Halo Nuclei and Multineutron Correlations

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Universality of Quantum Systems: From Cold Atoms, Nuclei, to Hadron Physics,
Tohoku University, Sep. 4-5, 2024

Outline

- Halo nuclei and Halo EFT
- Efimov physics in halo nuclei
  Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023)
- Nuclear reactions with neutrons
  HWH, Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021)
- Summary and Outlook

Halo Nuclei

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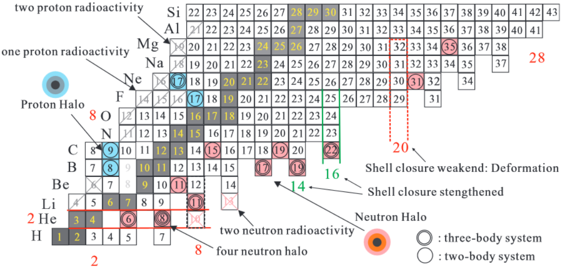
- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to “nucleon drip line” $\longrightarrow$ scale separation $\longrightarrow$ EFT



C.-B. Moon, Wikimedia Commons

- EFT for halo nuclei

  (Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003; ...)

Halo Effective Field Theory

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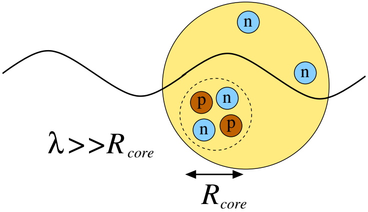
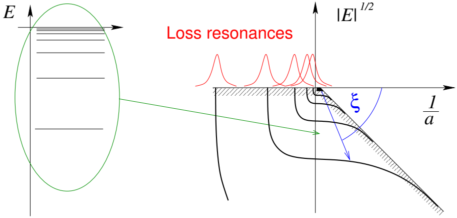
- Separation of scales:
  $1/k = \lambda \gg R_{core}$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR_{core}
  $\longrightarrow$ contact interactions



- Short-distance physics not resolved
  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly if present
- Systematic, model independent $\Longrightarrow$ universal properties
- Nucleon degrees of freedom: $\Longrightarrow$ pionless EFT
- Exploit cluster substructures $\Longrightarrow$ Halo EFT

Efimov Effect $\Longleftrightarrow$ Limit Cycle

- At least two pairs with resonant interactions $\Longrightarrow$ universal spectrum of three-body states (Efimov, 1970)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \xrightarrow{1/a\to 0} \left(e^{\pi/s_0}\right)^2 = 515.035...$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances

Efimov Physics in Halo Nuclei

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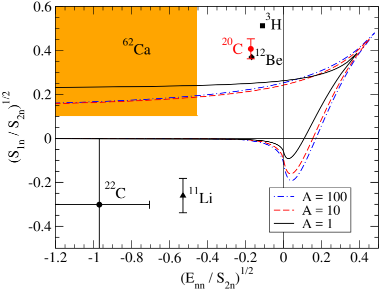
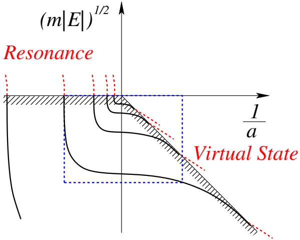
- Efimov effect in 2*n* halo nuclei? (Fedorov, Jensen, Riisager, 1994)

  $\Longrightarrow$ excited states obeying scaling relations
- Correlation plot: $E_{nn} \leftrightarrow S_{1n}$ (Amorin, Frederico, Tomio, 1997)





HWH, Ji, Phillips, JPG **44**, 103002 (2017)

- Alternative ways to observe Efimov physics in 2*n* halo nuclei?

Halo EFT

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- LO Halo EFT for two-neutron halos (resonant neutron-core interaction)

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

$$\mathcal{L}_1 = \boldsymbol{n}^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) \boldsymbol{n} + c^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_c} \right) c$$

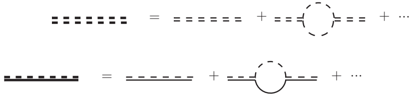
$$\begin{aligned}\mathcal{L}_2 = \; & s^\dagger \left[\Delta_s - \left(i\partial_0 + \frac{\nabla^2}{4m_n} \right) \right] s + \sigma_i^\dagger \left[\Delta_\sigma - \left(i\partial_0 + \frac{\nabla^2}{2m_\sigma} \right) \right] \sigma_i \\ & - g_s C^{00}_{1/2\alpha,1/2\beta} \left[s^\dagger n_\alpha n_\beta + \text{H.c.} \right] - g_\sigma \left[\sigma_i^\dagger n_i c + \text{H.c.} \right]\end{aligned}$$

$$\mathcal{L}_3 = g_s^2 D_0 (sc)^\dagger (sc)$$

- Dimer propagators

s :

σ :



Neutron Scattering

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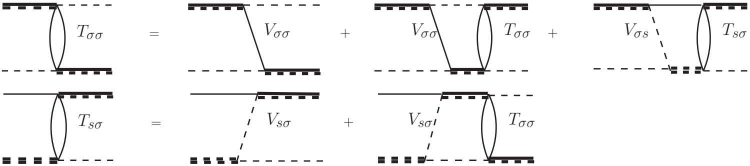
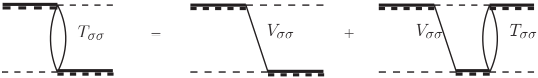
- Neutron scattering off $J^P = 1/2^+$ one-neutron halos (^{11}Be, ^{15}C, ^{19}C)
- $J = 0$ channel (three-body force not shown)



- $J = 1$ channel (no three-body force)



Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023)

$J = 1$ channel

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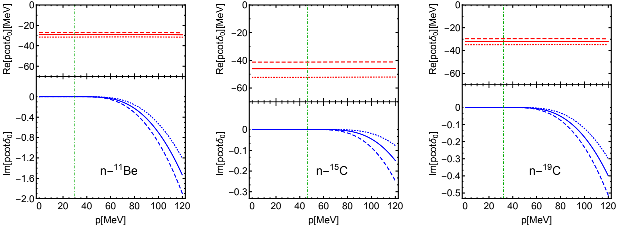
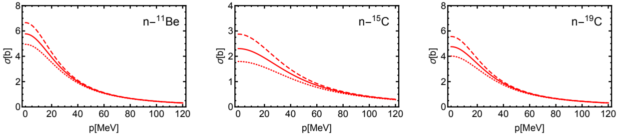
- *S*-wave scattering amplitude (Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023))

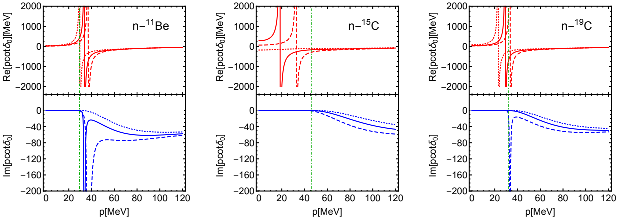


- Total *S*-wave cross section

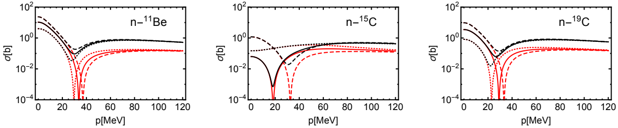


$J = 0$ **channel**

- *S*-wave scattering amplitude (Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023))

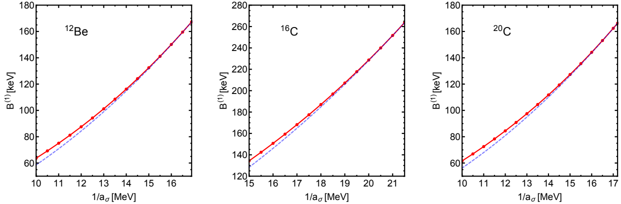


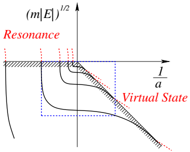
- Total cross section



Efimov physics in halo nuclei?

- Alternative ways to see Efimov physics in halo nuclei?
  - Pole in $p\cot\delta_0$ due to virtual Efimov state close to threshold
  - Real excited state appears for $1/a_\sigma$ smaller than some critical value





(Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023))

- Pole position directly correlated with virtual state energy

$\Longrightarrow$ pole position determined by Efimov physics

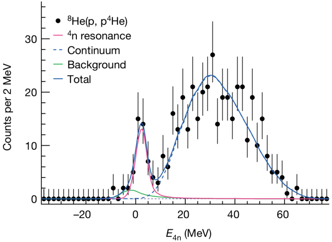
- Also present in deuteron-halo scattering?

Multineutron Correlations

- Multi-neutron systems: long history
- Most recently: resonance-like structure in $^{8}\mathrm{He}(p, p\alpha)4n$

  M. Duer et al., Nature **606** (2022) 678

  $E_R = 2.37 \pm 0.38(st) \pm 0.44(sy)$ MeV
  $\Gamma_R = 1.75 \pm 0.22(st) \pm 0.30(sy)$ MeV



- Genuine resonance or other effect?
- No resonance but threshold enhancement of density of states

  Higgins, Greene, Kievsky, Viviani, Phys. Rev. Lett. **125**, 052501 (2020)
- Dineutron correlations can produce peak

  Lazauskas, Hiyama, Carbonell, Phys. Rev. Lett. **130**, 102501 (2023)
- ...

Reactions with neutrons

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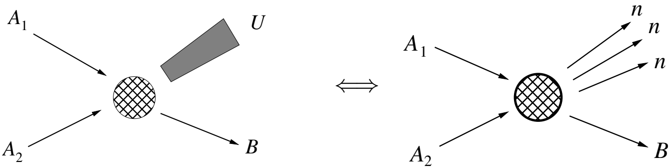
- High-energy nuclear reaction with multi-neutron final state
  (HWH, Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021))



$$E_{\text{kin}} = (M_{A_1} + M_{A_2} - M_B - M_{\mathcal{U}})c^2 + \frac{p_{A_1}^2}{2M_{A_1}} + \frac{p_{A_2}^2}{2M_{A_2}} = E_B + E_{\mathcal{U}}$$

- Assumption: energy scale of primary reaction $\gg E_{\mathcal{U}} - \frac{\boldsymbol{p}^2}{2M_{\mathcal{U}}} = E_n^{cms}$
- Factorization: $\dfrac{d\sigma}{dE} \sim |\mathcal{M}_{primary}|^2 \, \mathrm{Im}\, G_{\mathcal{U}}(E_{\mathcal{U}}, \boldsymbol{p})$
- Reproduces Watson-Migdal treatment of FSI for $2n$
  (Watson, Phys. Rev. **88**, 1163 (1952); Migdal, Sov. Phys. JETP **1**, 2 (1955))

Unitary limit

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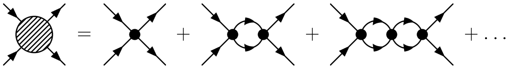
- Spin-1/2 Fermions with zero-range interactions ($|a| \gg r_e$)



- Renormalization group equation: $$\Lambda \frac{d}{d\Lambda}\tilde{g}_2 = \tilde{g}_2(1+\tilde{g}_2)$$
- Two fixed points:

  $-\ \tilde{g}_2 = 0 \ \Leftrightarrow\ a = 0 \quad \Rightarrow$ no interaction, free particles

  $-\ \tilde{g}_2 = -1 \ \Leftrightarrow\ 1/a = 0 \quad \Rightarrow$ unitary limit

  $\Rightarrow$ conformal/Schrödinger symmetry

  (Mehen, Stewart, Wise, PLB **474**, 145 (2000); Nishida, Son, PRD **76**, 086004 (2007); ...)

- Neutrons: $a \approx -18.6$ fm, $r_e \approx 2.8$ fm

  $\Rightarrow$ neutrons are close to the unitary limit

Conformal symmetry

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- Two-point function of primary field operator $\mathcal{U}$ ("unnucleus/unparticle") constrained by conformal/Schrödinger symmetry

$$G_{\mathcal{U}}(t, \boldsymbol{x}) = -i\langle T\mathcal{U}(t, \boldsymbol{x})\mathcal{U}^{\dagger}(0, \boldsymbol{0})\rangle = C\,\frac{\theta(t)}{(it)^{\Delta}}\exp\left(\frac{iM\boldsymbol{x}^2}{2t}\right)$$

- Determined by symmetry up to overall constant C
- Two-point function in momentum space

$$G_{\mathcal{U}}(\omega, \boldsymbol{p}) = -C\left(\frac{2\pi}{M}\right)^{3/2}\Gamma\left(\frac{5}{2}-\Delta\right)\left(\frac{\boldsymbol{p}^2}{2M}-\omega-i\epsilon\right)^{\Delta-\frac{5}{2}}$$

  - pole only for $\Delta = 3/2$ (free field)
  - branch cut for $\Delta > 3/2 \to$ continuous energy spectrum

- Value of Δ not determined by symmetry $\longrightarrow$ non-perturbative problem

Reactions with neutrons

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- Two ways to do experiments
  (a) detect recoil particle *B*

$$\frac{d\sigma}{dE} \sim (E_0 - E_B)^{\Delta-5/2}, \qquad E_0 = (1 + M_B/M_{\mathcal{U}})^{-1} E_{\text{kin}}$$

  (b) detect all final state particles including neutrons

$$\frac{d\sigma}{dE} \sim (E_{xn}^{cms})^{\Delta-5/2}$$

(HWH, D.T. Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021))

Reactions with neutrons

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$$\frac{d\sigma}{dE} \sim (E_{xn}^{cms})^{\Delta - 5/2}$$

(HWH, D.T. Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021))

- $2n$ case understood from dimer propagator in Halo/pionless EFT ($\Delta = 2$)

$$G_d(E_{nn}^{cms}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}^{cms}}} \quad \Rightarrow \quad \mathrm{Im}\, G_d(E_{nn}^{cms}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}^{cms}}}{(ma^2)^{-1} + E_{nn}^{cms}}$$

- $3n$ case consistent with previous experiments for $^3\mathrm{H}(\pi^-, \gamma)3n$
  (Miller et al., Nucl. Phys. A **343**, 347 (1980))

3n reaction calculations

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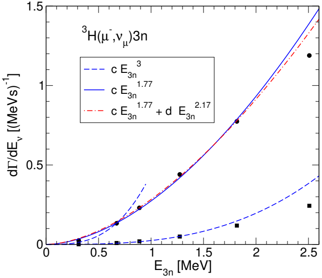
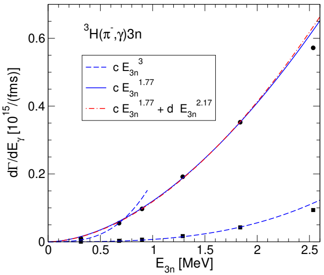
- Radiative muon/pion capture on the triton (AV18 + UIX)



Golak et al., PRC **98**, 054001 (2018)



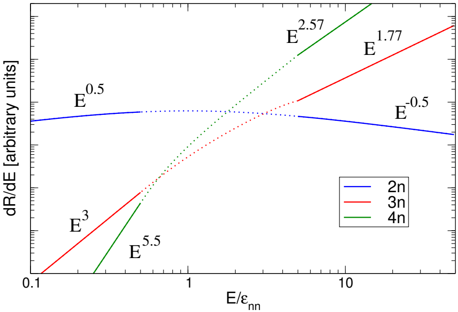
Golak et al., PRC **94**, 054001 (2016)

- Conformal prediction

$$\frac{d\Gamma}{dE} \sim (E_{3n})^{4.27272-5/2} \sim (E_{3n})^{1.77272}, \qquad 0.1\,\text{MeV} \ll E_{3n} \ll 5\,\text{MeV}$$

Predictions for relative energy distributions

- Power law behavior at low energies (predictions for $N \leq 6$ available)



Braaten, HWH, Phys. Rev. D **107**,034017 (2023)

- No resonance-like peak for 4*n*
- Structure of initial state?

Comparison to tetraneutron experiment

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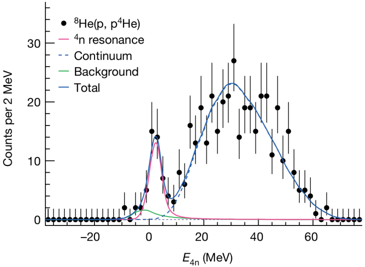
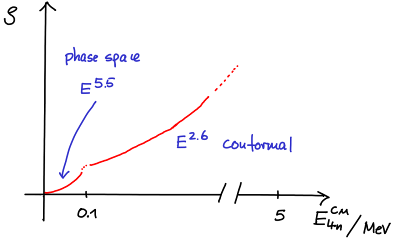
- Search for tetraneutron resonances in $^{8}\mathrm{He}(p, p\alpha)4n$





M. Duer et al., Nature **606** (2022) 678

- Dineutron correlations in $4n$ continuum included

Summary and Outlook

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- Efimov physics in halo nuclei
- High-energy nuclear reactions with final state neutrons

  $\Rightarrow$ (approximate) conformal symmetry
  $\Rightarrow$ power law behavior of observables determined by Δ

- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles

Summary and Outlook

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- Efimov physics in halo nuclei
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  $\Rightarrow$ (approximate) conformal symmetry
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- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles
- Other applications & extensions
  - Two-component Fermions in ultracold atom physics
  - Neutral charm mesons
    (Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022), Phys. Rev. D **107**, 034017 (2023))

Additional Slides

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EFT Framework

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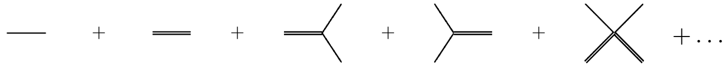
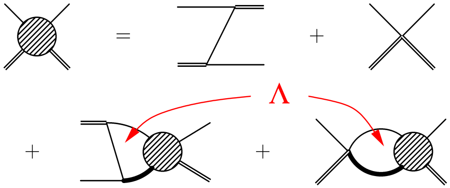
- Exploit scale separation in EFT framework
- Here: *S*-wave case, higher *L* states can also be treated
- Effective Lagrangian

$\mathcal{L}_{eff} =$



- 2-body amplitude:

- 2-body coupling g_2 near fixed point $(1/a = 0)$ $\Longleftrightarrow$ unitary limit
- 3-body amplitude:



$g_3(\Lambda)$ $\Rightarrow$ limit cycle

$\Rightarrow$ discrete scale inv.

Limit Cycle

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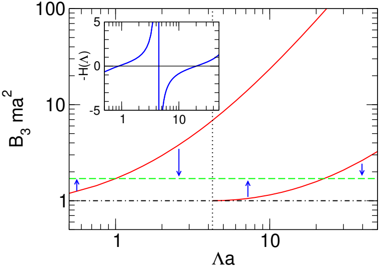
- RG invariance $\Longrightarrow$ running coupling $H(\Lambda) = g_3\Lambda^2/(9g_2^2)$
  - $H(\Lambda)$ periodic: limit cycle

    $$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

    (cf. Wilson, 1971)
  - Anomaly: scale invariance broken to discrete subgroup



$$H(\Lambda) \approx \frac{\cos(s_0\ln(\Lambda/\Lambda_*)+\arctan(s_0))}{\cos(s_0\ln(\Lambda/\Lambda_*)-\arctan(s_0))}\,, \quad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance

Conformal field theory

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- Imaginary part of propagator

$$\mathrm{Im}\, G_{\mathcal{U}}(\omega, \boldsymbol{p}) \sim \begin{cases} \delta\left(\omega - \frac{\boldsymbol{p}^2}{2M}\right), & \Delta = \frac{3}{2}\,, \\ \left(\omega - \frac{\boldsymbol{p}^2}{2M}\right)^{\Delta - \frac{5}{2}} \theta\left(\omega - \frac{\boldsymbol{p}^2}{2M}\right), & \Delta > \frac{3}{2} \end{cases}$$

- Examples of unnuclei
  - free field: $\mathcal{U} = \psi$, $M = m_\psi$, $\Delta = 3/2$
  - N free fields: $\mathcal{U} = \psi_1 \dots \psi_N$, $M = N m_\psi$, $\Delta = 3N/2$
  - N interacting fields: $\mathcal{U} = \psi_1 \dots \psi_N$, $M = N m_\psi$, $\Delta > 3/2$

- In our case: unnucleus is strongly interacting multi-neutron state with

$$1/(ma^2) \sim 0.1\,\text{MeV} \ll E_n^{cms} \ll 1/(mr_e^2) \sim 5\,\text{MeV}$$

- Corrections from finite a and r_0 (S. Dutta, R. Mishra, D.T. Son, arXiv:2309.15177)

$$\mathrm{Im}\, G_{\mathcal{U}}(\omega, 0) \sim \omega^{\Delta - \frac{5}{2}} \theta(\omega) \left(1 + \frac{c_1}{a\sqrt{m\omega}} + c_2 r_0 \sqrt{m\omega}\right)\,, \quad c_2 = 0$$

Scaling dimension

- How to calculate scaling dimension Δ?

  (1) Δ can be obtained from field theory calculation

  (2) Δ can be obtained from operator state correspondence

> Δ of primary operator = (Energy of state in HO)/$\hbar\omega$

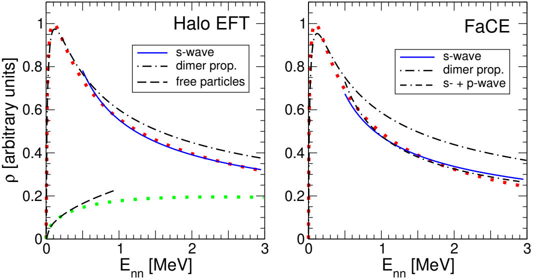
(Nishida, Son, Phys. Rev. D **76**, 086004 (2007))

N	S	L	$\mathcal{O}$	Δ
2	0	0	$\psi_1\psi_2$	2
3	1/2	1	$\psi_1\psi_2\nabla_j\psi_2$	4.27272
3	1/2	0	$\psi_1\nabla_j\psi_2\nabla_j\psi_2$	4.66622
4	0	0	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2$	5.07(1)
5	1/2	1	$\ldots$	7.6(1)

$\Rightarrow$ connection between Δ and energy of particles in a trap

Reaction calculations

- Two-neutron spectrum for ${}^{6}\mathrm{He}(p, p\alpha)2n$ (Göbel et al., Phys. Rev. C **104**, 024001 (2021))



- Can be understood from dimer propagator ($\Delta = 2$)

$$G_d(E_{nn}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}}} \quad \Rightarrow \quad \mathrm{Im}\, G_d(E_{nn}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}}}{(ma^2)^{-1} + E_{nn}}$$