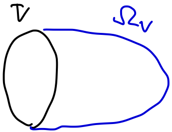
A more precise statement is...

On large gauge invariance of magnetic helicity (2/3)

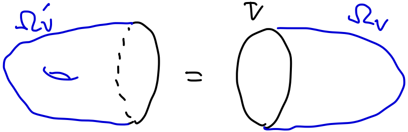
We assume that $\exp\left(i\alpha \int_V d^3\boldsymbol{x} \frac{1}{8\pi^2} \boldsymbol{A}\cdot\boldsymbol{B}\right)$ is a unitary operator.

- Problem: integrand is not gauge invariant.
- Integrand can be gauge invariant using Stokes theorem with $\partial\Omega_V = V$

$$\exp\left(i\alpha \int_V d^3\boldsymbol{x} \frac{1}{8\pi^2} \boldsymbol{A}\cdot\boldsymbol{B}\right) = \exp\left(i\alpha \int_{\Omega_V} d^4x \frac{1}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$



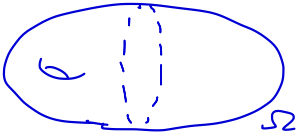
- RHS is manifestly gauge invariant, but has ambiguity of choice of Ω_V
- We require the absence of ambiguity



On large gauge invariance of magnetic helicity (3/3)

- The requirement means

$$\exp\left(-i\alpha\int_\Omega d^4x\,\frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)=1$$



- $e^{i\alpha}=1$ because $\int_\Omega d^4x\,\frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\in\mathbb{Z}$

$U\propto\exp\left(i\alpha\int_V d^3\boldsymbol{x}\,\frac{1}{8\pi^2}\boldsymbol{A}\cdot\boldsymbol{B}\right)$ does not generate any symmetry transf.

However...

We can modify magnetic helicity $\exp\left(i\alpha\int d^3\boldsymbol{x}\,\frac{1}{8\pi^2}\boldsymbol{A}\cdot\boldsymbol{B}\right)$ for $\alpha\in 2\pi\mathbb{Q}$ (e.g., $\alpha=\frac{2\pi}{q}$, $q\in\mathbb{Z}$)

in a gauge invariant way at the expense of invertibility (unitarity)!

Gauge invariant magnetic helicity [Choi, et al., '22; Córdova & Ohmori, '22]

Modification using partition function of Chern-Simons theory

$$\exp\left(\frac{i}{4\pi q}\int_V d^3\boldsymbol{x}\boldsymbol{A}\cdot\boldsymbol{B}\right) \to \int \mathcal{D}c \exp\left(i\int_V d^3\boldsymbol{x}\left(-\frac{q}{4\pi}\epsilon^{ijk}c_i\partial_j c_k + \frac{i}{2\pi}\epsilon^{ijk}c_i\partial_j A_k\right)\right)$$

- Essentially, it is a square completion $\frac{1}{q}x^2 \to -qy^2 + 2xy$ so that q is in numerator
- RHS: partition function of $U(1)$ Chern-Simons theory
  - c_μ: auxiliary $U(1)$ gauge field on V, Dirac quant. $\int \partial_\mu c_\nu dS^{\mu\nu} \in 2\pi\mathbb{Z}$
  - Large gauge invariant: q is in numerator
  - Magnetic helicity: naive expression obtained by EOM $F_{\mu\nu} = qc_{\mu\nu}$ only for trivial Dirac quantization $\int \boldsymbol{B}\cdot d\boldsymbol{S} = 0$
- Invertibility is lost
  - path integral (sum) over phase factors (e.g., $\cos\theta \sim e^{i\theta} + e^{-i\theta}$ is non-invertible)

Non-invertible symmetry [Choi, et al., '22; Córdova & Ohmori, '22]

We have conserved & gauge invariant quantity

Generator of non-invertible symmetry

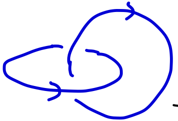
$$D = \int \mathcal{D}c \exp\left(i \int_V d^3\boldsymbol{x} \left(-\frac{q}{4\pi}\epsilon^{ijk} c_i \partial_j c_k + \frac{i}{2\pi}\epsilon^{ijk} c_i \partial_j A_k\right)\right) \times \exp\left(\frac{2\pi i}{q}\int_V d^3\boldsymbol{x}\partial_0\phi\right)$$

- Conservation law = EOM of axion
- Fractional rotation on axion: $De^{i\phi} = e^{\frac{2\pi i}{q}} e^{i\phi} D$
- Non-invertible transf. on magnetic monopole: $D|\text{monopole}\rangle = 0$ (depending on q and V)
- Stability of magnetic helicity = existence of non-invertible symmetry
- Generalization: e.g., $\int_S \phi \boldsymbol{B} \cdot d\boldsymbol{S} \to$ non-invertible 1-form symmetry [Choi, et al., '22; RY '22]

Magnetic helicity $=$ linking number [Yamamoto & RY, '23]

Non-invertible symmetry can capture linked magnetic fluxes

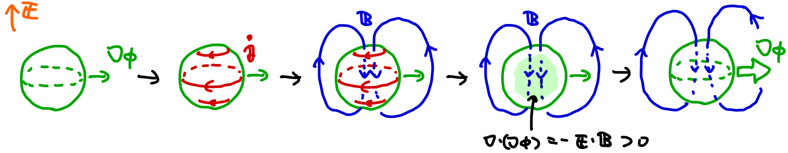
$$D\left[A = \right.$$



$$\left. \right] \propto \exp\left(\frac{2\pi i}{q} \Phi_1 \Phi_2 \, \mathrm{Link}(C_1, C_2) \right)$$

- Relation "$\int d^3\boldsymbol{x} \boldsymbol{A} \cdot \boldsymbol{B} \propto$ linking number" still holds (with some technical modification)

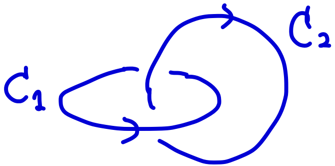
Summary



- Axion electrodynamics exhibits instability in the presence of background time dependent axion $\partial_t \phi$ or electric field.
- Generalized chiral instabilities: universal mechanism of these instabilities
  - Instabilities tend to be weakened.
  - $\boldsymbol{B}$ & $\nabla\phi$ with linking number are generated.
  - Stability of mag. fields is due to non-invertible symmetries.
- We can extend the mechanism to massless Abelian p-form gauge theories with cubic topological interactions (see our paper [2305.01234])
- Future work: non-linear analysis, final state, including gravity, applications,...

Magnetic helicity = linking number (1/3)

$$\int d^3\boldsymbol{x}\,\boldsymbol{A}\cdot\boldsymbol{B} = 2\Phi_1\Phi_2\,\mathrm{Link}\,(C_1, C_2)$$



- Magnetic field

$$\boldsymbol{B}(\boldsymbol{x}) = \Phi_1\boldsymbol{J}(C_1;\boldsymbol{x}) + \Phi_2\boldsymbol{J}(C_2;\boldsymbol{x}) \quad \text{with} \quad \boldsymbol{J}(C_1;\boldsymbol{x}) = \int_{C_1}\delta^3(\boldsymbol{x}-\boldsymbol{r})d\boldsymbol{r}$$

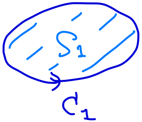
- $\boldsymbol{J}(C_1;\boldsymbol{x})$: delta function on C_1 line integral $\leftrightarrow$ volume integral

$$\int_{C_1}\boldsymbol{v}(\boldsymbol{r})\cdot d\boldsymbol{r} = \int d^3\boldsymbol{x}\int_{C_1}d\boldsymbol{r}\cdot\boldsymbol{v}(\boldsymbol{x})\delta^3(\boldsymbol{x}-\boldsymbol{r}) = \int d^3\boldsymbol{x}\boldsymbol{v}\cdot\boldsymbol{J}(C_1)$$

How can $\boldsymbol{A}$ be solved?

Magnetic helicity = linking number (2/3)

$$\boldsymbol{A} = \Phi_1 \boldsymbol{K}(S_1) + \Phi_2 \boldsymbol{K}(S_2) \quad \text{with} \quad \boldsymbol{K}(S_1) = \int_{S_1} \delta^3(\boldsymbol{x} - \boldsymbol{r}) d\boldsymbol{S}(\boldsymbol{r})$$



- $\boldsymbol{K}(S_1)$: delta function on S_1, $\quad \boldsymbol{J}(C_1) = \nabla \times \boldsymbol{K}(S_1)$

  Derivation: Stokes theorem & partial integral

$$\begin{aligned} \int d^3\boldsymbol{x}\boldsymbol{v} \cdot \boldsymbol{J}(C_1) &= \int_{C_1} \boldsymbol{v}(\boldsymbol{r}) \cdot d\boldsymbol{r} = \int_{S_1} \nabla \times \boldsymbol{v}(\boldsymbol{r}) \cdot d\boldsymbol{S} \\ &= \int d^3\boldsymbol{x}(\nabla \times \boldsymbol{v}) \cdot \boldsymbol{K}(S_1) = \int d^3\boldsymbol{x}\boldsymbol{v} \cdot \nabla \times \boldsymbol{K}(S_1) \end{aligned}$$

We can explicitly evaluate $\int d^3\boldsymbol{x}\boldsymbol{A} \cdot \boldsymbol{B}$

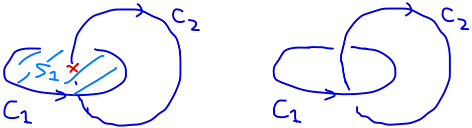
Magnetic helicity = linking number (3/3)

- Magnetic helicity

$$\int d^3\boldsymbol{x}\boldsymbol{A}\cdot\boldsymbol{B} = 2\Phi_1\Phi_2\int d^3\boldsymbol{x}\boldsymbol{K}(S_1)\cdot\boldsymbol{J}(C_2) = 2\Phi_1\Phi_2\int_{C_2}\boldsymbol{K}(S_1)\cdot d\boldsymbol{r}$$

- Using

$$\int_{C_2}\boldsymbol{K}(S_1)\cdot d\boldsymbol{r} = \text{intersection number of } S_1 \text{ \& } C_2 = \mathrm{Link}\,(C_1, C_2),$$



we have

$$\int d^3\boldsymbol{x}\boldsymbol{A}\cdot\boldsymbol{B} = 2\Phi_1\Phi_2\,\mathrm{Link}\,(C_1, C_2)$$

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