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Effective field theory of quantum Hall systems with Galilean invariance

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(TA and Y. Nishida, arXiv:2407.07578)

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Outline

1. Introduction
2. Microscopic theory
3. Effective action
4. Responses
5. Summary

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Introduction

[1] J. E. Avron, J. Stat. Phys. 92 543 (1998)
[2] X. G. Wen and A. Zee PRL 69, 953 (1992)
[3] C. Hoyos, Int. J. Mod. Phys. B 28, 1430007 (2014)
[4] C. Hoyos and D. T. Son, PRL 108, 066805 (2012)

✓ Hall conductivity

$$\sigma_H = \frac{\nu}{2\pi}$$

✓ Hall viscosity[1]

$$\eta_H = \frac{\kappa B}{8\pi} \qquad \kappa = \nu \mathcal{S}$$

Wen-Zee shift[2]

The QH system on a closed surface with genus g

$$Q = \nu(N_\phi + (1-g)\mathcal{S})$$

	ν	$\mathcal{S}$
Integer	N	N
Laughlin	1/q	q

(Cited from [3])

The Hall viscosity is related to the Hall conductivity[4].

$$\sigma_H(q) = \frac{\nu}{2\pi} + \frac{q^2}{B}\left[\frac{\eta_H}{B} - \frac{m}{B}\mathcal{E}''(B)\right] + \dots$$

Introduction

✓ Motivation

- We will explore the universal relations between responses.
- We will discuss all responses including { an energy current / nonlinear responses.

✓ Prescription of EFT

1. Check symmetries of the microscopic theory.

2. Determine the power counting scheme of the derivative expansion.

3. Write down all possible terms allowed by the symmetries up to $\mathcal{O}(\partial^n)$

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Microscopic theory

✓ Microscopic action

$$S = S_0 + S_{\rm int}$$

$$S_0 = \int dt d^2x \left(\Psi^\dagger i \overleftrightarrow{D}_t \Psi - \frac{D_i \Psi^\dagger D_i \Psi}{2m} + \frac{gB}{4m} \Psi^\dagger \Psi \right)$$

$$\Psi^\dagger \overleftrightarrow{D}_\mu \Psi = [\Psi^\dagger (D_\mu \Psi) - (D_\mu \Psi^\dagger)\Psi]/2$$

$$D_\mu \Psi = (\partial_\mu - i A_\mu)\Psi \quad D_\mu \Psi^\dagger = (\partial_\mu + i A_\mu)\Psi^\dagger$$

g : g-factor (not genus)

✓ Currents

$$\mathcal{J}_\mu \sim \frac{\delta S}{\delta A_\mu}$$

$$\mathcal{E}_\mu \sim \frac{\delta S}{\delta ?}$$

cf. relativistic systems

$$\mathcal{T}_{\mu\nu} \sim \frac{\delta S}{\delta g^{\mu\nu}}$$

➡ **Newton-Cartan geometry** (Non-relativistic curved spacetime)

Microscopic theory

✓ Newton-Cartan (NC) geometry[5]

[5] e.g., D. T. Son, arXiv:1306.0638

NC spacetime

= clock covector n_μ ⟷ Energy current
spatial metric $h^{\mu\nu}$ ⟷ Stress
velocity vector v^μ ⟷ Momentum

- $h^{\mu\nu} n_\mu = 0$
- $n_\mu v^\mu = 1$

A lower spatial metric is defined by requiring

$$h_{\mu\nu} v^\mu = 0 \qquad h^{\mu\lambda} h_{\lambda\nu} = P^\mu_{\ \nu} = \delta^\mu_{\ \nu} - v^\mu n_\nu$$

Microscopic theory

✓ Microscopic action on NC spacetime

$$S_0 = \int dt d^2x \sqrt{\gamma} \left[\Psi^\dagger i v^\mu \overleftrightarrow{D}_\mu \Psi - \left(\frac{h^{\mu\nu}}{2m} + \frac{i g \varepsilon^{\lambda\mu\nu} n_\lambda}{4m} \right) D_\mu \Psi^\dagger D_\nu \Psi \right] \quad \begin{aligned} \gamma_{\mu\nu} &= n_\mu n_\nu + h_{\mu\nu}, \\ \gamma &= \det(\gamma_{\mu\nu}) \end{aligned}$$

Flat spacetime: $n_\mu = v^\mu = (1,0,0)$ and $h^{\mu\nu} = h_{\mu\nu} = \mathrm{diag}(0,1,1)$

✓ Symmetries

- U(1) gauge transformation

$$\Psi \to e^{i\chi}\Psi, \; A_\mu \to A_\mu + \partial_\mu \chi$$

- General coordinate transformation

$$x^\mu \to x'^\mu = x'^\mu(x)$$

- **Milne boost**[6]

[6] e.g., K. Jensen, SciPost Phys. 5 011 (2018)

$$v^\mu \to v^\mu + h^{\mu\nu}\psi_\nu,$$

$$h_{\mu\nu} \to h_{\mu\nu} - n_{(\mu} P^\lambda_{\ \nu)} \psi_\lambda + n_\mu n_\nu h^{\rho\sigma} \psi_\rho \psi_\sigma,$$

$$A_\mu \to A_\mu + m P^\nu_{\ \mu} \psi_\nu - \frac{m}{2} n_\mu h^{\rho\sigma} \psi_\rho \psi_\sigma + \frac{g}{4} n_\mu \varepsilon^{\nu\rho\sigma} \partial_\nu (n_\rho P^\lambda_{\ \sigma} \psi_\lambda)$$

Symmetries

✓ Interaction[4]

- Coulomb (propagates in three spatial dimensions)

$$S_{\text{Coulomb}} = \int d^3x \sqrt{\gamma} a_0 \Psi^\dagger \Psi + \frac{1}{2} \int d^3x dz \sqrt{\gamma} [h^{\mu\nu} \partial_\mu a_0 \partial_\nu a_0 + (\partial_z a_0)^2]$$

- Yukawa

$$S_{\text{Yukawa}} = \int d^3x \sqrt{\gamma} \left(\lambda \Psi^\dagger \Psi \phi - \frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{m_\phi^2}{2} \phi^2 \right)$$

They are { general coordinate invariant
Milne invariant.
} if the auxiliary fields transform as scalars.

[4] C. Hoyos and D. T. Son, PRL 108, 066805 (2012)

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Effective action

✓ EFT

Prescription of EFT

1. ~~Check symmetries of the microscopic theory.~~

➡ U(1), general coordinate, Milne

2. Determine the power counting scheme of the derivative expansion.

$$\partial_\mu \sim \mathcal{O}(\partial) \quad n_\mu, v^\mu, h^{\mu\nu} \sim \mathcal{O}(1) \quad A_\mu \sim \mathcal{O}(\partial^{-1})$$

3. Write down all possible terms allowed by the symmetries up to $\mathcal{O}(\partial^2)$

Effective action

✓ Milne invariant objects[7]

A Milne invariant vector $u^\mu \to u'^\mu = u^\mu$

(normalized as $n_\mu u^\mu = 1$)

$u_\mu = h_{\mu\nu} u^\nu$ and $u^2 = u_\mu u^\mu$ are not Milne invariant.

➡ $\tilde{h}_{\mu\nu} = h_{\mu\nu} - n_{(\mu} u_{\nu)} + n_\mu n_\nu u^2$

$\tilde{A}_\mu = A_\mu + m u_\mu - \frac{m}{2} n_\mu u^2 + \frac{\mathrm{g}}{4} n_\mu \varepsilon^{\nu\rho\sigma} \partial_\nu (n_\rho u_\sigma)$ are Milne invariant.

We construct u^μ order by order in derivative expansion.

➡ $u^\mu = \frac{\varepsilon^{\mu\nu\lambda} \partial_\nu A_\lambda}{B} + \mathcal{O}(\partial) \quad B = \varepsilon^{\mu\nu\lambda} n_\mu \partial_\nu A_\lambda$

[7] K. Jensen, J. High Energy Phys. 2015, 123 (2015)

Effective action

✓ Vielbein and spin connection

$$h^{\mu\nu} = e^{a\mu} e^{a\nu}$$

The spatial metric is invariant under a local SO(2) rotation,

$$e^{x\mu} \pm i e^{y\mu} \to e^{\pm i\theta}(e^{x\mu} \pm i e^{y\mu}) \,.$$

$\omega_\mu = \frac{1}{2}\epsilon^{ab} h_{\lambda\nu} e^{a\lambda} \nabla_\mu e^{b\nu}$: an abelian gauge field

under the local rotation $\omega_\mu \to \omega_\mu + \partial_\mu \theta$

Covariant derivative: $\nabla_\mu e^{a\nu} = \partial_\mu e^{a\nu} + \Gamma^\nu{}_{\mu\lambda} e^{a\lambda}$

$$\Gamma^\lambda{}_{\mu\nu} = v^\lambda \partial_\mu n_\nu + \frac{1}{2} h^{\lambda\rho}(\partial_{(\mu} h_{\nu)\rho} - \partial_\rho h_{\mu\nu})$$

Milne invariant spin connection $\tilde{\omega}_\mu$ can be obtained by replacing $(v^\mu, h_{\mu\nu}) \to (u^\mu, \tilde{h}_{\mu\nu})$

Effective action

✓ Effective action up to $\mathcal{O}(\partial^2)$

$$S_{\text{eff}} = \int d^3x \sqrt{\gamma} \sum_{i=-1}^{1} \mathcal{L}_i,$$

$$\mathcal{L}_{-1} = \frac{\nu}{4\pi} \varepsilon^{\mu\nu\lambda} \tilde{A}_\mu \partial_\nu \tilde{A}_\lambda,$$

Chern-Simons

$$\mathcal{L}_0 = -\mathcal{E}(\tilde{B}),$$

$$\mathcal{L}_1 = \frac{\kappa}{4\pi} \varepsilon^{\mu\nu\lambda} \tilde{A}_\mu \partial_\nu \tilde{\omega}_\lambda - \mathcal{M}(\tilde{B}) \varepsilon^{\mu\nu\lambda} n_\mu \partial_\nu n_\lambda$$

Wen-Zee

$$\tilde{B} = \varepsilon^{\mu\nu\lambda} n_\mu \partial_\nu \tilde{A}_\lambda$$

- CS and WZ are gauge invariant up to surface.

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Responses

✓ In equilibrium (E=0, static but inhomogeneous B)

$$\mathcal{J}^t = \frac{\nu}{2\pi} B + \partial_i \left[\left(m\mathcal{E}''(B) + \frac{\nu \mathrm{g}}{8\pi} - \frac{\kappa}{8\pi} \right) \frac{\partial_i B}{B} \right]$$

$$\mathcal{J}^i = -\epsilon^{ij} \mathcal{E}'(B)$$ → Magnetization

$$\mathcal{E}^t = \mathcal{E}(B)$$ → Energy density

$$\mathcal{E}^i = \epsilon^{ij} \partial_j \mathcal{M}(B)$$ → Energy magnetization

$$\mathcal{T}_{ij} = \delta_{ij} [B\mathcal{E}'(B) - \mathcal{E}(B)]$$ → Pressure

Responses

✓Out of equilibrium (spacetime-dependent E, constant B)

$$\mathcal{T}_{ij} = \delta_{ij}\left[B\mathcal{E}'(B) - \mathcal{E}(B) - \left(m\mathcal{E}''(B) + \frac{\nu g}{8\pi}\right)\partial_k E_k\right] + \frac{\kappa}{8\pi}\left[\partial_{(i}E_{j)} - \delta_{ij}\partial_k E_k\right] + \mathcal{O}(E^2)$$

$$= -\frac{\kappa B}{8\pi}\frac{1}{2}\epsilon^{(ik}\delta^{jl)}\partial_{(k}u_{l)}$$

→ Hall viscosity

$$\mathcal{J}^t = \frac{\nu}{2\pi}\left(B - m\frac{\partial_i E_i}{B}\right) + \mathcal{O}(E^2)$$

$$\mathcal{J}^i = \frac{\nu}{2\pi}\left(\epsilon^{ij}E_j + m\frac{\partial_t E_i}{B} - m^2\frac{\epsilon^{ij}\partial_t^2 E_j}{B^2}\right) + \left[m\mathcal{E}''(B) + \frac{\nu g}{8\pi} - \frac{\kappa}{8\pi}\right]\frac{\epsilon^{ij}\partial_j\partial_k E_k}{B} + \mathcal{O}(E^2)$$

Hall conductivity

Hoyos-Son with g

New prediction 1: The Hall conductivity determine the longitudinal conductivity at nonzero frequency!

Responses

✓Out of equilibrium (spacetime-dependent E, constant B)

$$\mathcal{E}^t = \mathcal{E}(B) - \mathcal{E}'(B)\frac{\partial_i E_i}{B} + \frac{\nu m}{2\pi}\left(\frac{E^2}{2B} - m\frac{\epsilon^{ij}E_i\partial_t E_j}{B^2}\right) + \mathcal{O}(E^3)$$

$$\mathcal{E}^i = \mathcal{E}'(B)\left(\epsilon^{ij}E_j + m\frac{\partial_t E_i}{B} - \frac{m}{2}\frac{\epsilon^{ij}\partial_j E^2}{B^2}\right) - \left[m\mathcal{E}''(B) + \frac{\nu g}{8\pi}\right]\frac{\epsilon^{ij}E_j\partial_k E_k}{B}$$
$$- \boxed{\frac{\kappa}{8\pi}\frac{\epsilon^{ij}E_k\partial_k E_j + \epsilon^{jk}E_j\partial_k E_i}{B}} + \mathcal{O}(E^3)$$

New prediction 2: The Hall viscosity contributes to
the nonlinear thermoelectric response
at nonzero wave number !
= Energy current version of Hoyos-Son

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Summary

✓ The EFT of quantum Hall systems with Galilean invariance is studied.

✓ The charge, energy and momentum currents are computed.

✓ Important findings

1. Reproducing Hoyos-Son

$$\mathcal{J}^y \sim -\left[m\mathcal{E}''(B) + \frac{g}{4}\sigma_H + \frac{\eta_H}{B}\right]\frac{\partial_x^2 E_x}{B}$$

2. Energy version of Hoyos-Son

$$\mathcal{E}^y \sim -\left[m\frac{\mathcal{E}'(B)}{B} + m\mathcal{E}''(B) + \frac{g}{4}\sigma_H + \frac{\eta_H}{B}\right]\frac{\partial_x E_x^2}{2B}$$

3. Longitudinal conductivity at nonzero frequency

$$\mathcal{J}^x \sim \frac{\sigma_H m}{B}\partial_t E_x$$