Uncovering the mechanism of chiral three-nucleon force in driving spin-orbit splitting

Tokuro Fukui

4/September/2024

TOMOE

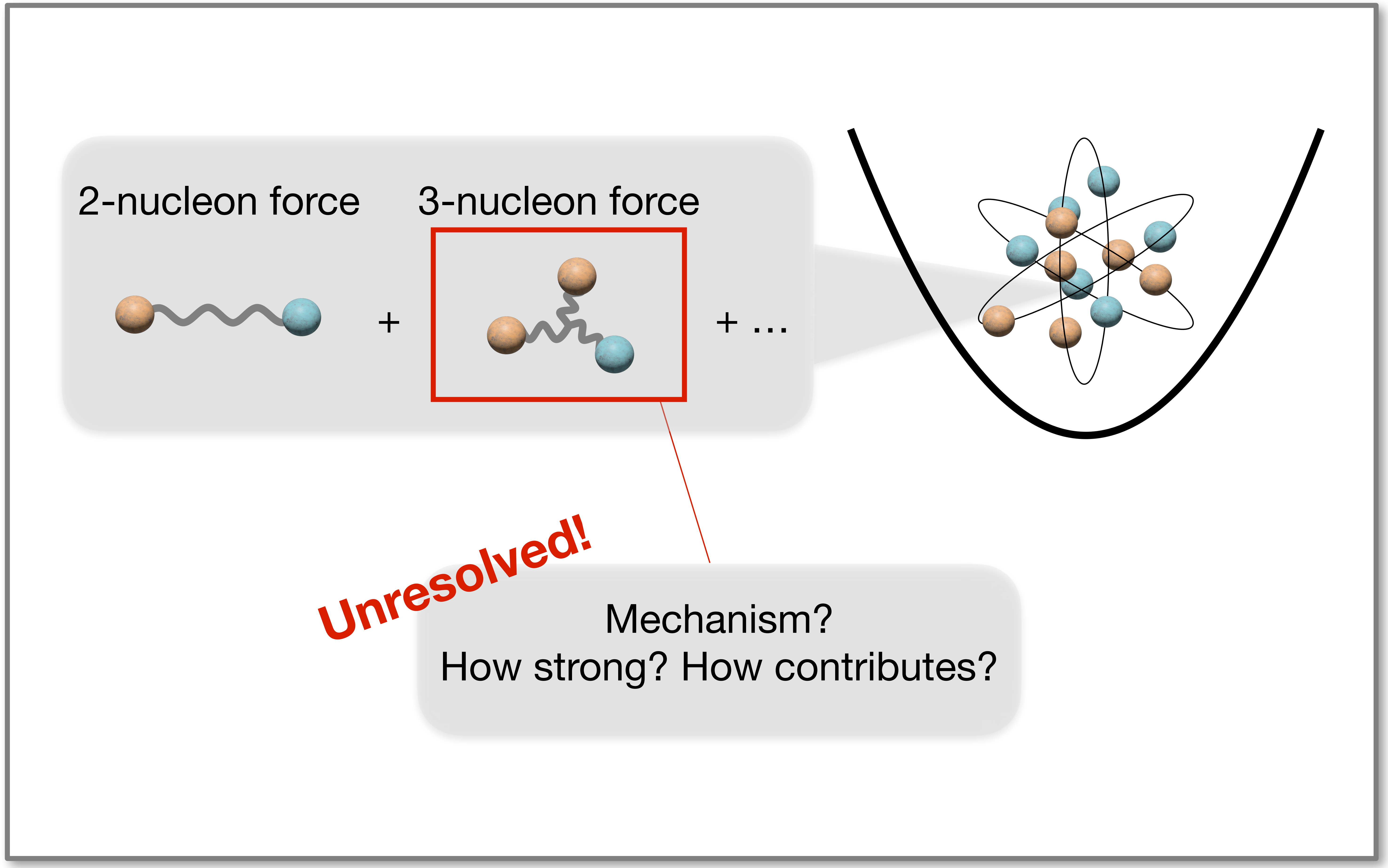
Collaborators

A. Gargano

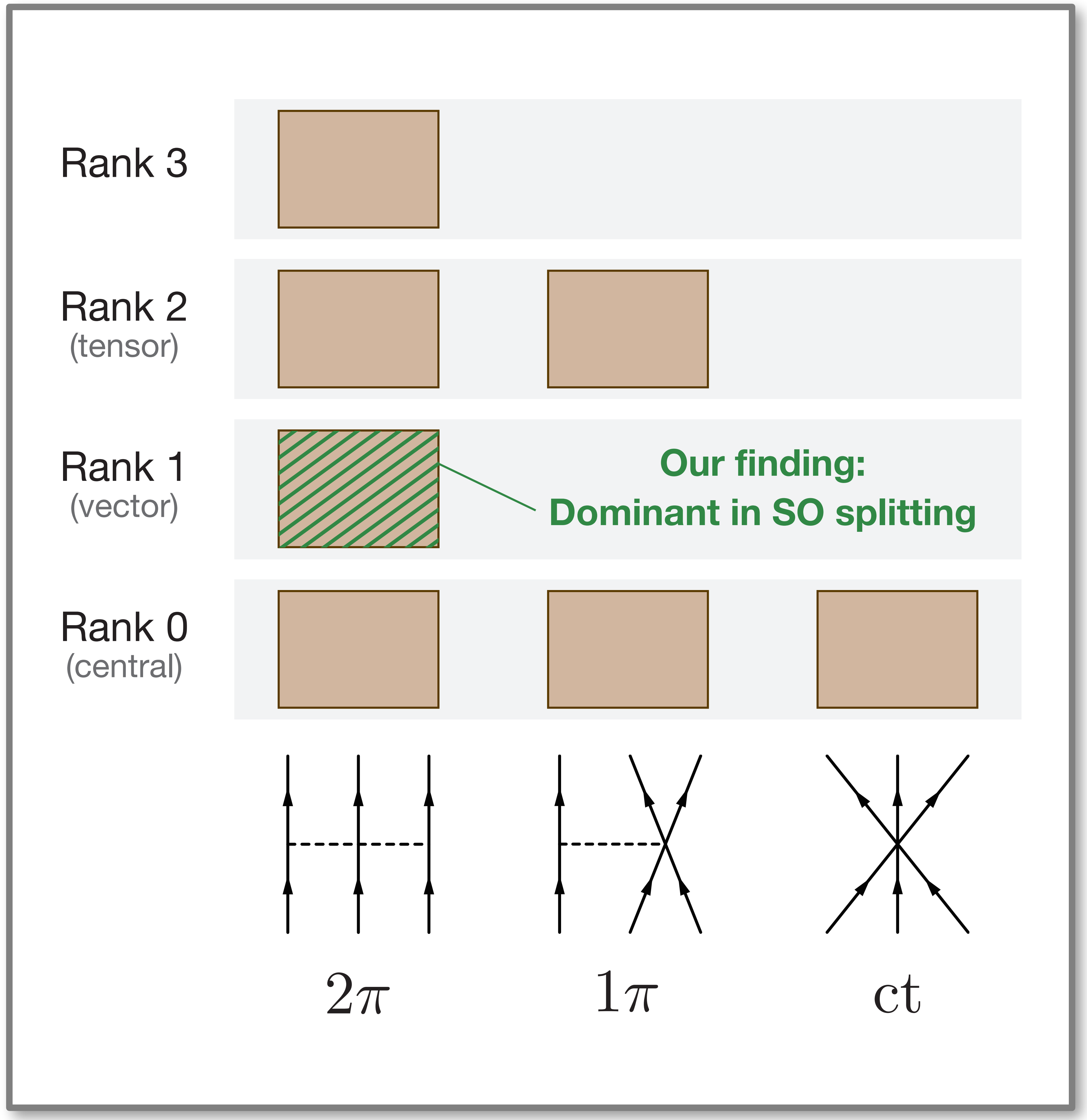
G. De Gregorio

Main message | 3NF and shell-structure manifestation

Target



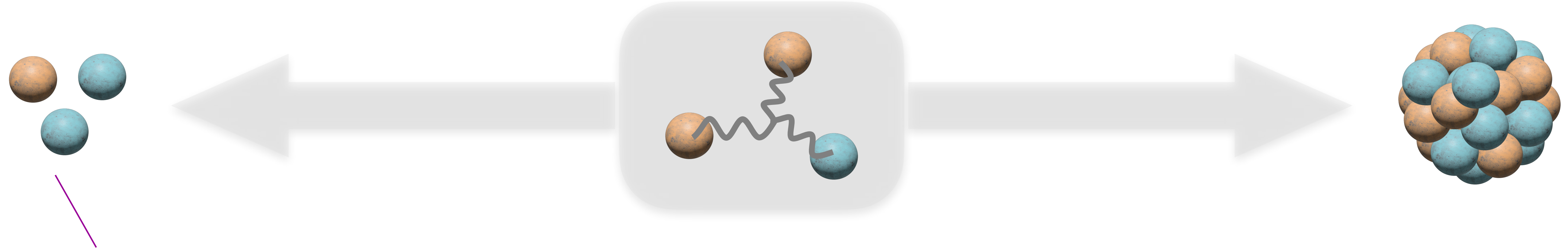
Main finding



Fukui +, PLB **855**, 138839 (2024)

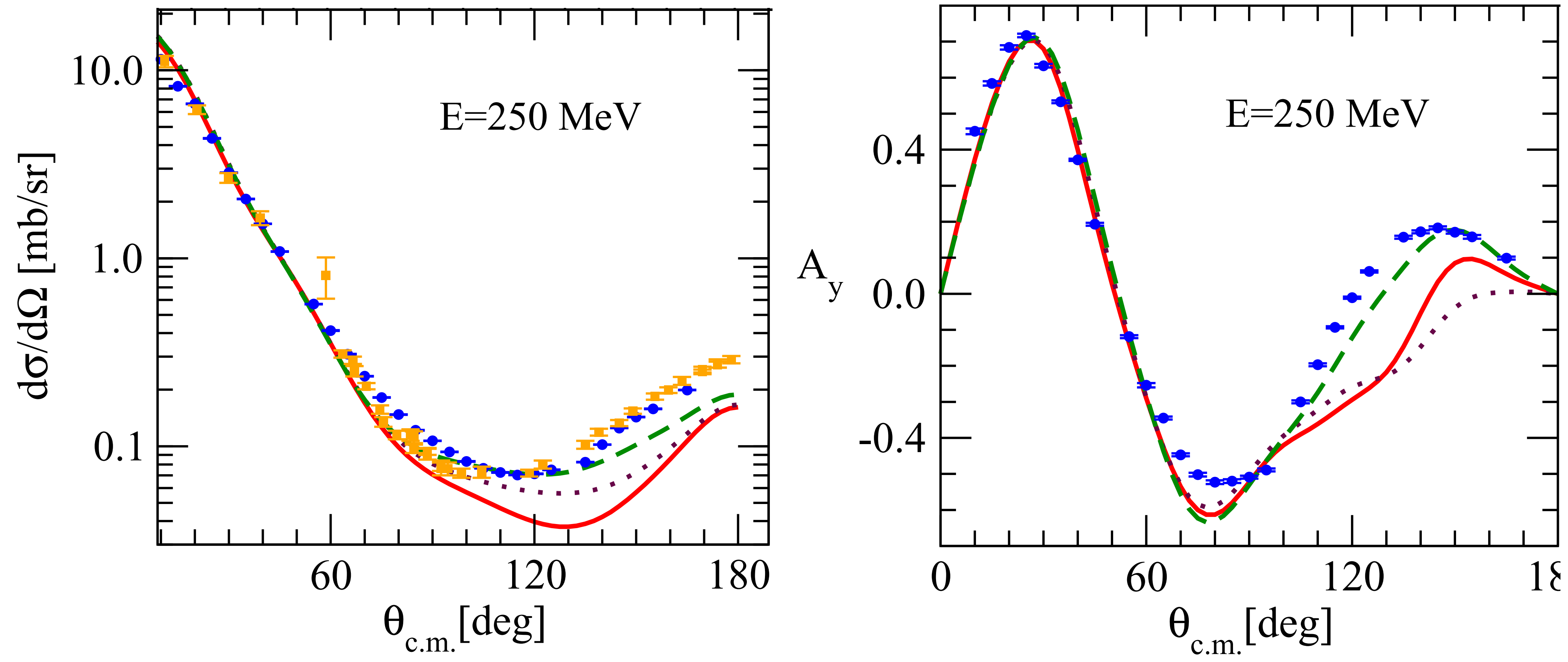
3NF | Few- and many-body systems

Few-body | 3NF | **Many-body** (my interest)



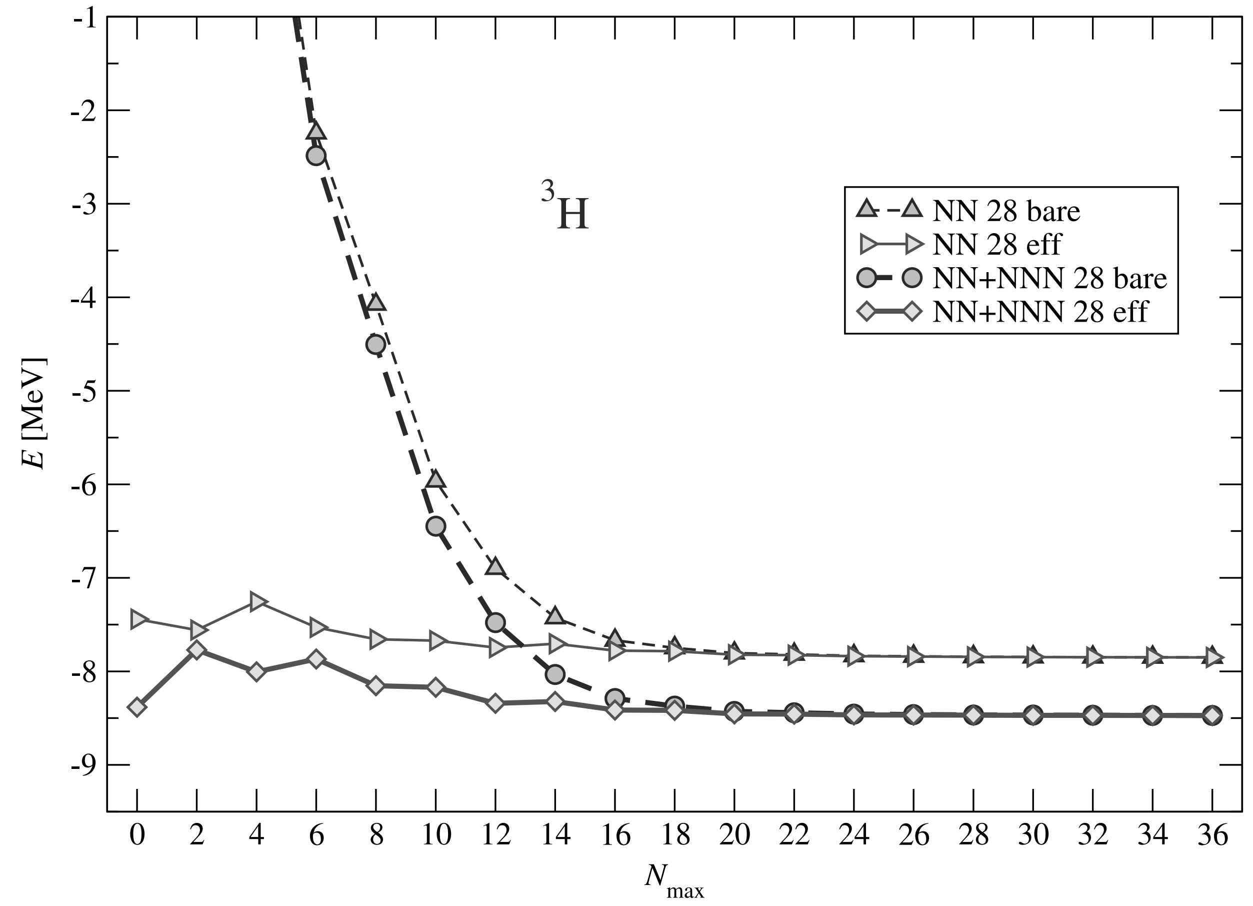
Fundamentals of 3NF

d + *p* scattering



Witała +, PRC **105**, 054004 (2022)

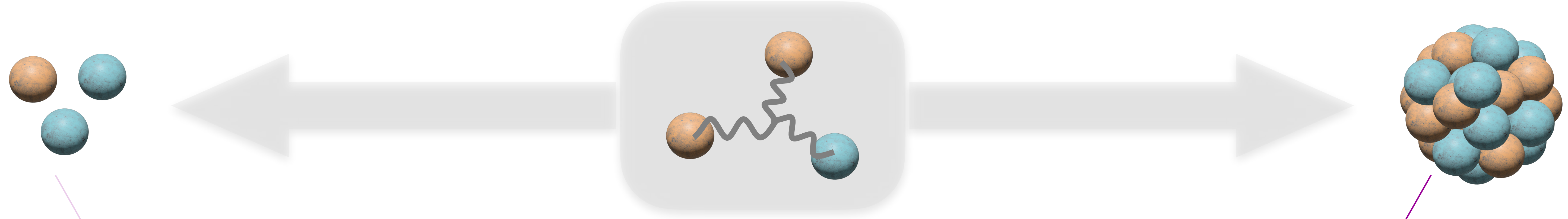
$^3H/^3He$ structure



Navrátil, FBS **41**, 117 (2007)

3NF | Few- and many-body systems

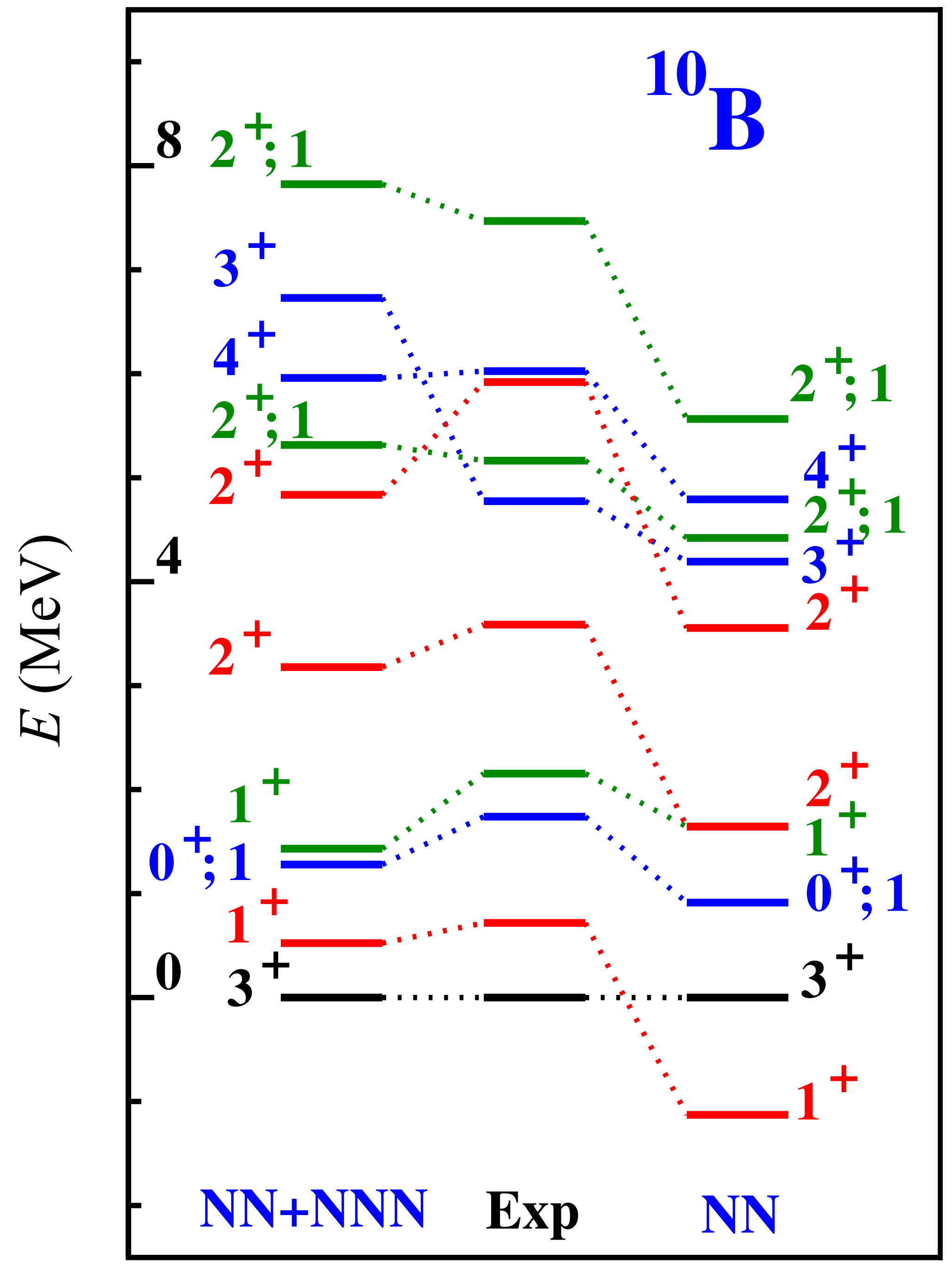
Few-body — **3NF** — **Many-body** (my interest)



Fundamentals of 3NF

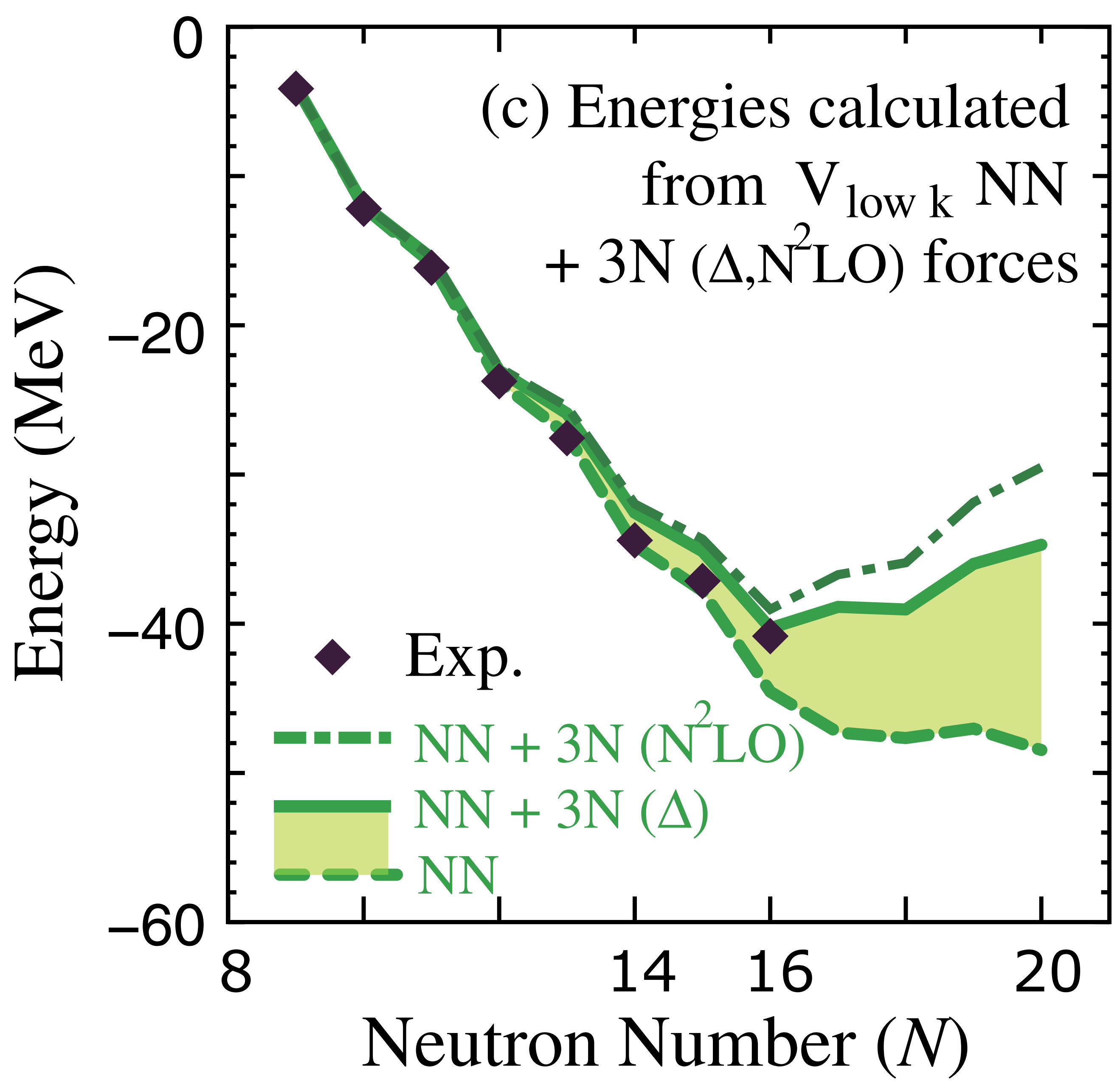
Major playground of 3NF

^{10}B spectra



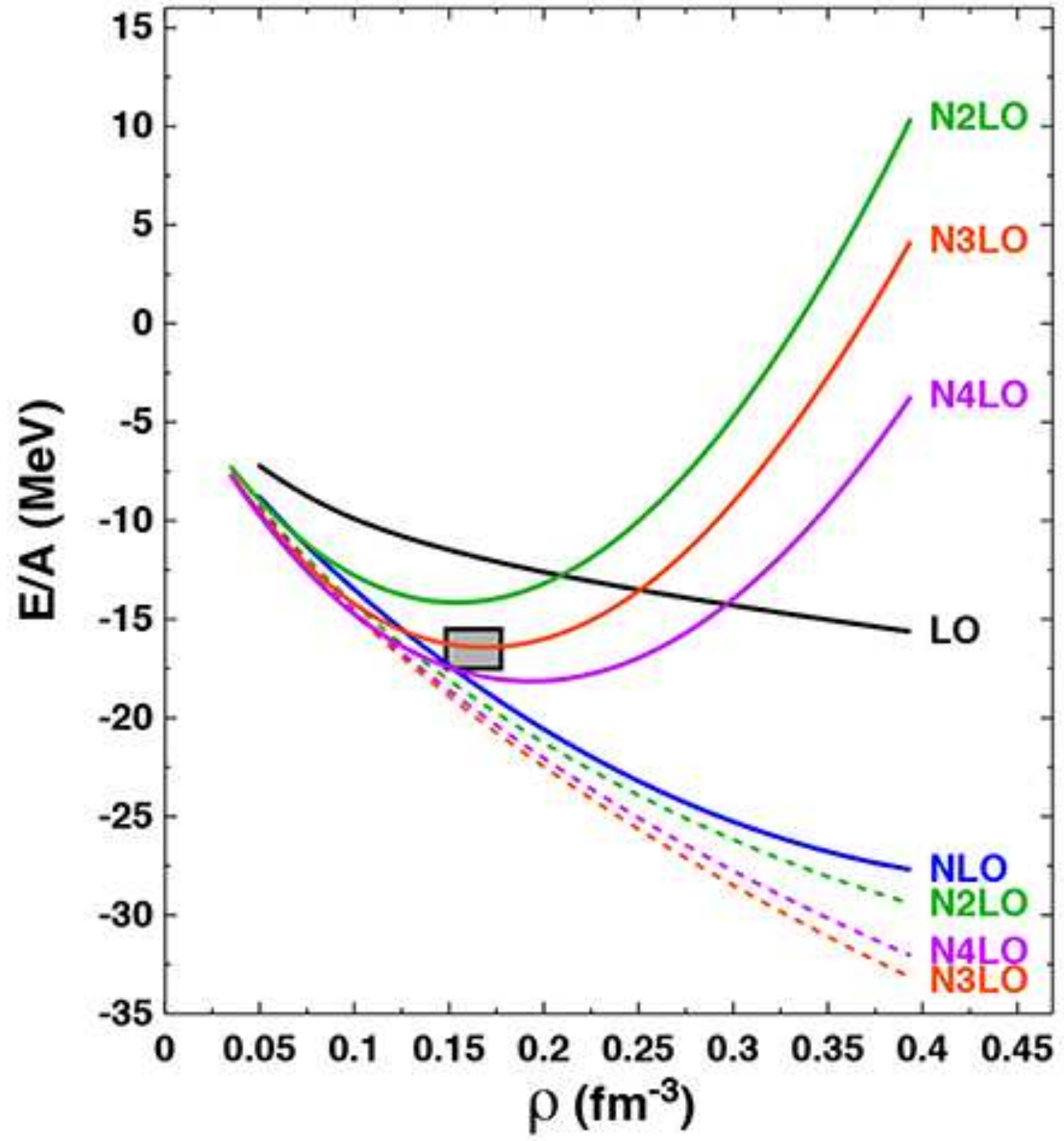
Navrátil +, PRL **99**, 042501 (2007)

Oxygen-drip line



Otsuka +, PRL **105**, 032501 (2010)

Nuclear matter saturation

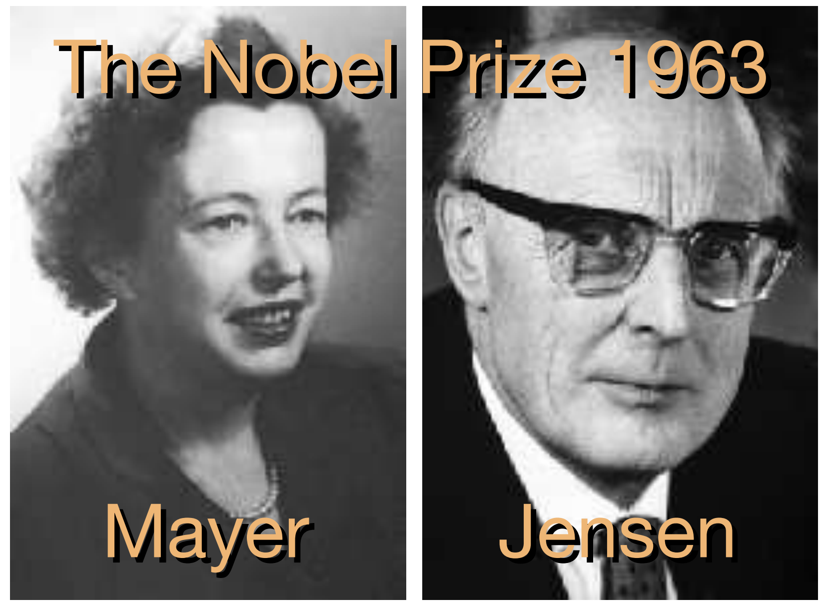


Sammarruca & Millerson, Front. Phys. **7**, 00213 (2019)

Nuclear shell structure

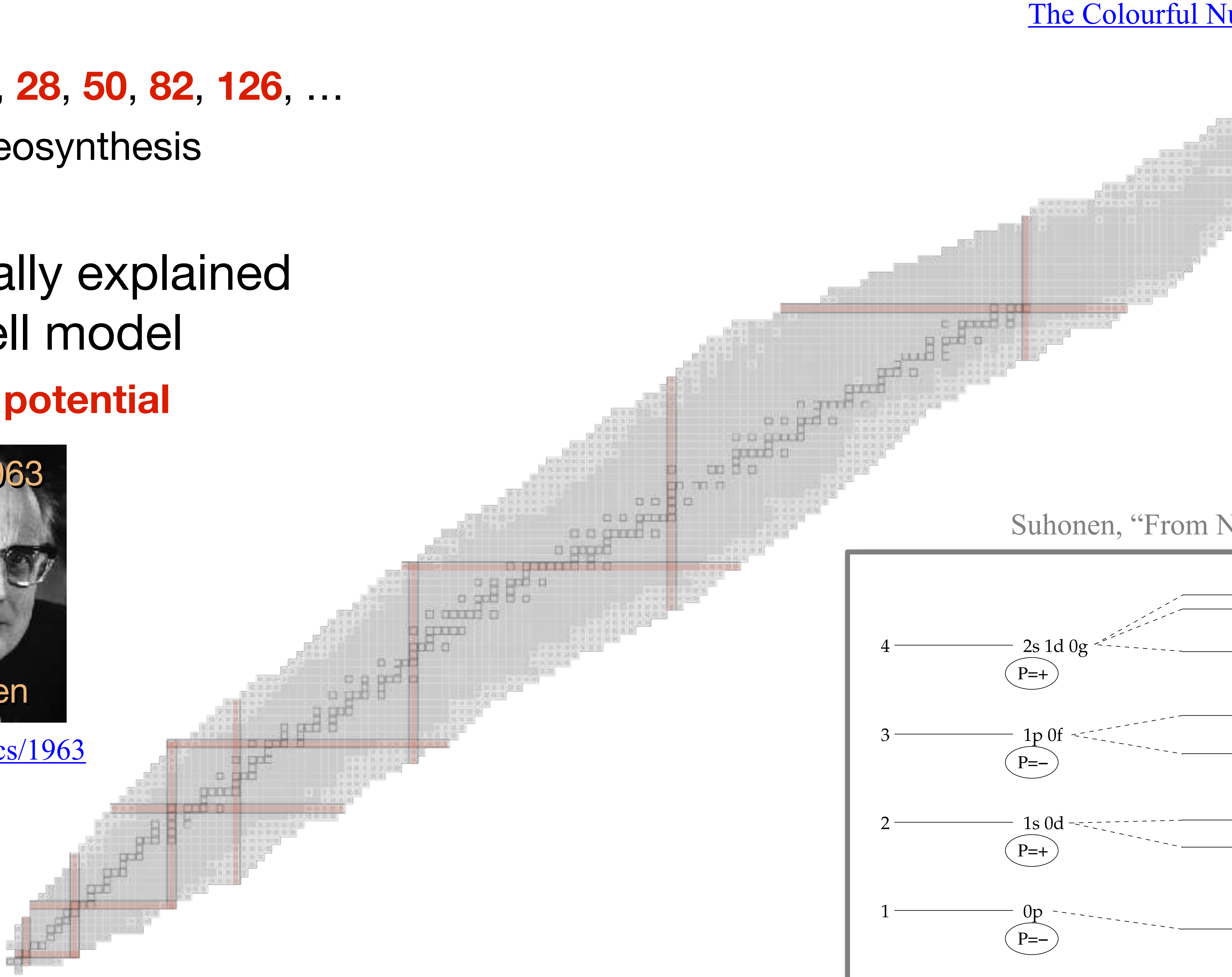
- Magic numbers
  - Stable nuclei: 2, 8, 20, **28**, **50**, **82**, **126**, …
  - Waiting points in nucleosynthesis

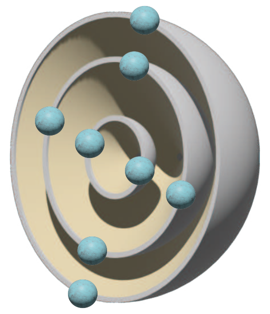
- Phenomenologically explained by *jj*-coupling shell model
  - **One-body spin-orbit potential**



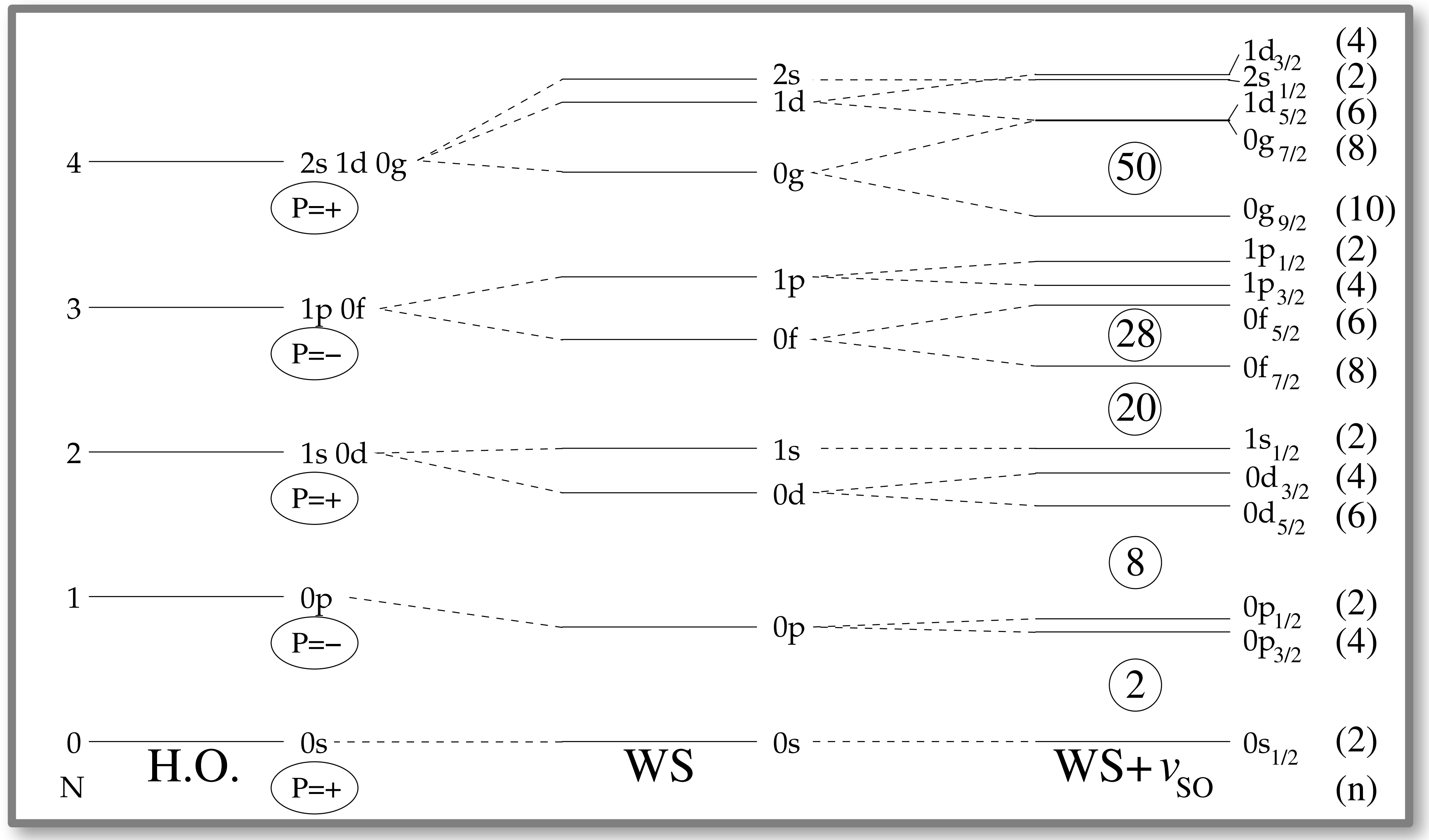
nobelprize.org/prizes/physics/1963

The Colourful Nuclide Chart





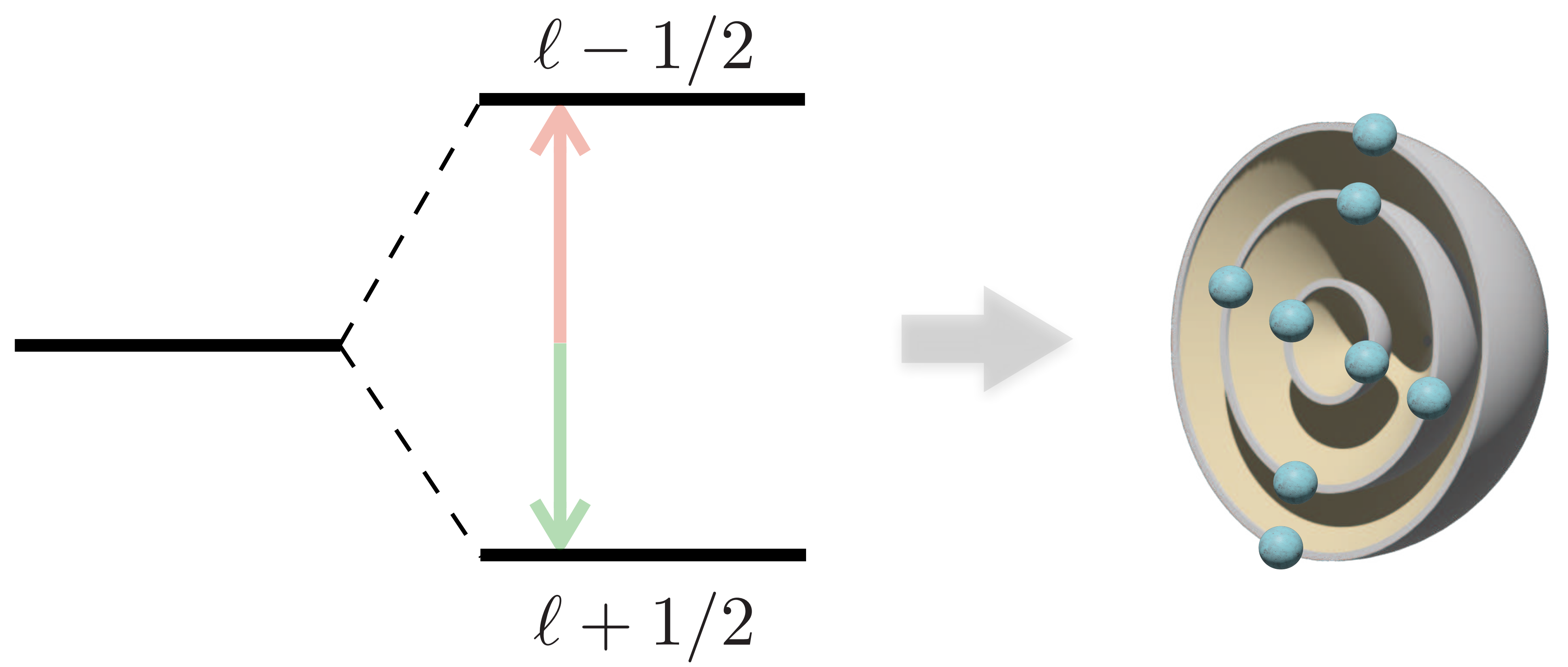
Suhonen, "From Nucleons to Nucleus", Springer (2007)



How 3NF contributes to spin-orbit splitting?

Microscopic origin remains elusive

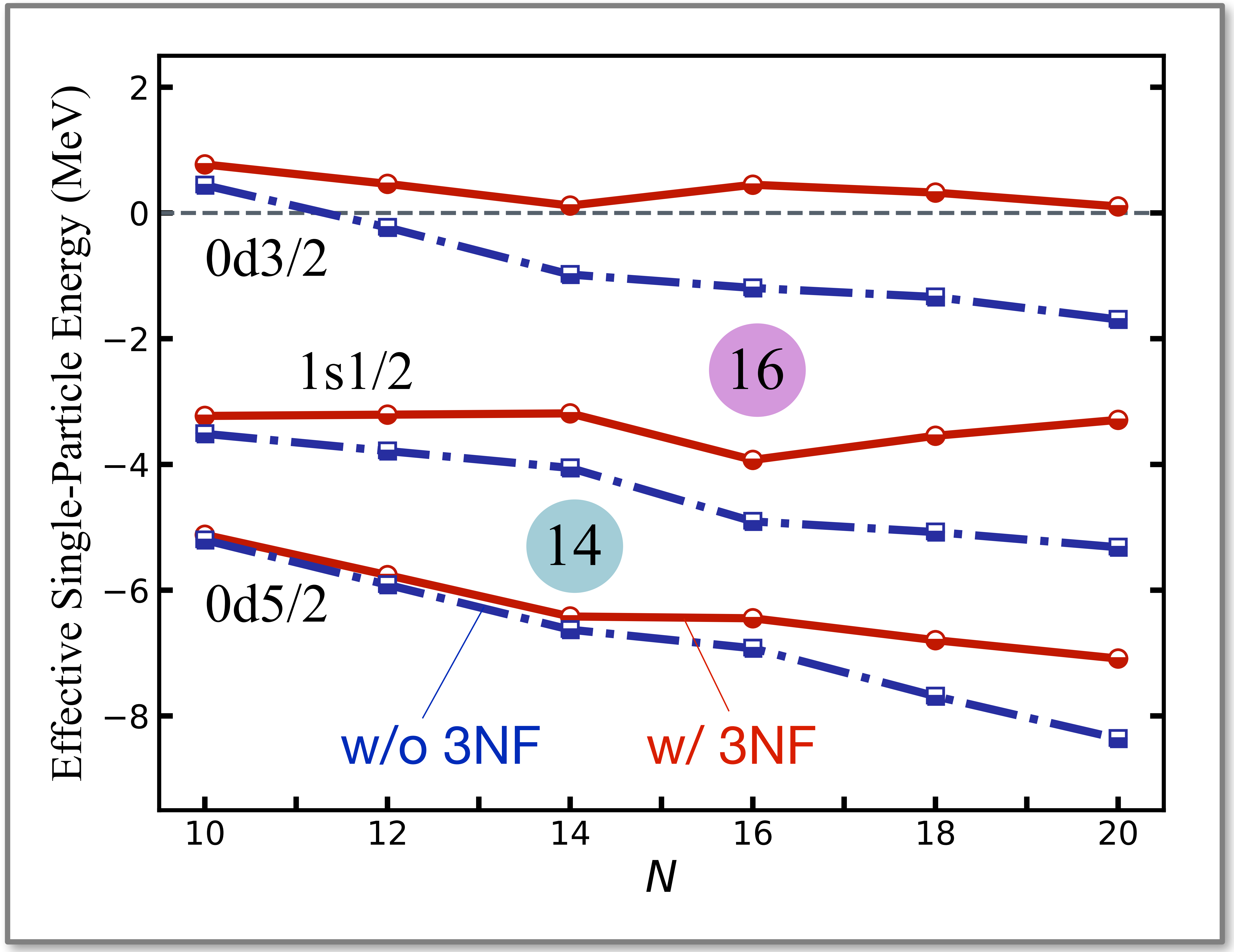
SO splitting & shell structure



Pioneering studies

cf. Fujita & Miyazawa, PTP 17, 366 (1957)
Andō & Bandō, PTP **66**, 227 (1981)
Kohno, PRC 86, 061301(R) (2012)
Uesaka, EPJ Plus **131**, 403 (2016)

e.g.) Oxygen isotopes



Ma +, PLB **802**, 135257 (2020)

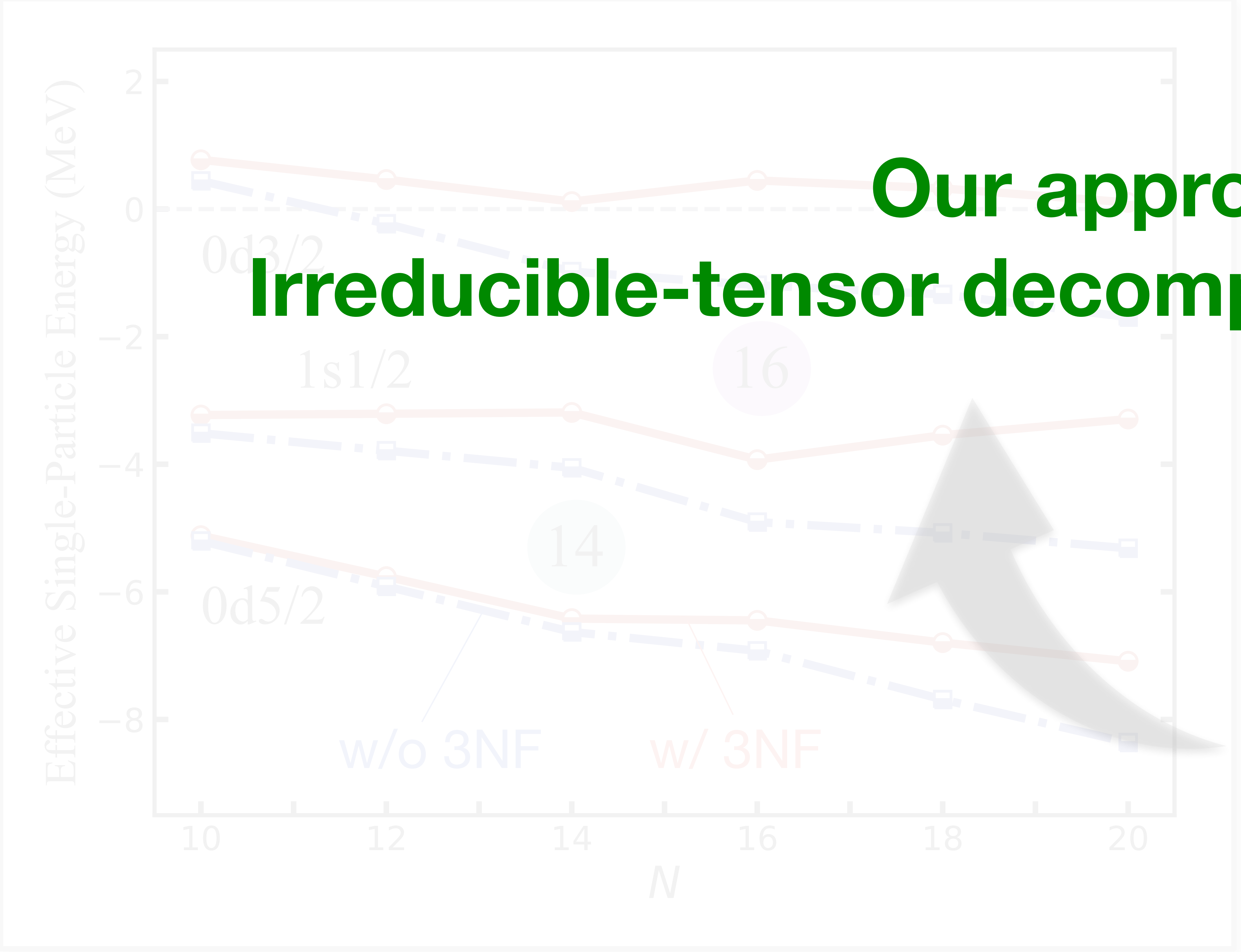
Similar results

- p-shell nuclei
  Fukui +, PRC **98**, 044305 (2018)
- Oxygen isotopes
  Otsuka +, PRL 105, 032501 (2010)
- pf-shell nuclei
  Holt +, JPG **39** 085111 (2012)
  Ma +, PRC **100**, 034324 (2019)

3NF enhances the SO splitting

3NF & SO splitting | Shell-model studies

e.g.) Oxygen isotopes



Ma +, PLB 802, 135257 (2020)

Other studies

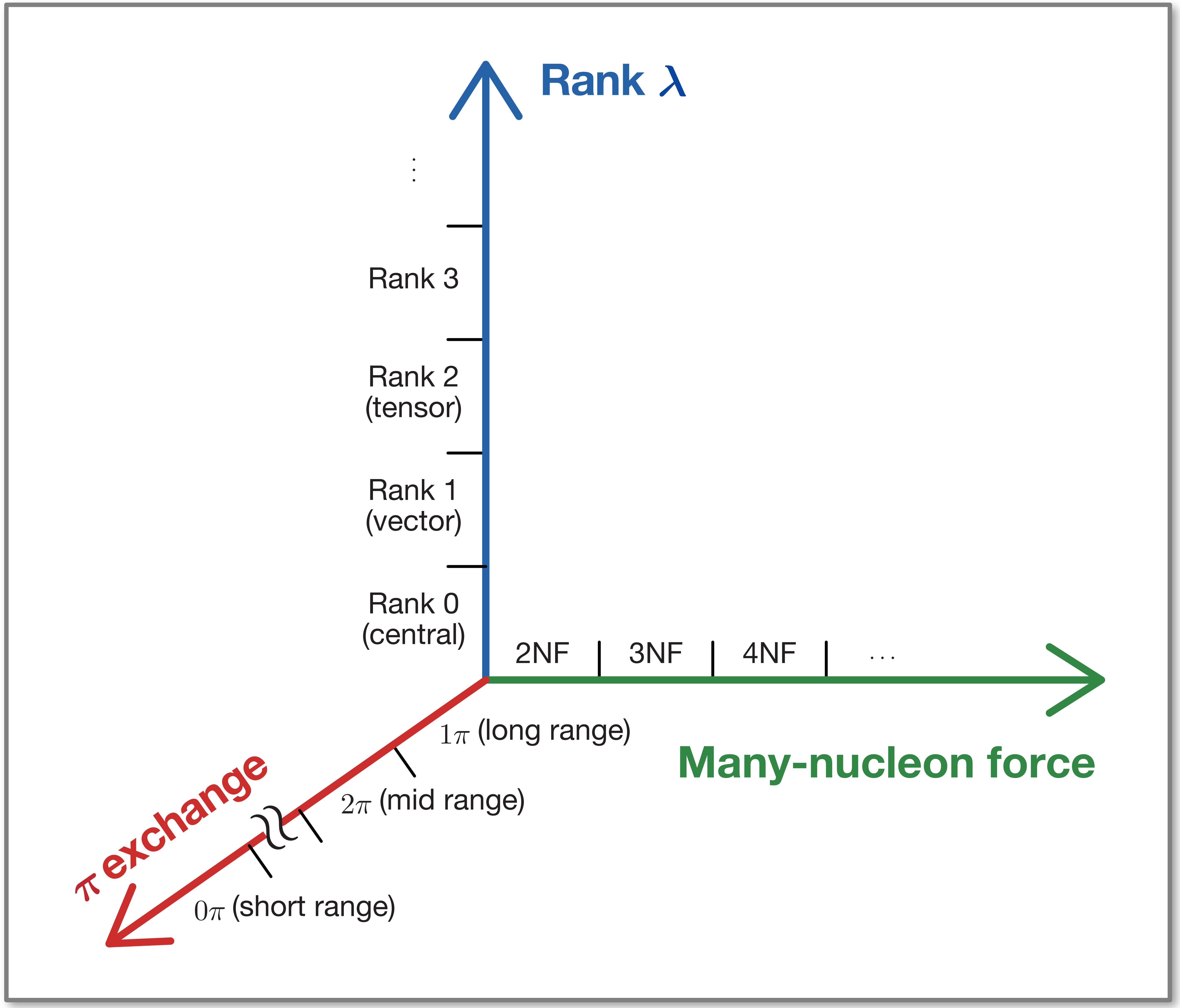
- p-shell nuclei
  Fukui +, PRC 98, 044305 (2018)
- Oxygen isotopes
- pf-shell nuclei
  Holt +, JPG 39 085111 (2012)
  Ma +, PRC 100, 034324 (2019)

Our approach:
Irreducible-tensor decomposition of chiral 3NF

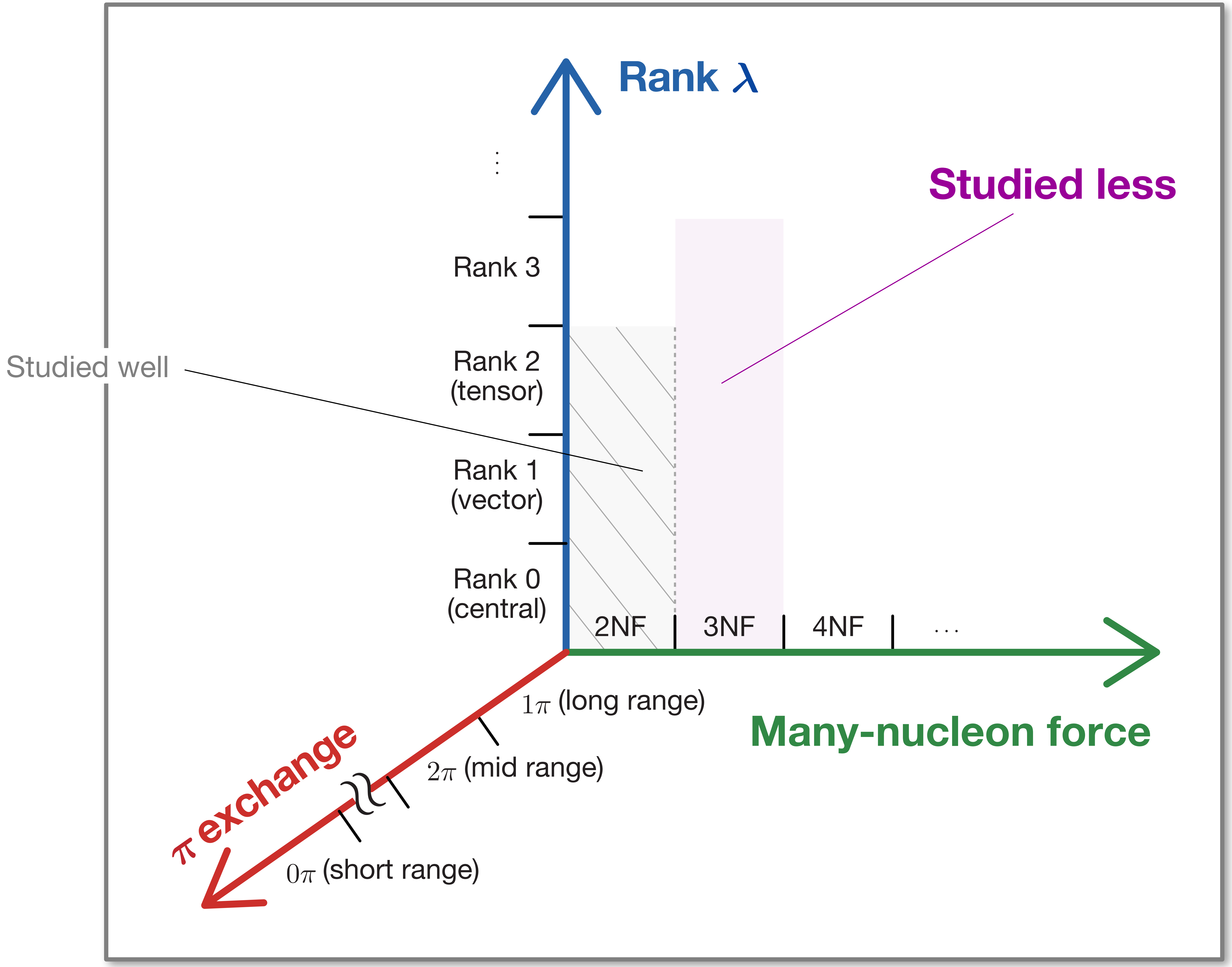
Mechanism?

3NF enhances
the SO splitting

Multidim. point of view | Classification of 3NF by rank of tensors

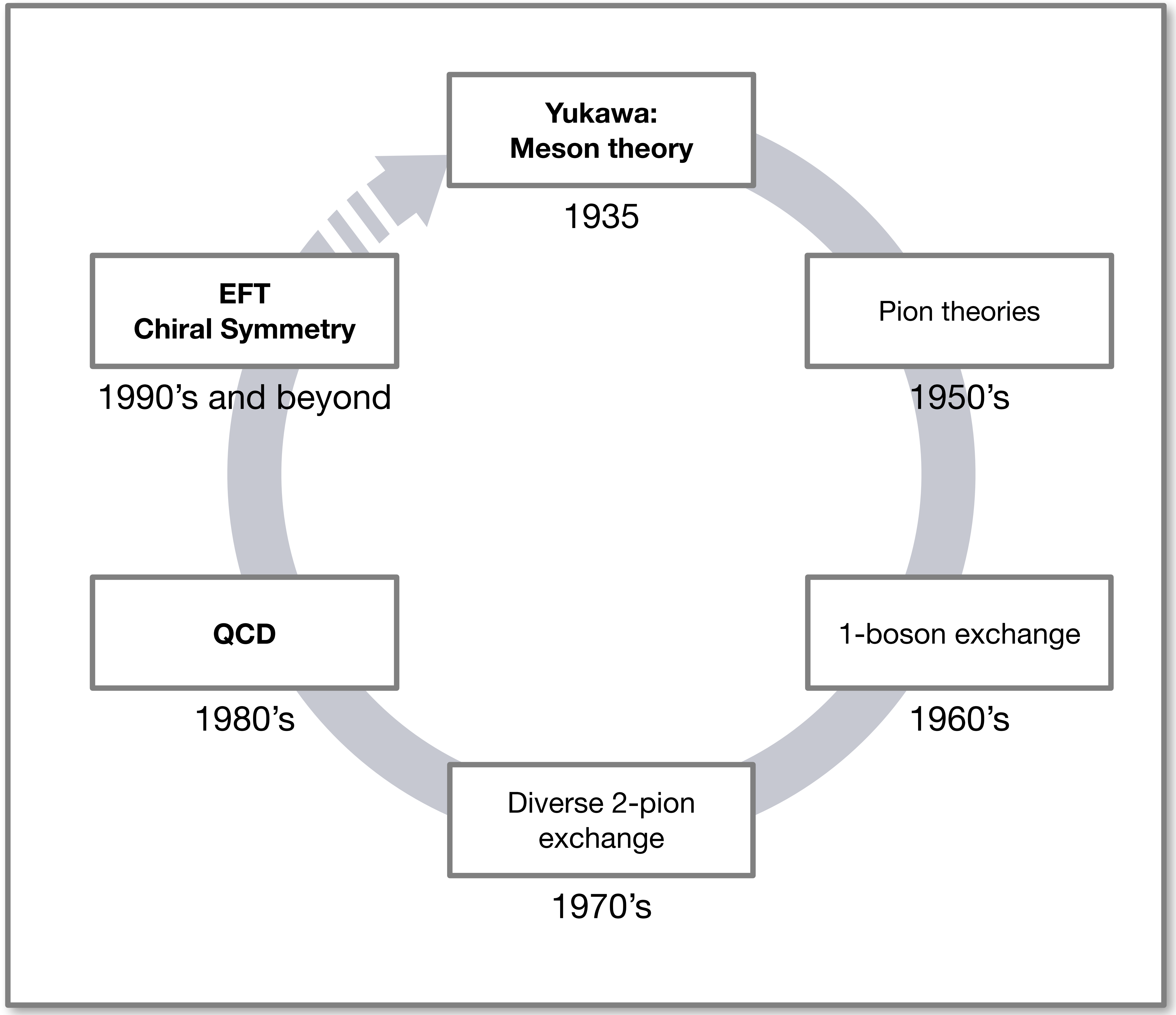


Multidim. point of view | Classification of 3NF by rank of tensors



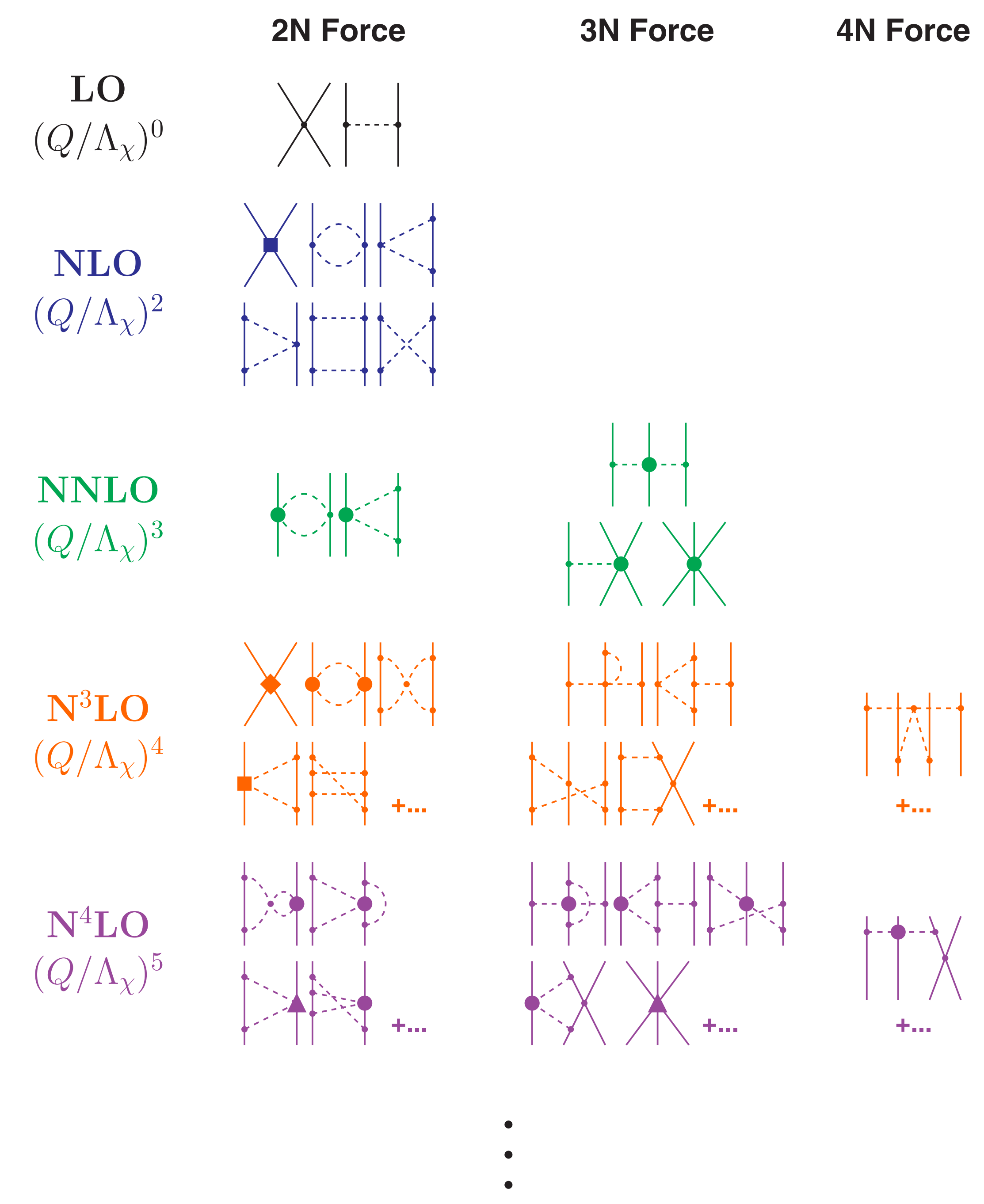
3NF & state-of-the-art theory | Opportune moment to tackle the problem

"the circle of history is closing"



Machleidt & Entem, PR **503**, 1 (2011)

Chiral EFT

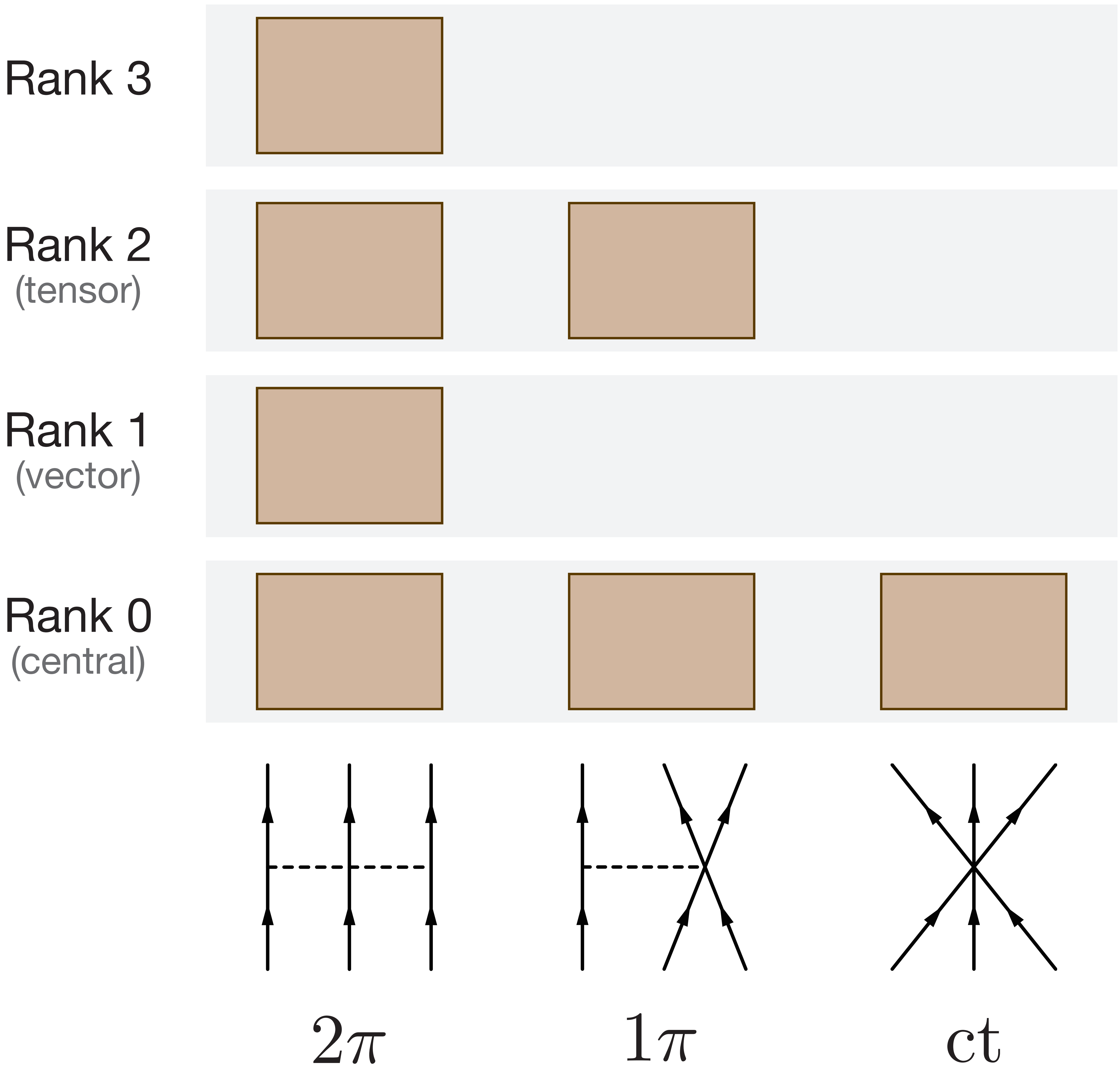


- Hierarchical structure
- Realistic force ($\chi^2 \lesssim 1$)
- Consistent many-body forces

Entem +, PRC **96**, 024004 (2017)

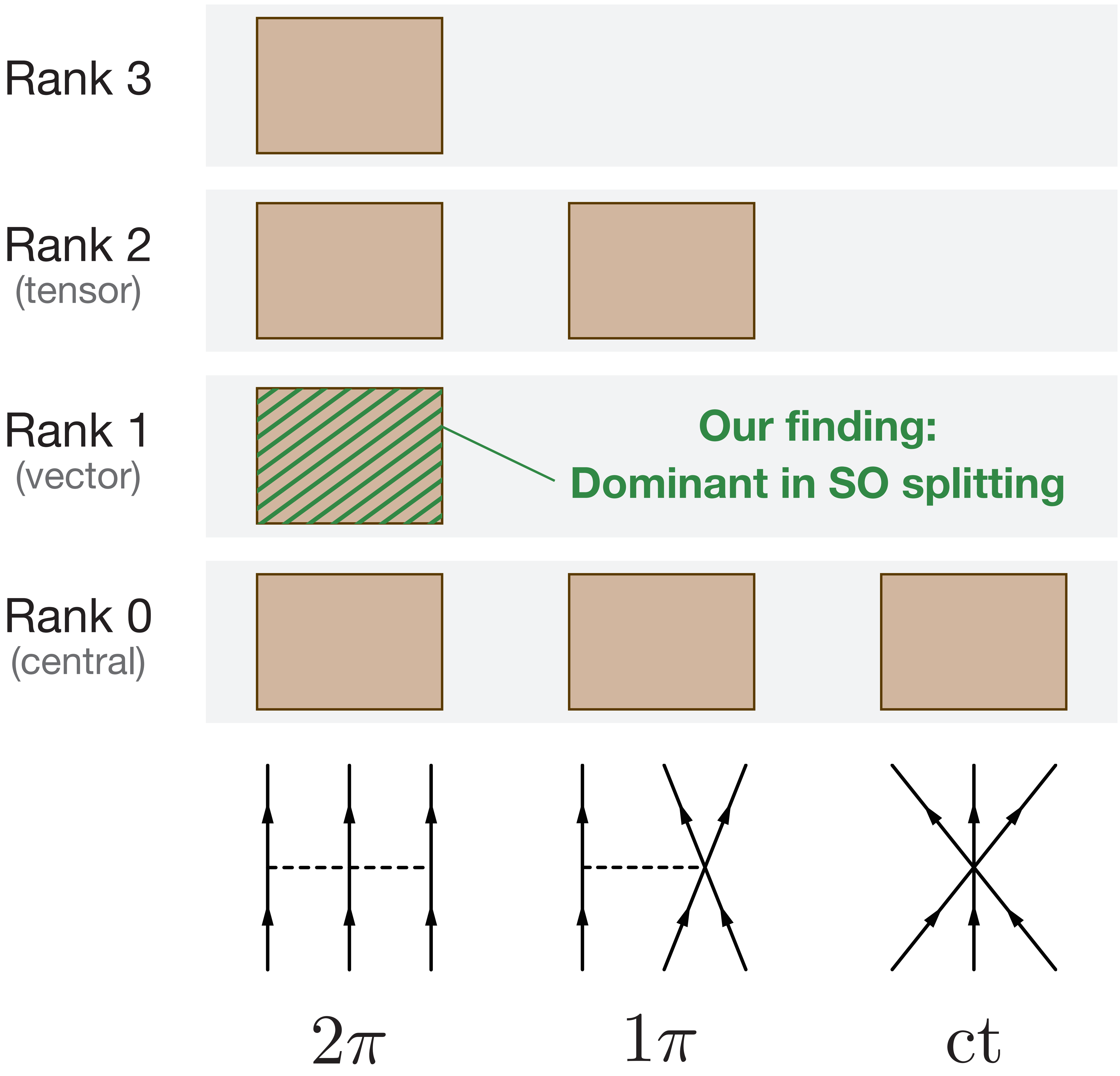
Multidim. point of view | Tensorial structure of chiral-N^{2}LO 3NF

Fukui +, PLB **855**, 138839 (2024)



Multidim. point of view | Tensorial structure of chiral-N²LO 3NF

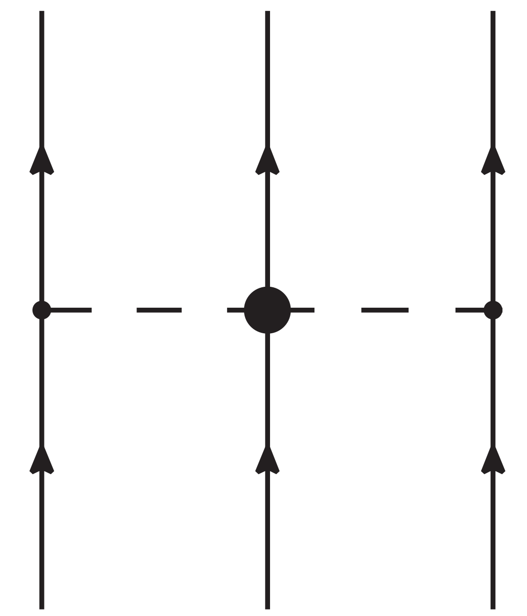
Fukui +, PLB **855**, 138839 (2024)



Irreducible-tensor decomposition | e.g. 2π-c_3 term

Fukui +, PLB **855**, 138839 (2024)

$$v_{3N}^{(c_3)} = \frac{g_A^2 c_3}{4 f_\pi^4} \sum_{i \neq j \neq k} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \frac{q_i^2 q_j^2}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} \left[\mathcal{O}_{ij}^{(0)} + \mathcal{O}_{ij}^{(1)} + \mathcal{O}_{ij}^{(2)} \right]$$



$$\mathcal{O}_{ij}^{(\lambda)} = \begin{cases} \frac{1}{3} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) (\hat{\boldsymbol{q}}_i \cdot \hat{\boldsymbol{q}}_j)^2 & (\lambda = 0), \\ \frac{1}{2} (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \cdot (\hat{\boldsymbol{q}}_i \times \hat{\boldsymbol{q}}_j) (\hat{\boldsymbol{q}}_i \cdot \hat{\boldsymbol{q}}_j) & (\lambda = 1), \\ \frac{1}{3} (\hat{\boldsymbol{q}}_i \cdot \hat{\boldsymbol{q}}_j) \mathcal{T}_{ij} (\hat{\boldsymbol{q}}_i, \hat{\boldsymbol{q}}_j) & (\lambda = 2). \end{cases}$$

Rank 0 (central)

Rank 1 (antisymmetric SO)

Rank 2 (tensor)

Generalized tensor operator

$$\mathcal{T}_{12}(\hat{\boldsymbol{q}}, \hat{\boldsymbol{q}}') = \frac{3}{2} \left[(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{q}}) (\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{q}}') + (\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{q}}) (\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{q}}') \right] - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) (\hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{q}}')$$

$$\mathcal{T}_{12}(\hat{\boldsymbol{q}}, \hat{\boldsymbol{q}}) = \mathcal{S}_{12}(\hat{\boldsymbol{q}})$$

Fujiwara +, PTP **97**, 587 (1997)

Shell-model calculations | Numerical details

Low-energy constants ($\Lambda = 500$ MeV)

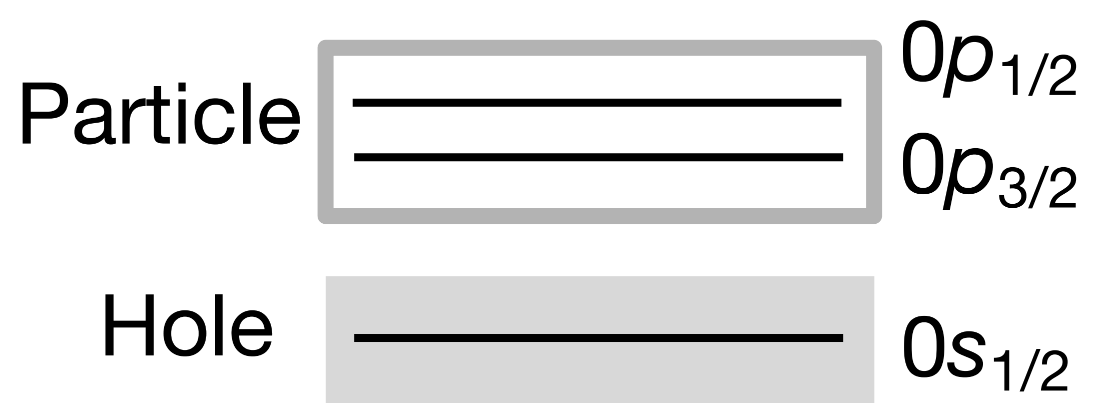
2NF (N³LO): Determined from
NN scattering up to 300 MeV

Entem & Machleidt, PRC **68**, 041001(R) (2003)

3NF (N²LO): Determined from
^{3}H- and ^{3}He binding energies

Navrátil +, PRL **99**, 042501 (2007)

Model space (p-shell)



$$\hbar\omega = 19 \text{ MeV}$$

Many-body perturbation theory

$$H^{(1B)} + H^{(2B)} \to H_{\text{eff}}$$

1-body and 2-body:
Up to 3rd-order folded-diagram expansion

Coraggio + AP **327**, 2125 (2012)

\+

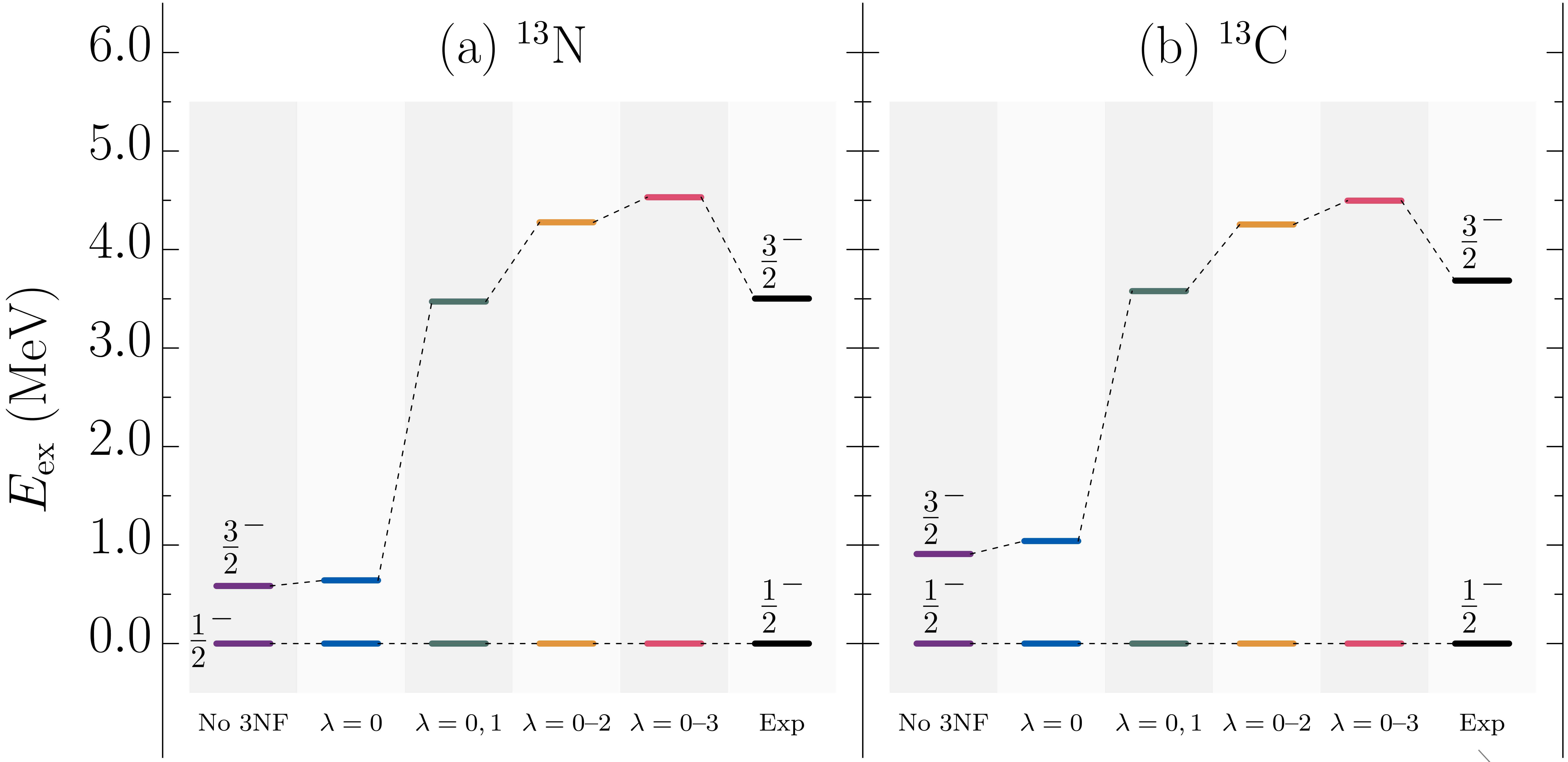
Normal-order approx.

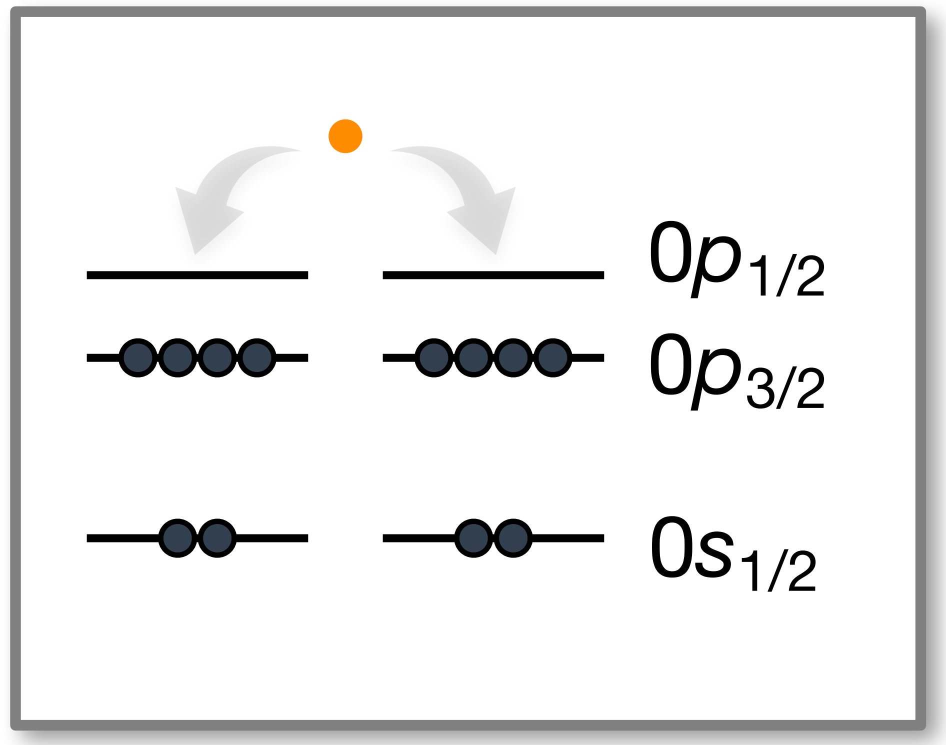
$$H_{3NF} \approx H_{3NF}^{(1B)} + H_{3NF}^{(2B)}$$

3NF:
Up to 1st-order

Roth +, PRL **109** , 052501 (2012)

Shell-model results | Spectra of ^{13}N and ^{13}C





N³LO 2NF
Entem & Machleidt, PRC **68**, 041001(R) (2003)

N²LO 3NF
Navrátil +, PRL **99**, 042501 (2007)

Numerical detail
Fukui +, PLB **855**, 138839 (2024)

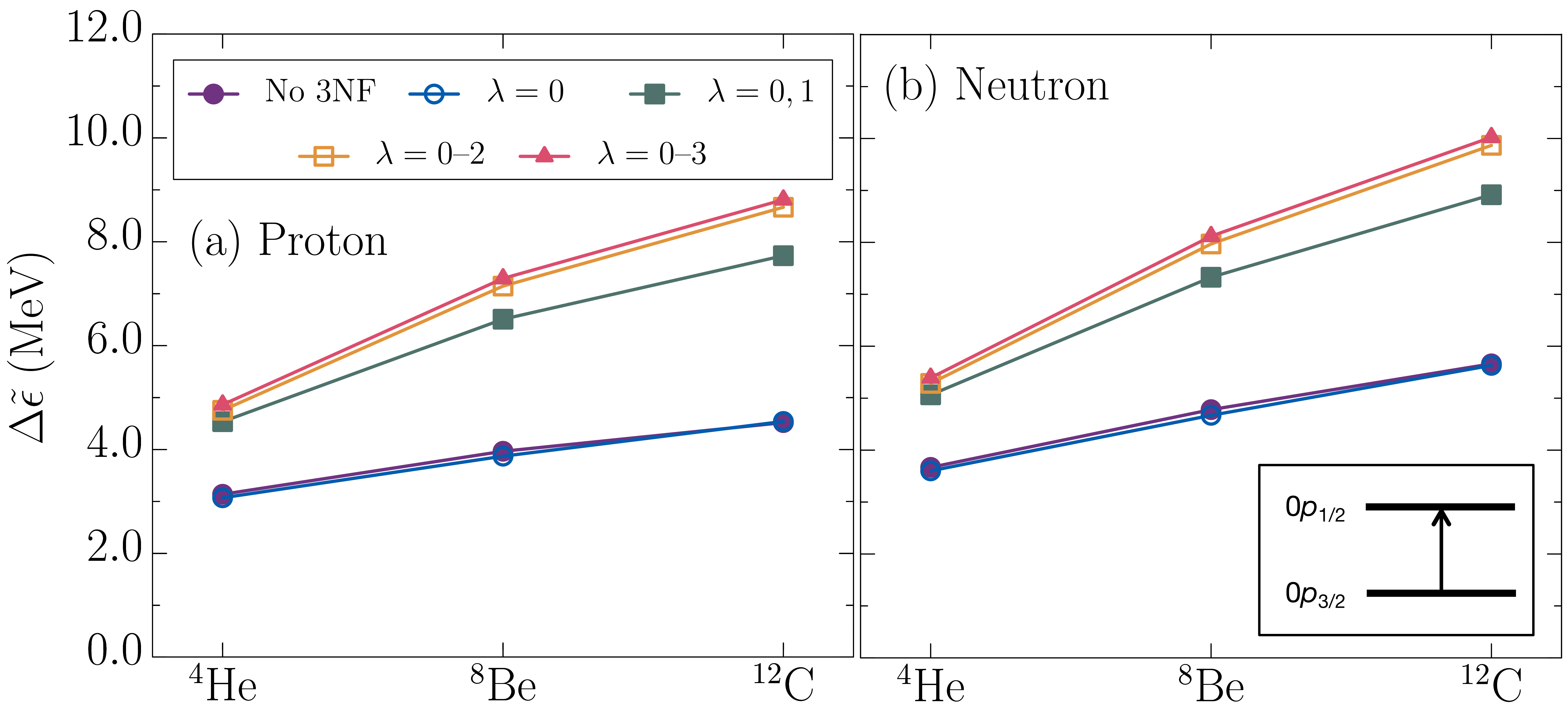
Ajzenberg-Selove (ENSDF), NPA **523**, 1 (1991)

- $3/2^-$ energies: Enhanced by noncentral 3NF
- Rank-1: Dominant contribution
- Experimental $3/2^-$: Not perfectly consistent w/ calc.

Shell-model results | Effective single-particle energies

Evolution of ESPEs ($N = Z$ nuclei)

Fukui +, PLB **855**, 138839 (2024)



- Whole 3NF: ~Doubling SO splitting (^{12}C)

Consistent

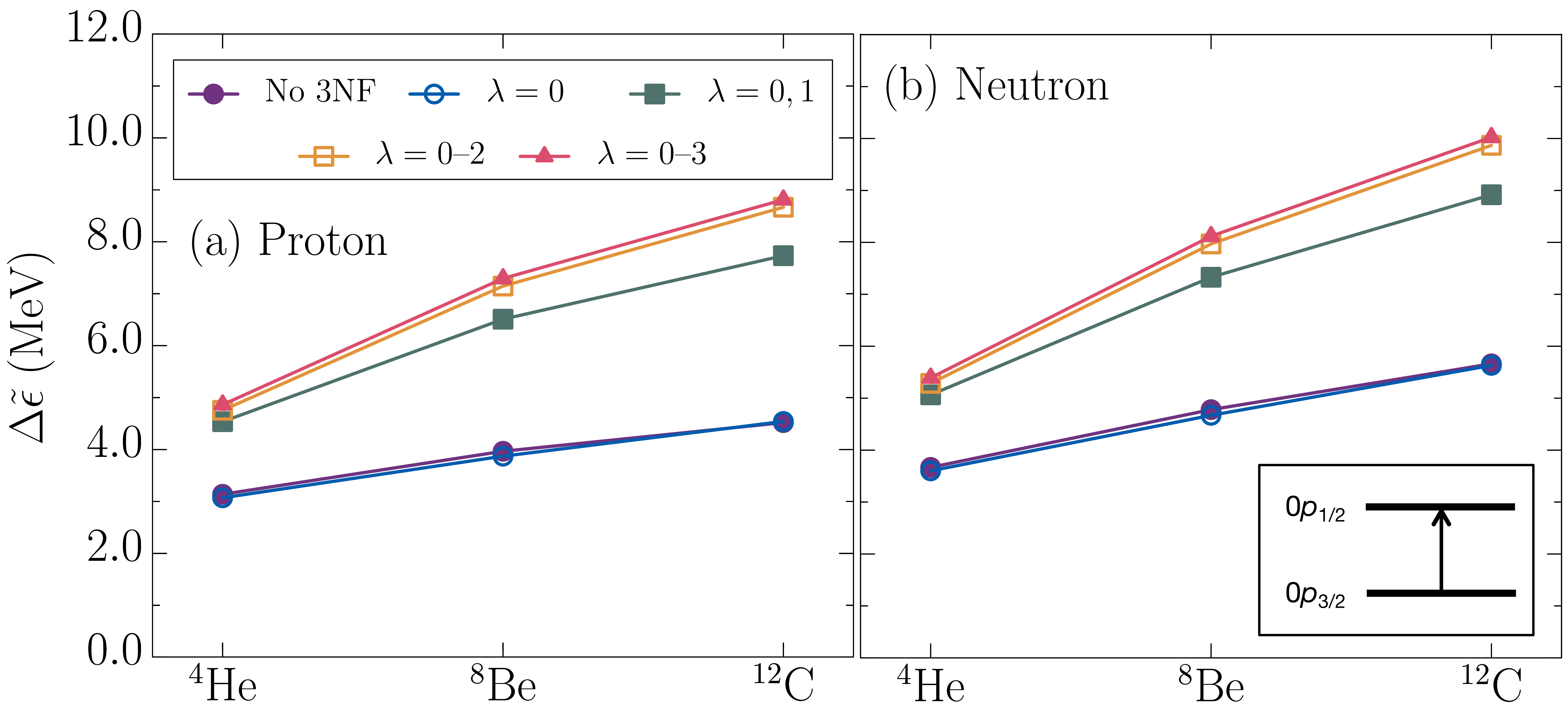
Pieper & Pandharipande

^{15}N-SO splitting: Half from 3NF (Urbana-VII 3NF)

Pieper & Pandharipande, PRL **70**, 2541 (1993)
Schiavilla +, NPA **449**, 219 (1986)

Evolution of ESPEs (*N* = *Z* nuclei)

Fukui +, PLB **855**, 138839 (2024)



- Whole 3NF: ~Doubling SO splitting (^{12}C)
- Rnak-1: ~75% contribution (^{8}Be, ^{12}C)
- Rank-2: ~20% contribution (^{8}Be, ^{12}C)

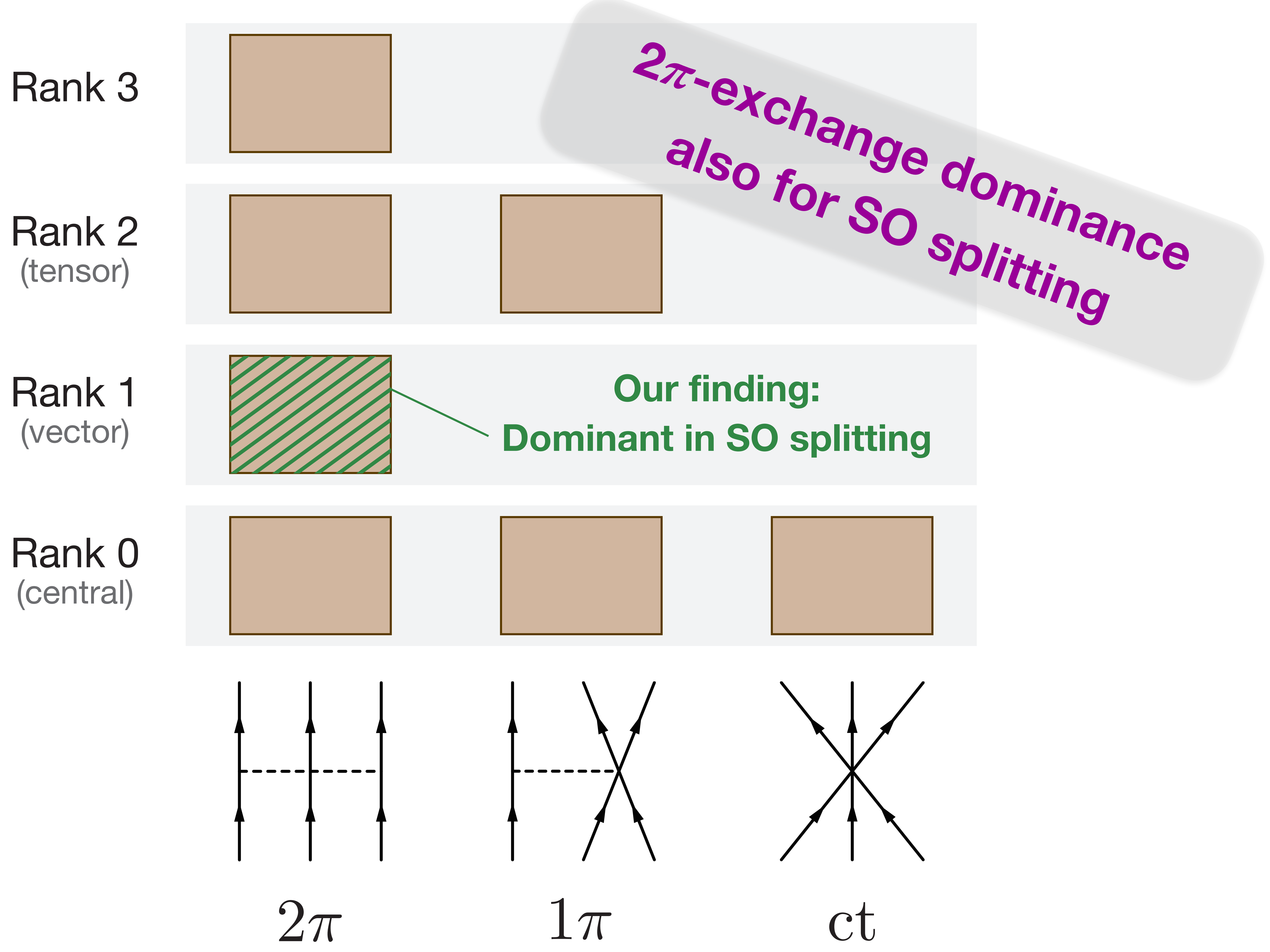
Consistent

Pieper & Pandharipande

^{15}N-SO splitting: Half from 3NF (Urbana-VII 3NF)

Pieper & Pandharipande, PRL **70**, 2541 (1993)
Schiavilla +, NPA **449**, 219 (1986)

2π-exchange dominance | Rank-1 component exclusively from 2π

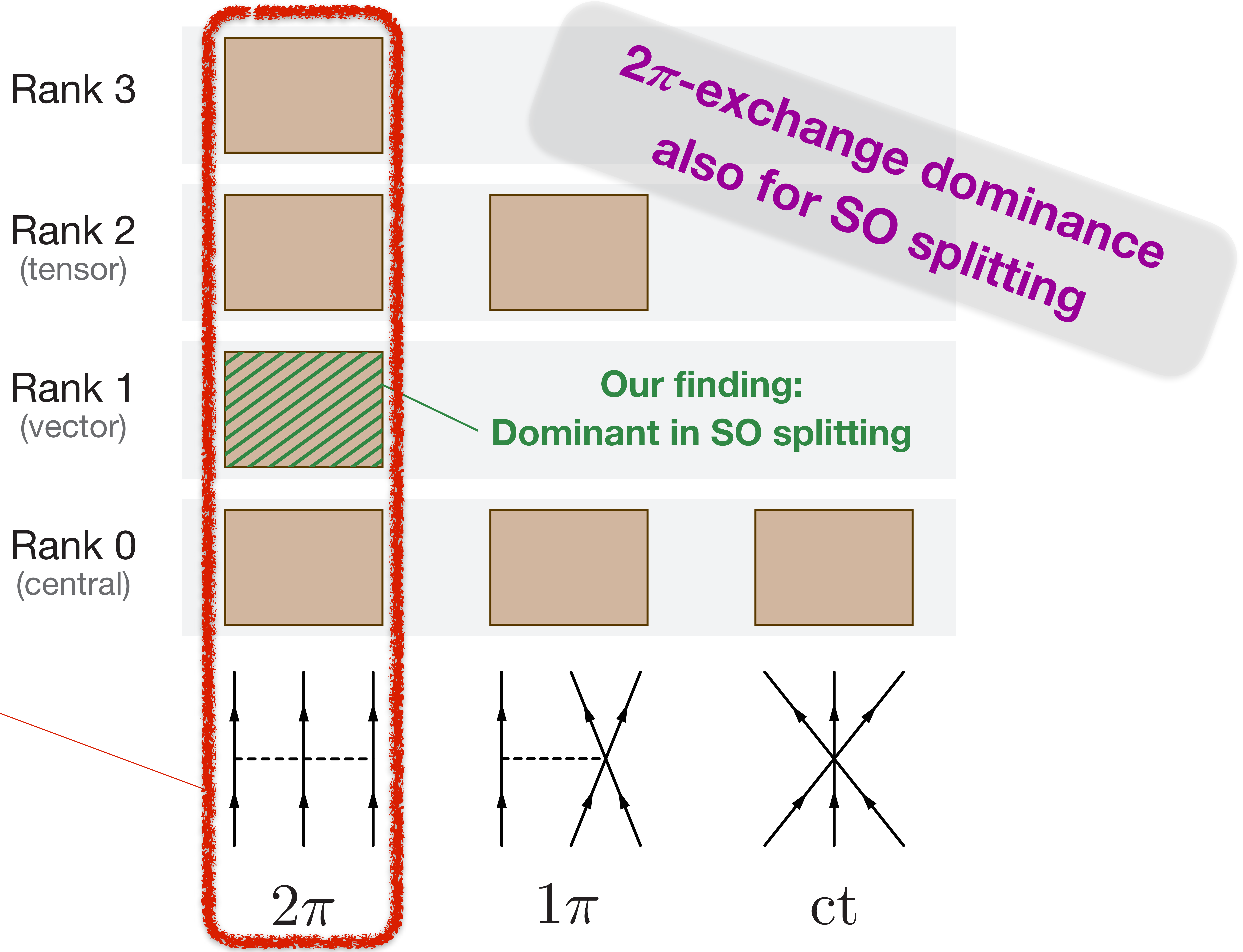


2π-exchange dominance | Rank-1 component exclusively from 2π

$\pi - N$ LECs:
Well constrained
by Roy-Steiner equation analysis

Hoferichter +, PRL **115**, 192301 (2015)

Our conclusions:
Probably independent of contact LECs



One-body SO potential from rank-1 3NF

$$\mathcal{O}_{ij}^{(1)} = \frac{1}{2}\left(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j\right) \cdot \left(\hat{\boldsymbol{q}}_i \times \hat{\boldsymbol{q}}_j\right)\left(\hat{\boldsymbol{q}}_i \cdot \hat{\boldsymbol{q}}_j\right)$$

Andō & Bandō, PTP **66**, 227 (1981)

$$\tilde{M}^{\mathrm{Sx}}_{\mathrm{TBF}}(a) \simeq \langle \phi_a | B_p \rho(r)(1/r)(d\rho(r)/dr)\,\boldsymbol{l}\cdot\boldsymbol{\sigma} | \phi_a \rangle$$

One-body SO potential from rank-1 3NF

Andō & Bandō

Rank-1 3NF
(Fujita—Miyazawa 3NF/
Tucson—Melbourne 3NF)
and SO splitting

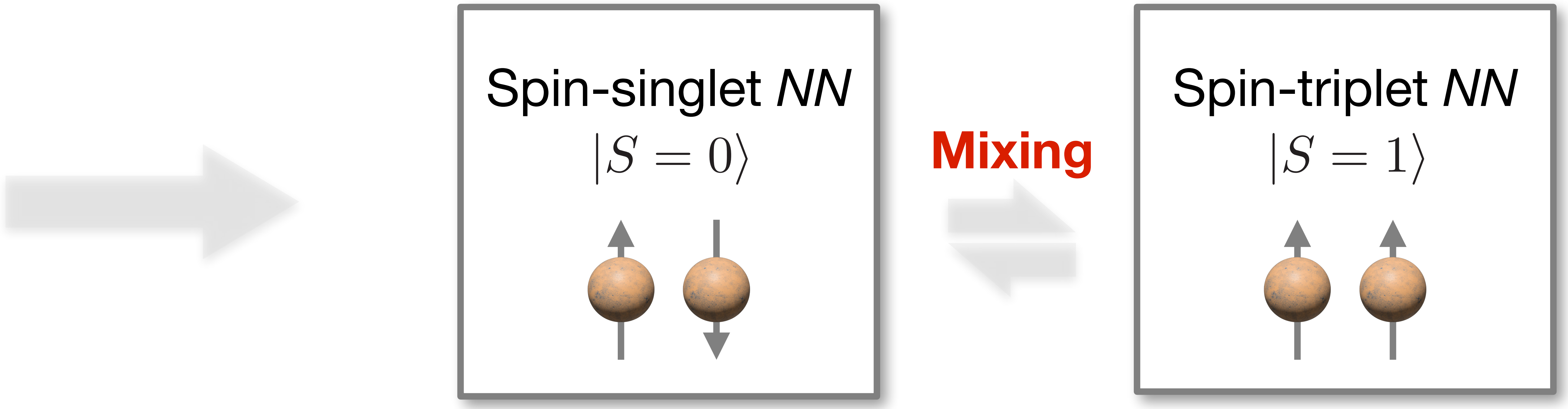
Fujita & Miyazawa, PTP **17**, 360 (1957)
Coon, NPA **317**, 242 (1979)

Rank-1 3NF

$$\mathcal{O}_{ij}^{(1)} = \frac{1}{2} \left(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j\right) \cdot \left(\hat{\boldsymbol{q}}_i \times \hat{\boldsymbol{q}}_j\right) \left(\hat{\boldsymbol{q}}_i \cdot \hat{\boldsymbol{q}}_j\right)$$

Antisymmetric-SO 3NF!

(Spin conservation locally violated)



3NF & single-triplet mixing | Antisymmetric 3NF

Rank-1 3NF

$$\mathcal{O}_{ij}^{(1)} = \frac{1}{2} (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \cdot (\hat{\boldsymbol{q}}_i \times \hat{\boldsymbol{q}}_j) (\hat{\boldsymbol{q}}_i \cdot \hat{\boldsymbol{q}}_j)$$

Antisymmetric-SO 3NF!

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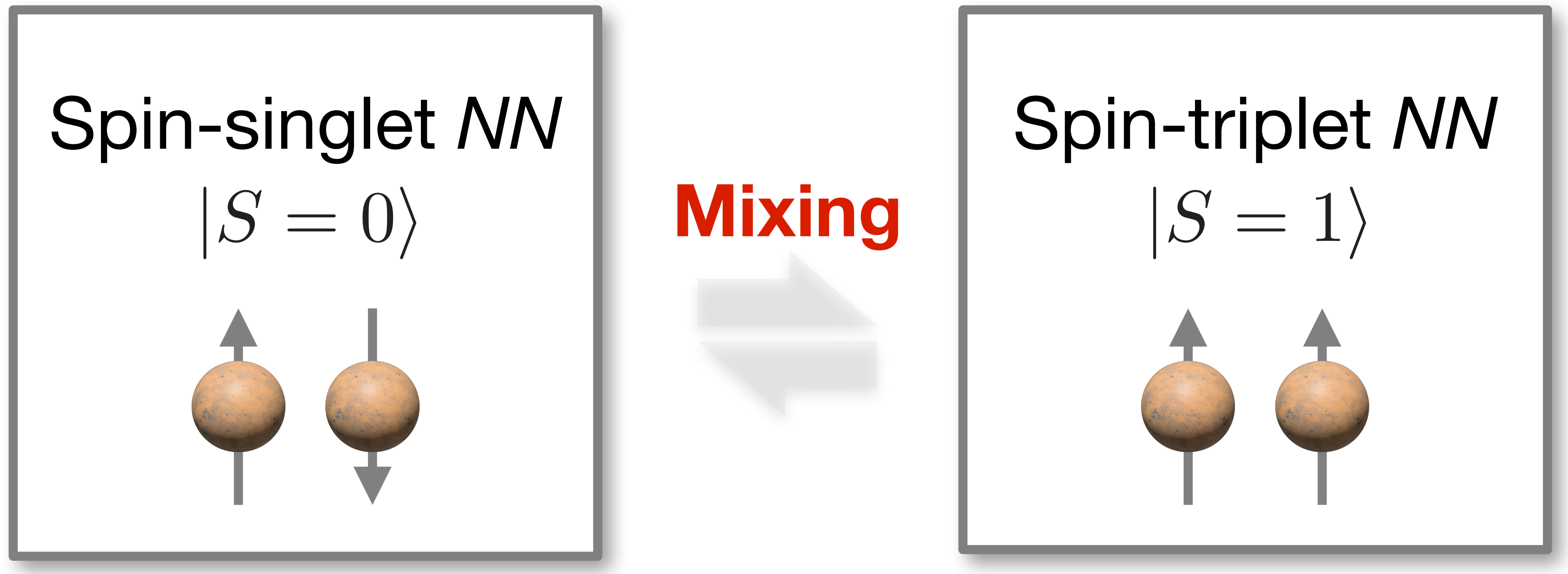
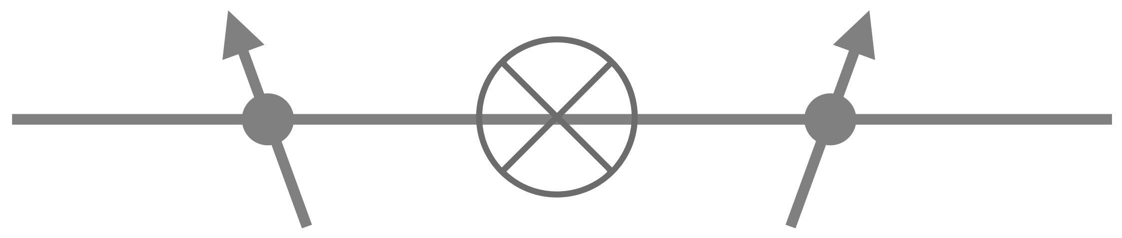


Analogy

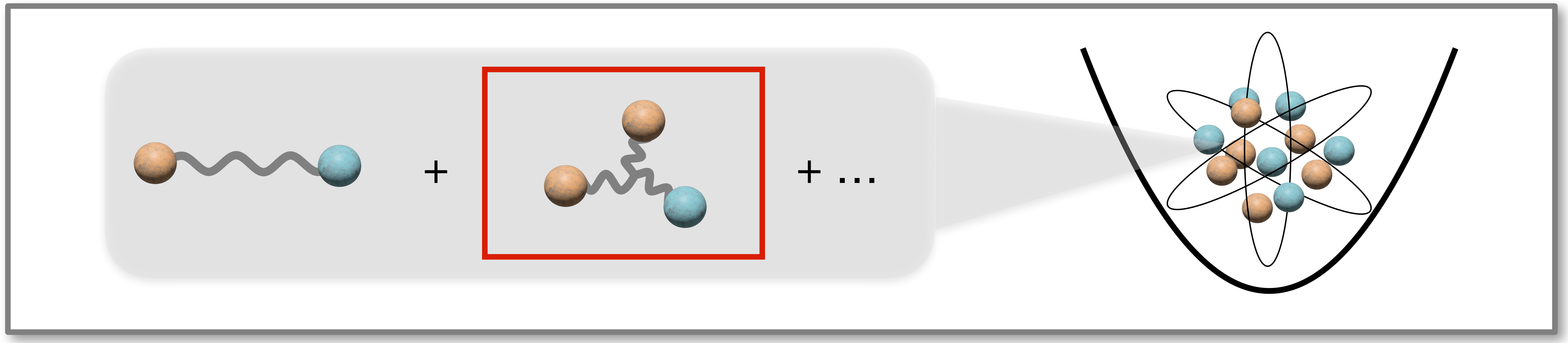
Spin canting of magnetic ions
by Dzyaloshinsky—Moriya interaction



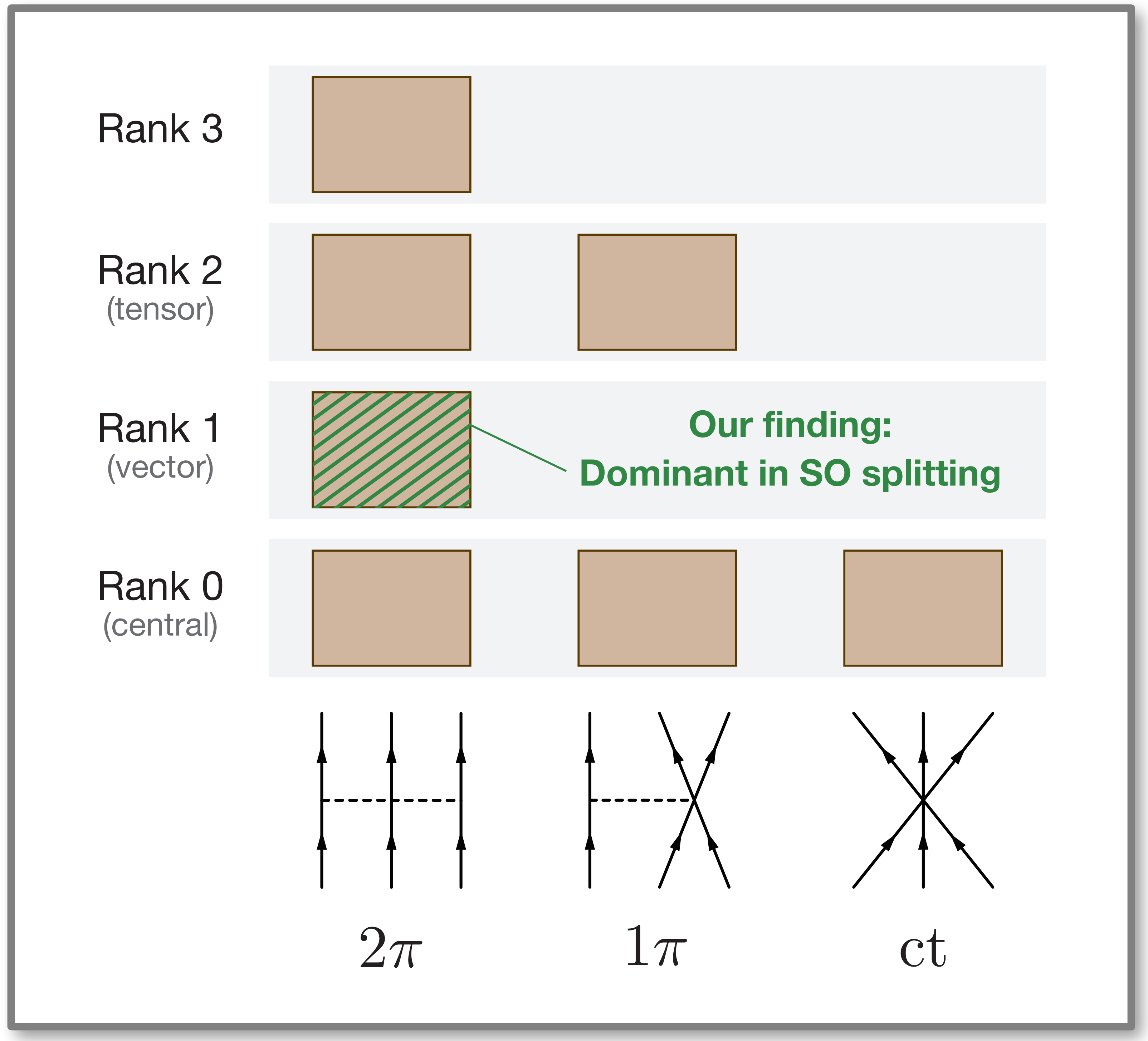
Dzyaloshinsky, JPCS 4, **241** (1958)
Moriya, PRL **4**, 228 (1960)
Moriya, PR **120**, 91 (1960)

Summary | 3NF & SO splitting

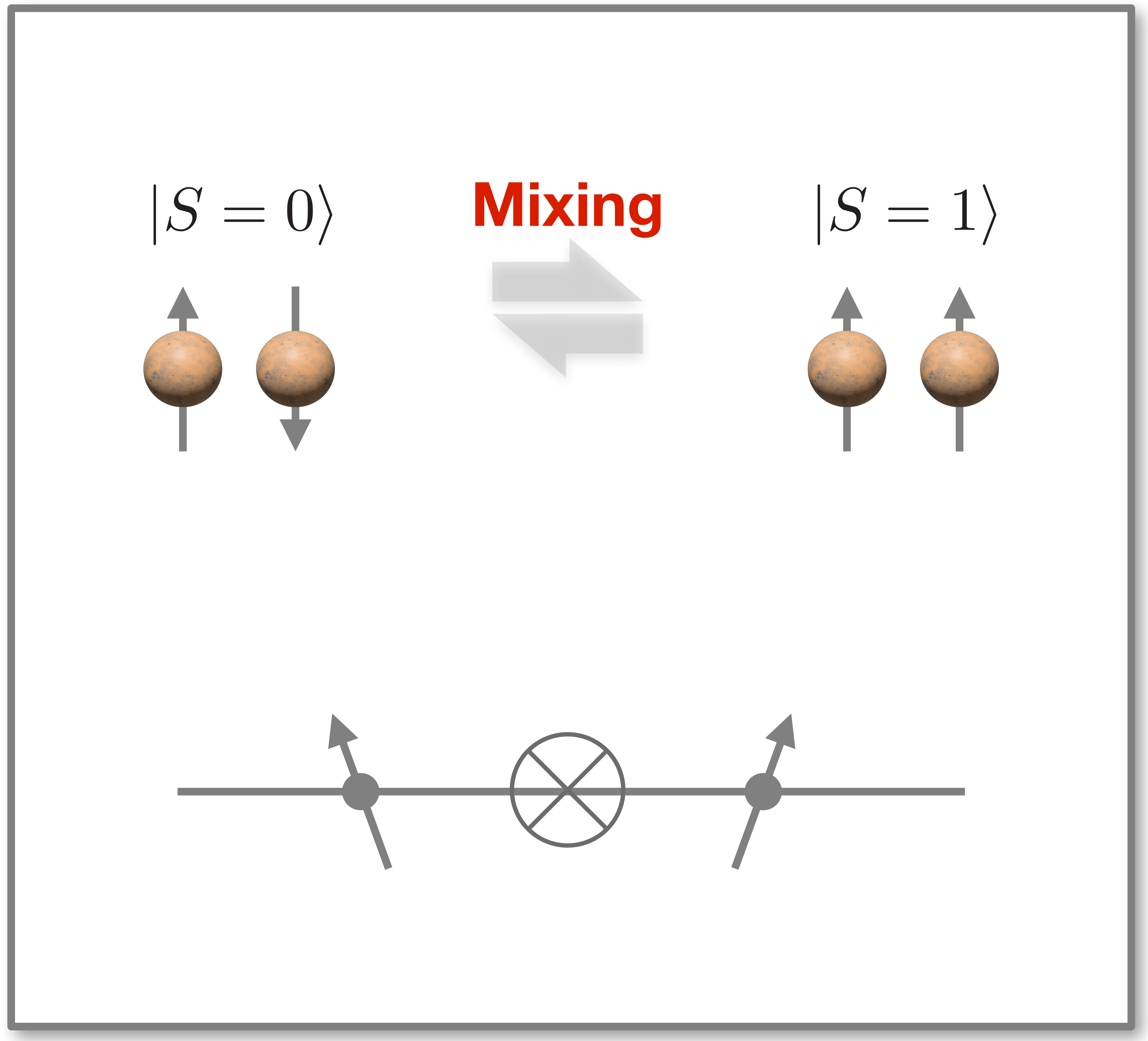
Chiral EFT + Shell model



Main finding



Singlet-triplet mixing & antisym. 3NF



Future

- Heavier nuclei: Rank-2 3NF becomes relevant?
- *2n* correlation, entanglement, and 3NF