Lifetimes of three-body resonances: dimensionality and mass ratio

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2024/09/04, Sendai (Japan)

Few-Body Syst **65**, 38 (2024)
arXiv:2312.04080

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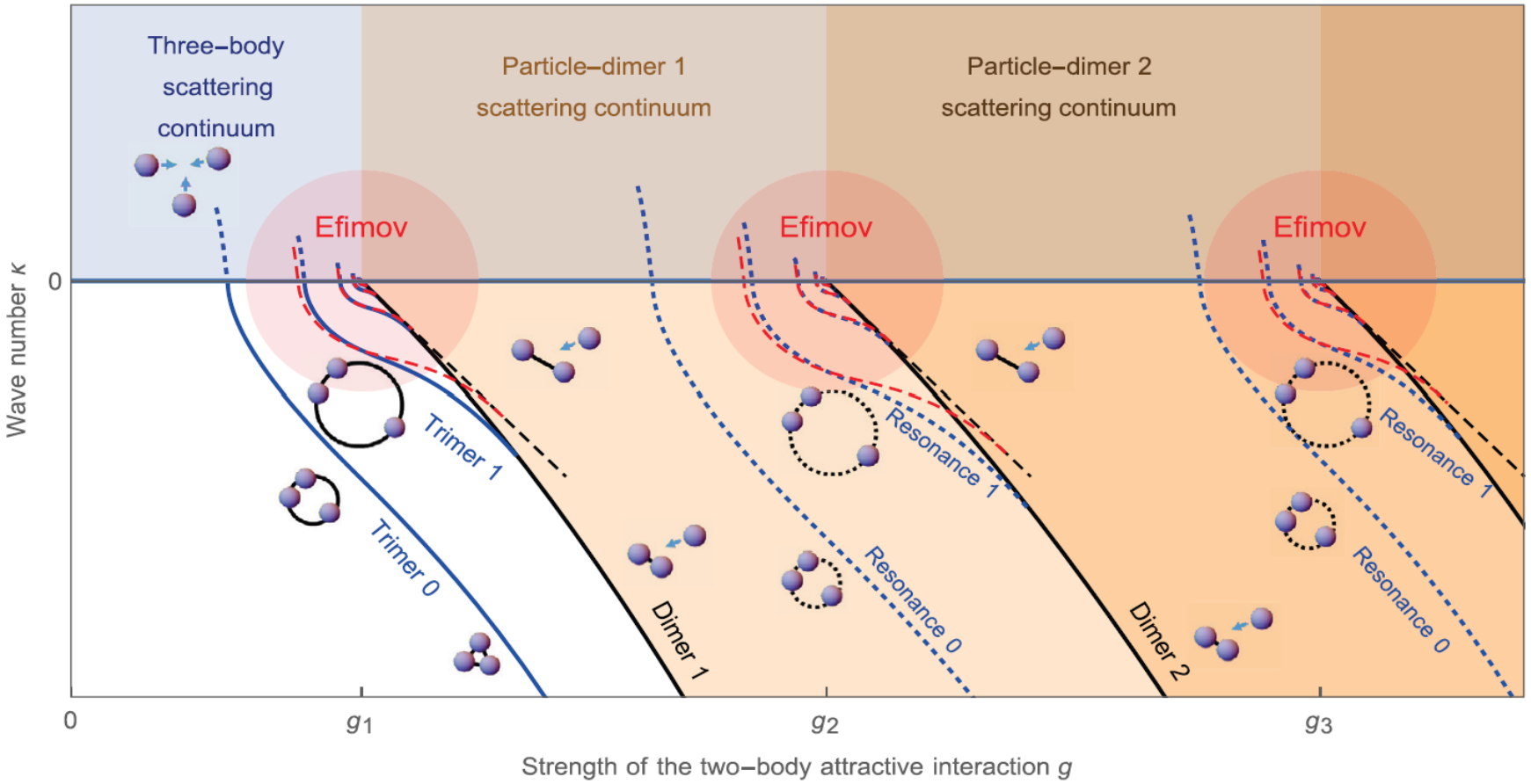
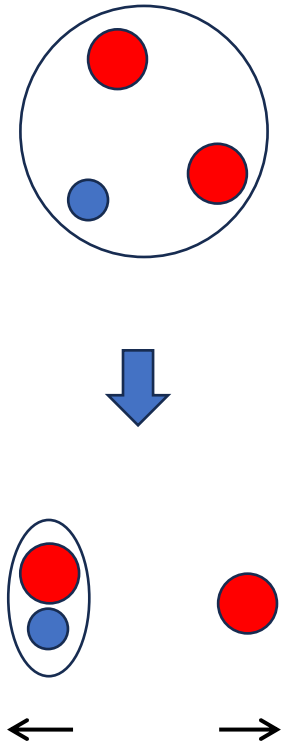
UQS-Tohoku-2024
08/26 – 09/27 2024

Three-body resonances





Naidon, Endo, *Rep. Prog. Phys.* **80** 056001 (2017)

complex energy

$$E = E_R - \frac{i}{2}\Gamma$$

Lifetime

$$\tau = \frac{\hbar}{\Gamma} = \frac{-\hbar}{2\,Im(E)}$$

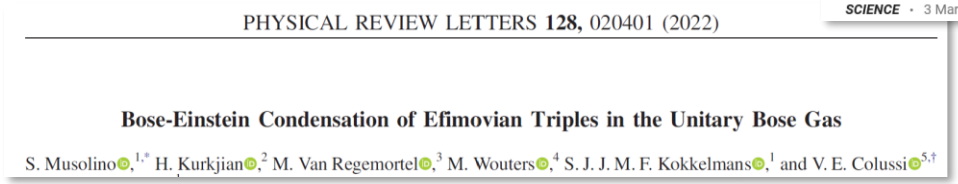
Three-body resonances

- **Ultracold atoms**


Control of reactive collisions by quantum interference

HYUNGMOK SON, JULIANA J. PARK, YU-KUN LU, ALAN O. JAMISON, TIJS KARMAN, AND WOLFGANG KETTERLE Authors Info & Affiliations

SCIENCE · 3 Mar 2022 · Vol 375, Issue 6584 · pp. 1006-1010 · DOI: 10.1126/science.abl7257

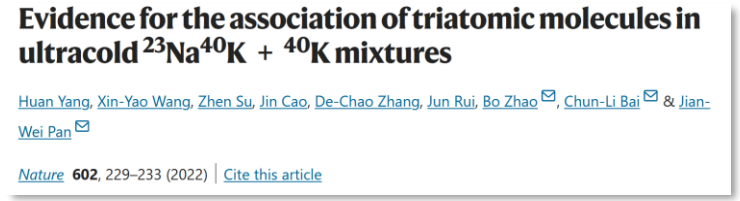

PHYSICAL REVIEW LETTERS 128, 020401 (2022)

Bose-Einstein Condensation of Efimovian Triples in the Unitary Bose Gas

S. Musolino, H. Kurkjian, M. Van Regemortel, M. Wouters, S. J. J. M. F. Kokkelmans, and V. E. Colussi


Reshaped three-body interactions and the observation of an Efimov state in the continuum

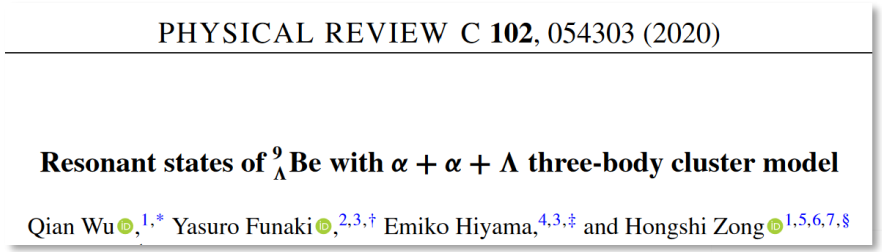
Yaakov Yudkin, Roy Elbaz, José P. D'Incao, Paul S. Julienne & Lev Khaykovich


Evidence for the association of triatomic molecules in ultracold $^{23}Na^{40}K + {}^{40}K$ mixtures

Huan Yang, Xin-Yao Wang, Zhen Su, Jin Cao, De-Chao Zhang, Jun Rui, Bo Zhao, Chun-Li Bai & Jian-Wei Pan

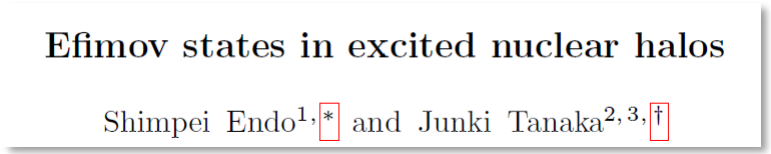
Nature **602**, 229–233 (2022) | Cite this article

- **Nuclear physics**


PHYSICAL REVIEW C 102, 054303 (2020)

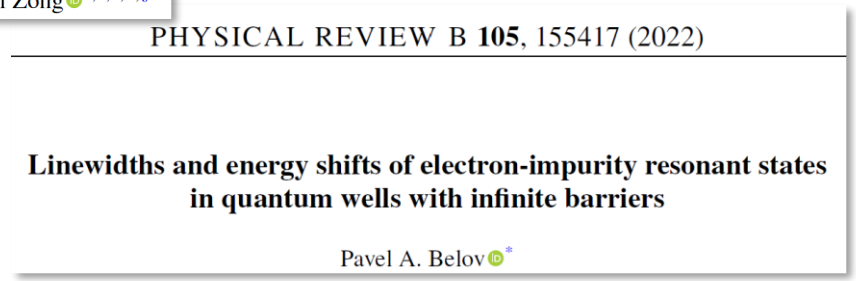
Resonant states of $^{9}_{\Lambda}$Be with $\alpha + \alpha + \Lambda$ three-body cluster model

Qian Wu, Yasuro Funaki, Emiko Hiyama, and Hongshi Zong


Efimov states in excited nuclear halos

Shimpei Endo, * and Junki Tanaka, †

- **Excitons in semiconductors, …**

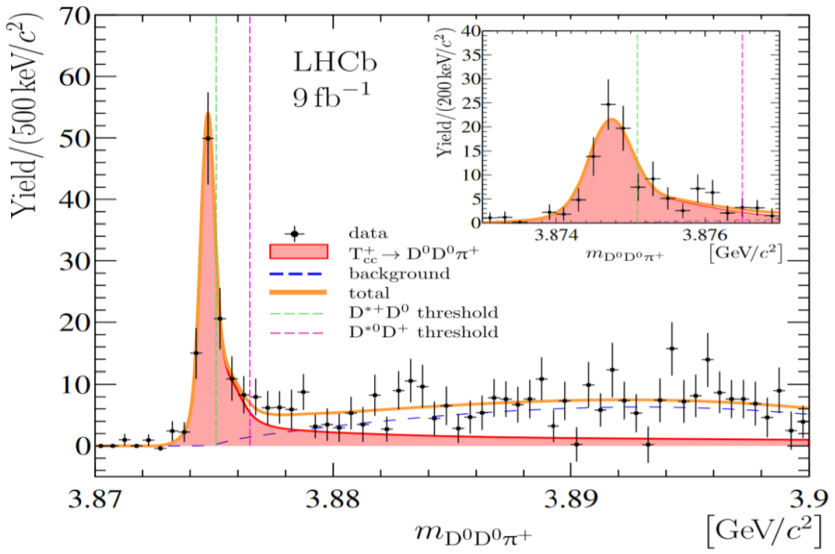

PHYSICAL REVIEW B 105, 155417 (2022)

Linewidths and energy shifts of electron-impurity resonant states in quantum wells with infinite barriers

Pavel A. Belov *

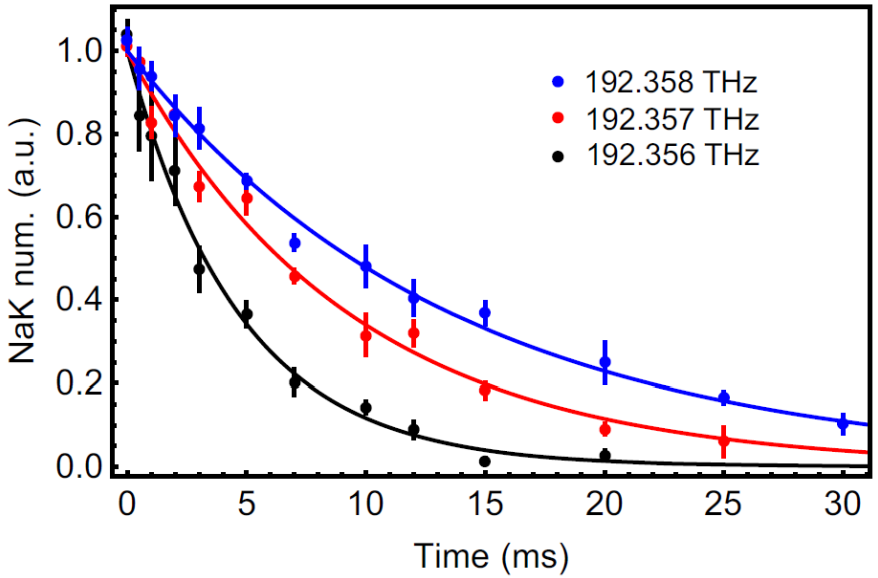
Resonances: Observables

Width Γ from scattering cross section



LHCb collab, *Nat Comm* **13**, 3351 (2022)

Lifetime τ from decaying population

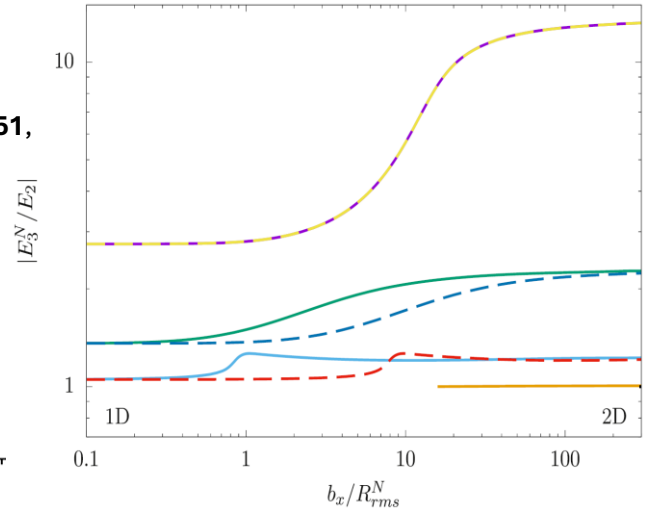


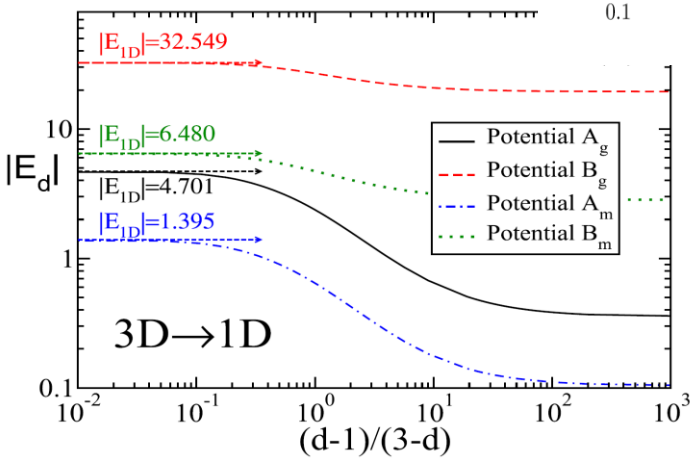
Cao et al., *PRL* **132**, 093403 (2024)

Motivation: Lifetime in low dimensions

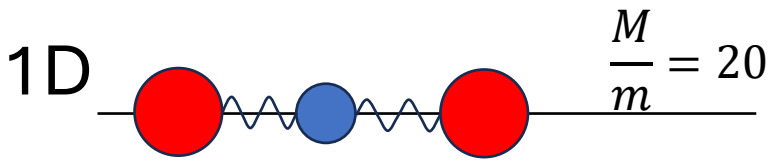
Spectral flow of (Efimov) trimers with dimensionality:

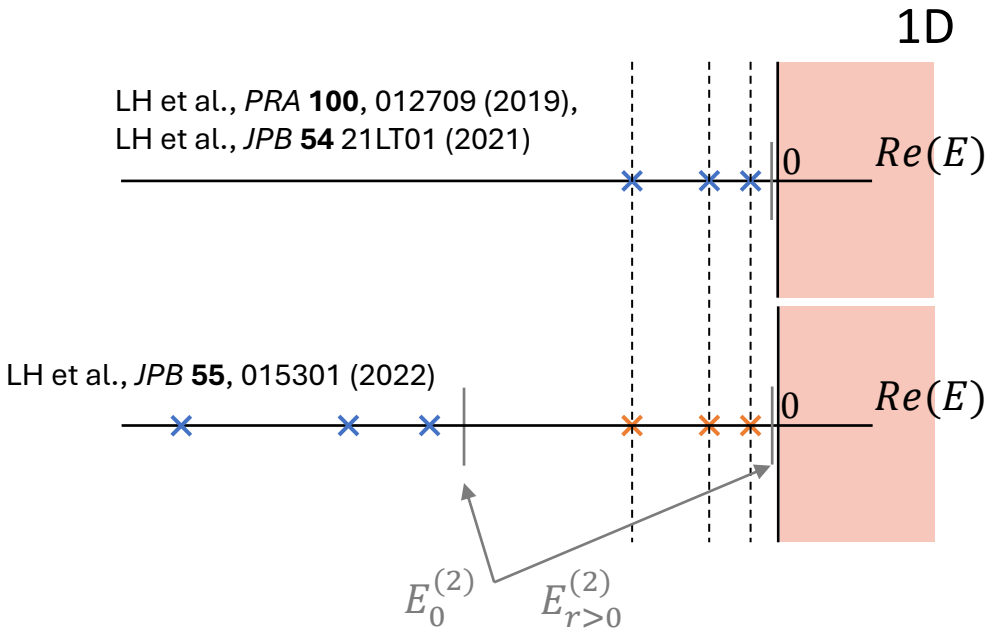
Sandoval et al., *JPB* **51**, 065004 (2018)





Garrido et al., *FBS* **65**, 35 (2024)



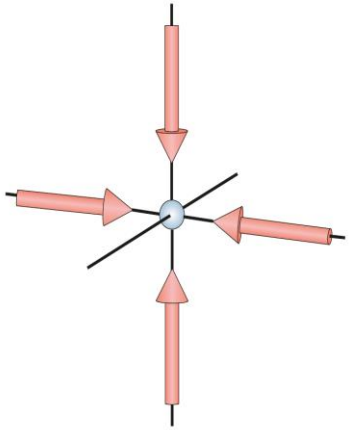


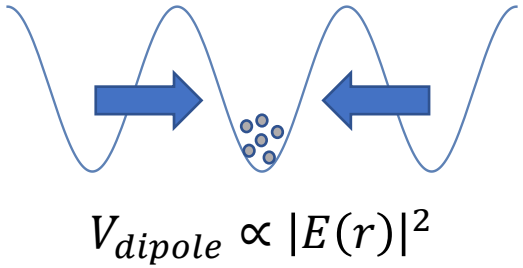
Universality: Resonances ≈ bound states

➢ Does dimensionality affect lifetimes?

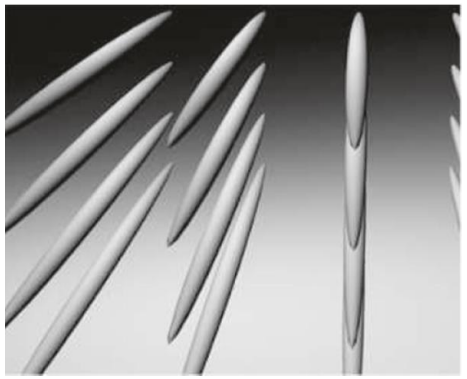
Low dimensions: Realization

Realization of low-dimensional geometries (quasi-1D)





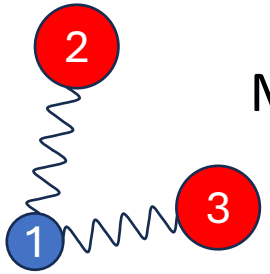
$$V_{dipole} \propto |E(r)|^2$$



Bloch, *Nat. Phys.* **1**, 23 (2005)

The three-body system

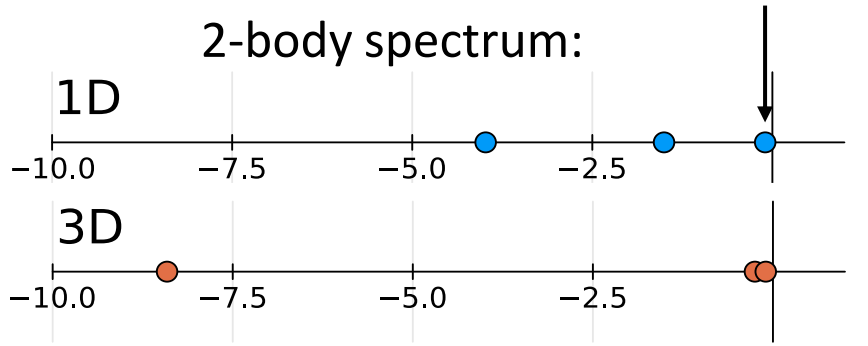
- Two identical bosons (2,3), one distinguishable (1)



Mass ratio $\beta = \frac{m_B}{m_X}$

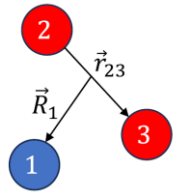
- No interaction between identical particles

$a \cong 3\, r_0$



- Gaussian pair-interactions $V_{ij}(r) = v_0 e^{-(r_{ij}/r_0)^2}$

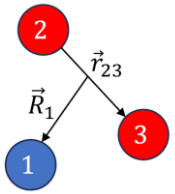
$$\left[-\frac{\hbar^2}{2\mu_{ij}}\nabla^2_{\vec{r}_{ij}} - \frac{\hbar^2}{2\mu_k}\nabla^2_{\vec{R}_k} + V(r_{12}) + V(r_{31})\right]\Psi(\vec{r}_{ij}, \vec{R}_k) = E\Psi(\vec{r}_{ij}, \vec{R}_k)$$



Method

Gaussian Expansion Method (GEM)

$$|\Psi\rangle = |\Phi^{(1)}\rangle + |\Phi^{(2)}\rangle + |\Phi^{(3)}\rangle$$



$$\Phi^{(c)}(\vec{r}_c, \vec{R}_c) = \sum_i \phi^{(c)}_{n_i,l_i}(r_c)\psi^{(c)}_{N_i,L_i}(R_c) \times \left[Y_{l_i,m_i}(\hat{\vec{r}}_c)Y_{L_i,M_i}(\hat{\vec{R}}_c)\right]_{J\mathcal{M}}$$

$$\phi_{n,l}(r) = N_{n,l} r^l e^{-\nu_n r^2}$$

$$\psi_{N,L}(R) = N_{N,L} R^L e^{-\lambda_N R^2}$$

Hiyama et al, *Prog. Partcl. Nucl. Phys.* **351**, 223 (2003)

3D: only s-wave ($l = L = 0$)

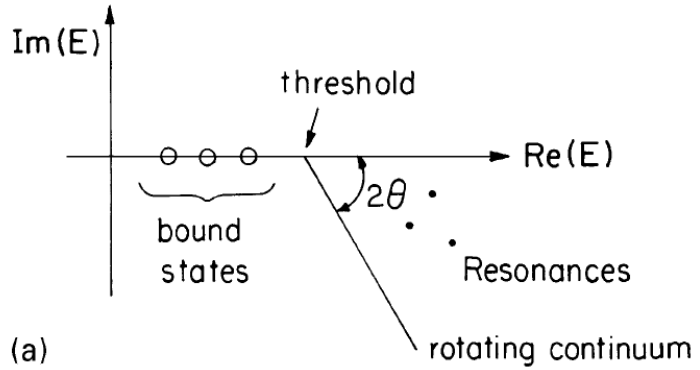
1D: no $Y_{l,m}$; $l, L \in \{0,1\}$

Complex Rotation Method (CSM)

$$r \to r\exp(i\theta)$$

$$H(0) = T + V(r)$$
$$H(\theta) = T\exp(-2i\theta) + V(r\exp(i\theta))$$

→ Resonances can be found via bound-state methods

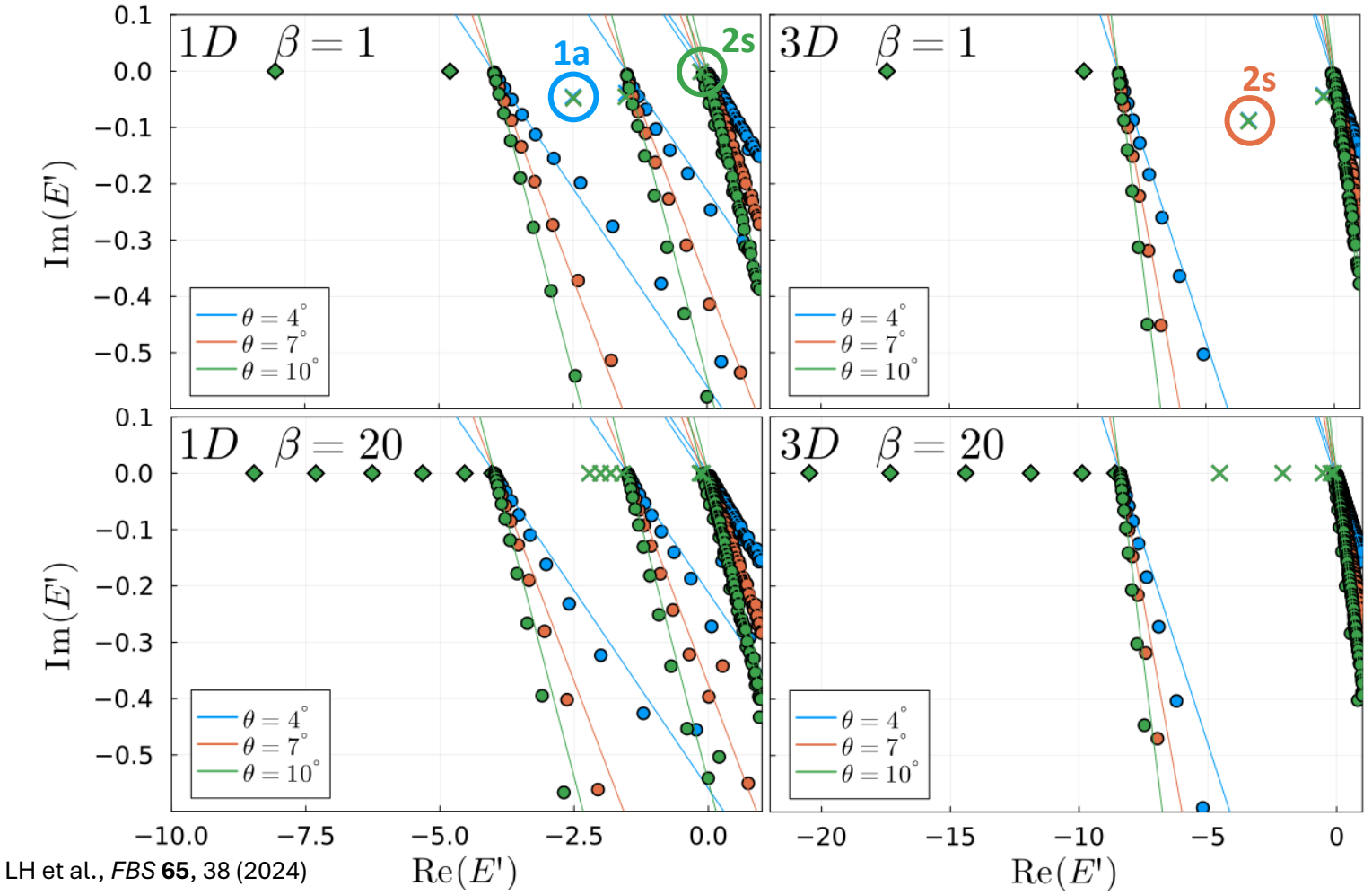


Moiseyev, *Phys. Rep.* **302**, 211 (1998)

Complex-rotated spectra

$\frac{M}{m} = 1$

$\frac{M}{m} = 20$



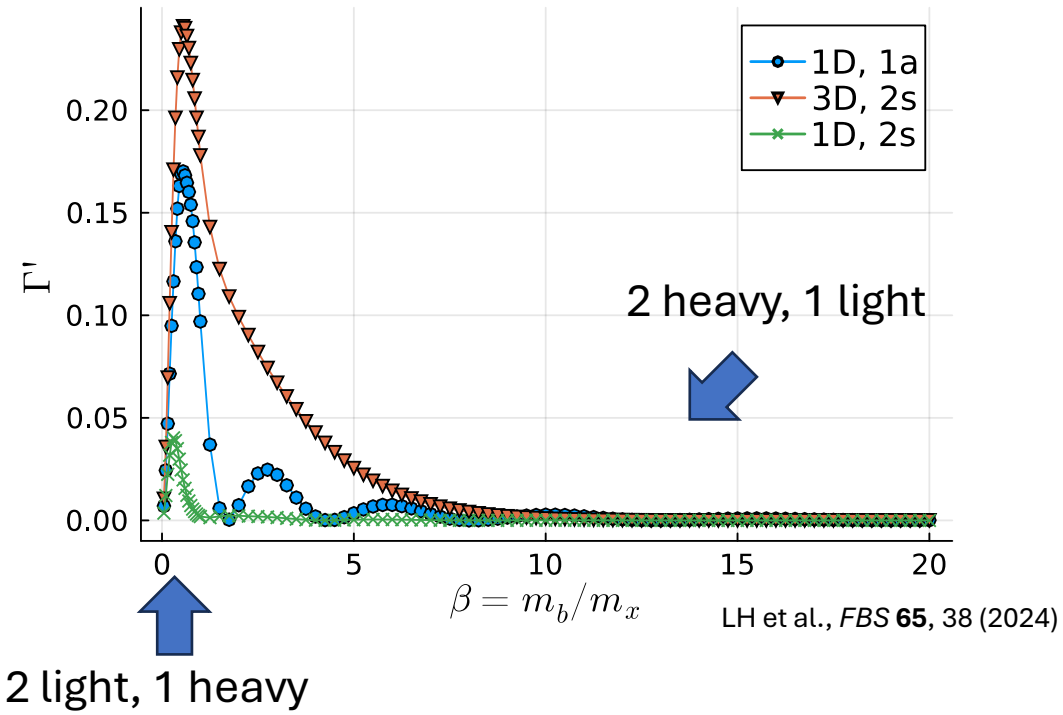
LH et al., *FBS* **65**, 38 (2024)

- Bound state
- Resonance
- Continuum

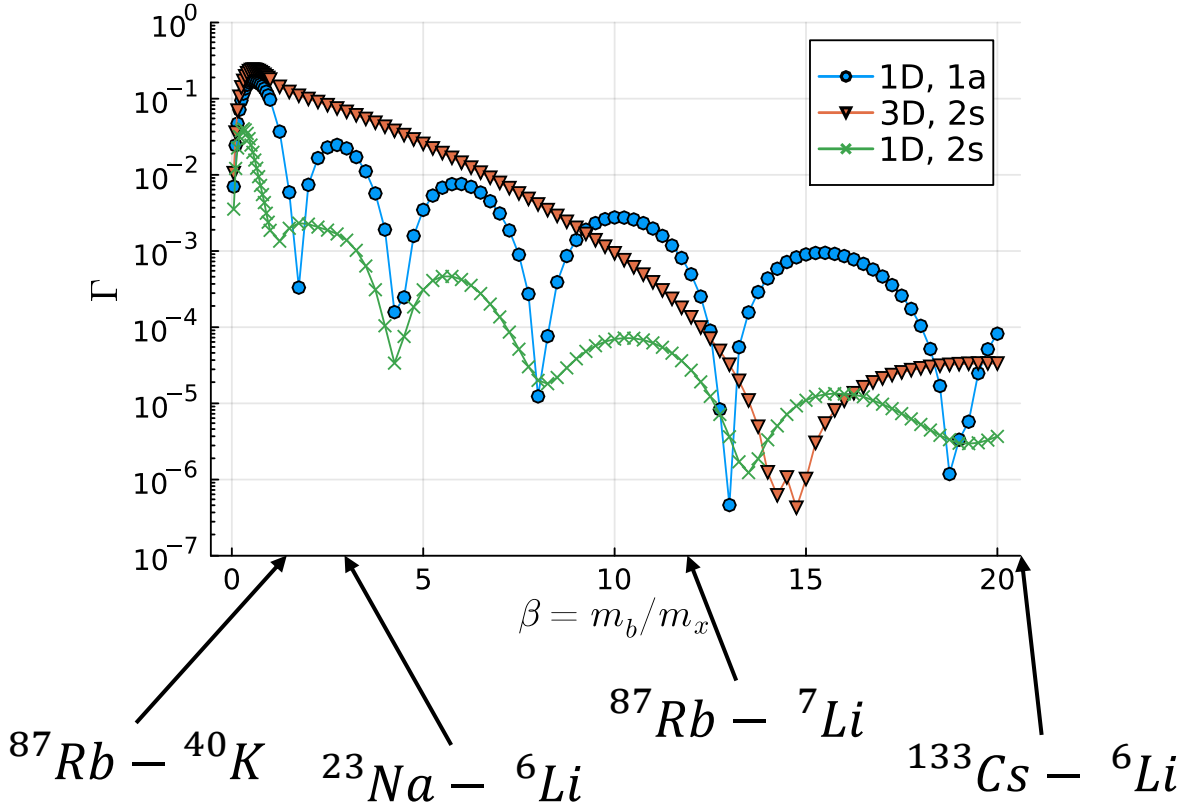
$Im(E') \simeq 0 \pm 10^{-5}$

- Similar result for both 1D & 3D
- Γ decreases with β

Width vs mass ratio



LH et al., *FBS* **65**, 38 (2024)



- Damped-oscillatory behavior
- Specific points of stability (BIC)

Summary & Outlook

Summary:

- Γ shows damped-oscillatory dependence on the mass-ratio
- Specific mass-ratios with exceptional stability

Few-Body Syst **65**, 38 (2024)
arXiv:2312.04080

Outlook:

- Validity of theory: deep resonances (Cao et al., *PRL* **132**, 093403 (2024))?
- Universality? Other systems?
- Realistic potentials

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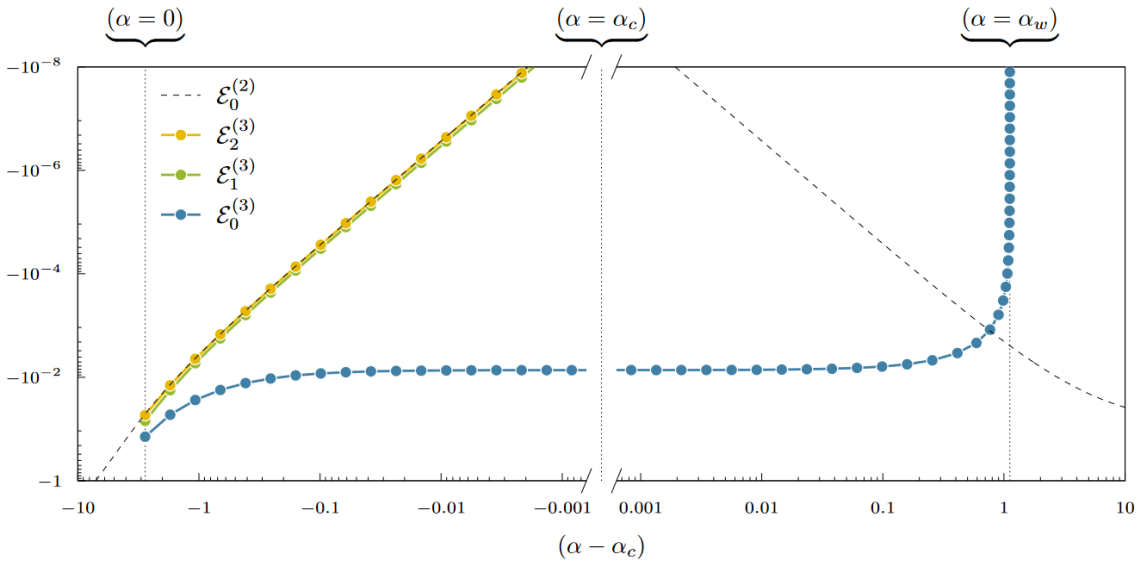
Borromean three-body states in 1D

T. Schnurrenberger et al., arXiv:2405.14865



Borromean window

1D (and 2D): attractive potentials always have a 2B bound state → no Borromean region

→ 2B potential with both attractive and repulsive parts

$$v(x) = -v_0 \left[\delta\left(x - \frac{1}{2}\right) - \alpha\delta\left(x + \frac{1}{2}\right) \right]$$

Overall repulsive ($\alpha > 1$): no 2B bound state

Addition of a third particle:
→ Borromean 3B state despite “added repulsion”

Borromean three-body states in 1D

3D: attractive potential has a 2B bound state only if sufficiently strong

→ Borromean region of 3B bound states in the absence of 2B states