Compositeness of near-threshold s-wave resonances

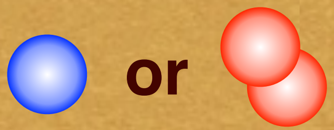
Tetsuo Hyodo

Tokyo Metropolitan Univ.

2024, Sep. 4th

Contents

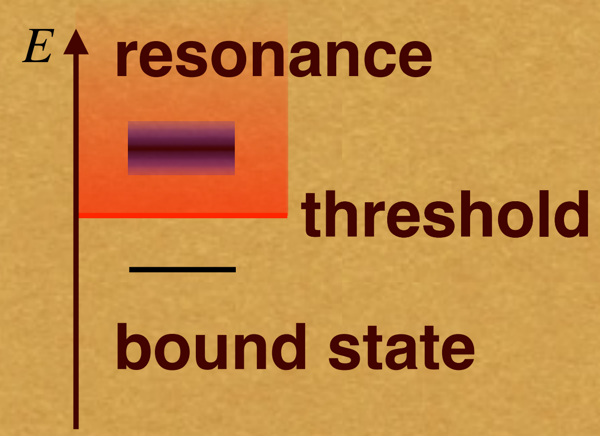
- Introduction — threshold rule?
- Compositeness



S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

- Near-threshold bound states

T. Hyodo, PRC90, 055208 (2014);
T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)



- Near-threshold resonances

T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

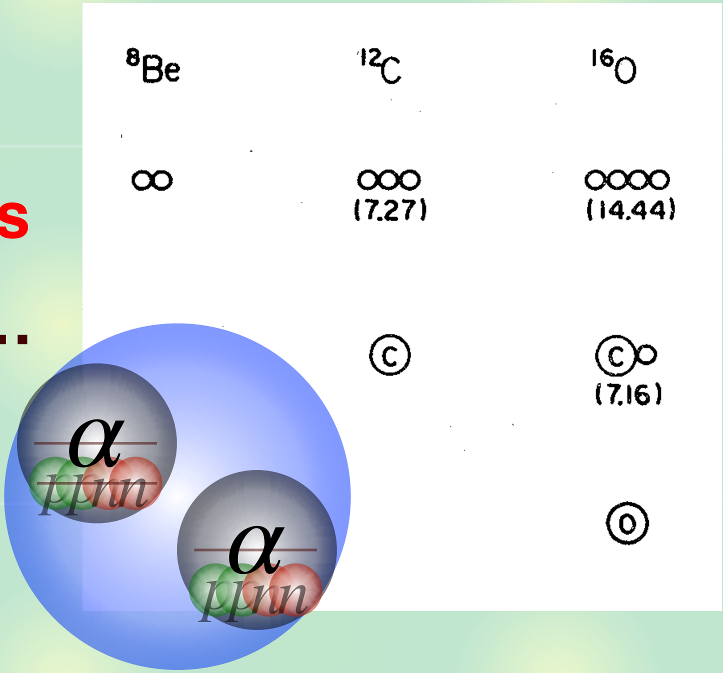
- Summary

Molecule-like structure and threshold rule

α cluster in nuclei

H. Horiuchi, K. Ikeda, Y. Suzuki, PTPS 55, 89 (1972)

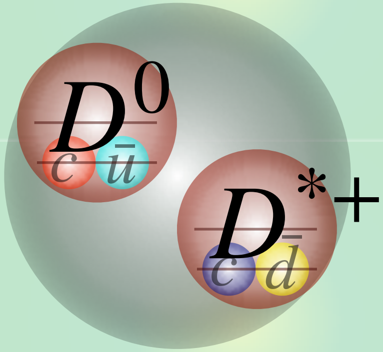
- cluster formation near $n\alpha$ thresholds
- e.g.) $^{8}\mathrm{Be} \sim \alpha\alpha$, $^{12}\mathrm{C}$ Hoyle state $\sim \alpha\alpha\alpha$, …
- molecule-like structure



Hadronic molecules

F.K. Guo, et al., RMP 90, 015004 (2018)

- Exotic hadrons near two-hadron thresholds
- e.g.) $T_{cc} \sim D^0 D^{*+}$, $X(3872) \sim D^0 \bar{D}^{*0}$, …



Threshold rule:

- molecule-like structure appears near threshold

Questions on threshold rule

Mechanism which leads to threshold rule

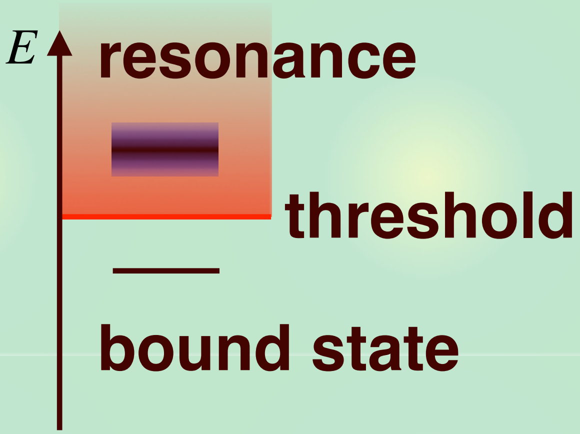
- low-energy universality ($|a_0| \to \infty$ for $B \to 0$)

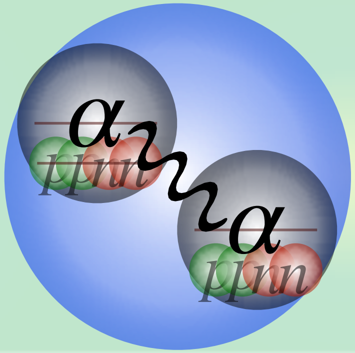
E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006);
P. Naidon, S. Endo, Rept. Prog. Phys. 80, 056001 (2017)

- spatially spread wavefunction —> molecule

Typical example: $^{8}\mathrm{Be} \sim \alpha\alpha$

- $E_R \sim +0.1$ MeV, above threshold
- wavefunction of unstable resonance?
- $Z_\alpha = 2$: Coulomb interaction?



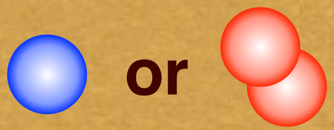


—> Next talk by Kinugawa-san

Contents

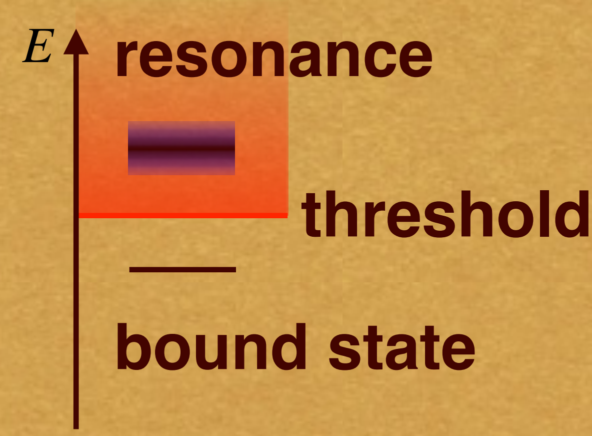
- Introduction — threshold rule?
- Compositeness

  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)



- Near-threshold bound states

  T. Hyodo, PRC90, 055208 (2014);
  T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)



- Near-threshold resonances

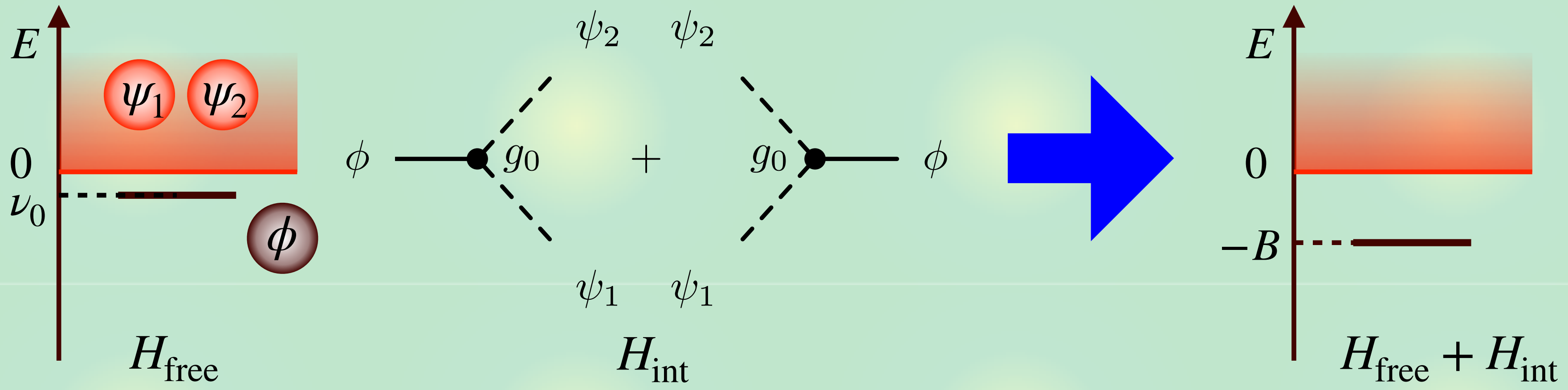
  T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

- Summary

EFT setup

EFT with bare state ϕ + scattering states $\psi_1\psi_2$

T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)



- eigenstates of free/full Hamiltonian

$$H_{\rm free}|B_0\rangle = \nu_0|B_0\rangle, \quad H_{\rm free}|\boldsymbol{p}\rangle = \frac{\boldsymbol{p}^2}{2\mu}|\boldsymbol{p}\rangle, \quad (H_{\rm free}+H_{\rm int})|B\rangle = -B|B\rangle$$

- $\psi_1\psi_2$ scattering amplitude: effective range expansion

$$f(p) = -\frac{\mu}{2\pi}\frac{1}{v(E)^{-1} - G(E)} = \frac{1}{-\frac{1}{a_0} + \frac{r_e}{2}p^2 - ip}$$

Compositeness and elementairty

Compositeness: quantitative measure of internal structure

- **Normalization of $|B\rangle$ + completeness of free eigenstates**

$$\langle B|B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\boldsymbol{p}}{(2\pi)^3} |\boldsymbol{p}\rangle\langle \boldsymbol{p}|$$

- **Definition**

$$1 = Z + X, \quad Z \equiv |\langle B_0|B\rangle|^2, \quad X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3} |\langle \boldsymbol{p}|B\rangle|^2$$

"elementarity" **compositeness**

- Z, X **: real and nonnegative —> interpreted as probability**

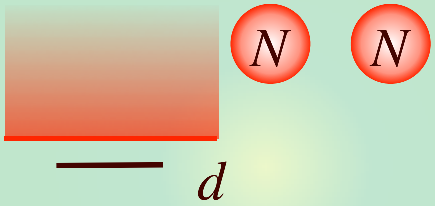
- **Closed-form expressions**

$$X = \left. \frac{G'(E)}{[v^{-1}(E)]' - G'(E)} \right|_{E=-B} = 1 - \left. \frac{1}{1 - \Sigma'(E)} \right|_{E=-B}$$

Weak-binding relation

Compositeness X of stable bound state

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)



$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{Z}\,|\text{others}\rangle, \quad X + Z = 1, \quad 0 \le X \le 1$$

range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length — radius of bound state

- $X < 1$ gives violation of universality $a_0 = R$
- for shallow bound state $R \gg R_{\text{typ}}$, $X \leftarrow$ observables (a_0, B)

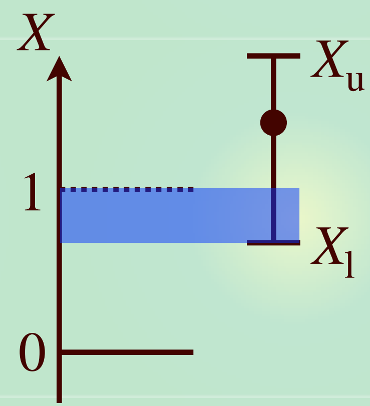
Problem: $a_0 = 5.42$ fm, $R = 4.32$ fm $\Rightarrow X = 1.68 > 1$?

Uncertainty and interpretation

Uncertainty estimation with $\mathcal{O}(R_{\rm typ}/R)$ term

Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

$$X_{\rm u} = \frac{a_0/R + \xi}{2 - a_0/R - \xi}, \quad X_{\rm l} = \frac{a_0/R - \xi}{2 - a_0/R + \xi}, \quad \xi = \frac{R_{\rm typ}}{R}$$



Interpretation (with finite range correction)

T. Kinugawa, T. Hyodo, PRC 106, 015205 (2022)

- exclude region outside $0 \leq X \leq 1$

$$R_{\rm typ} = \max\{R_{\rm int}, R_{\rm eff}\}$$

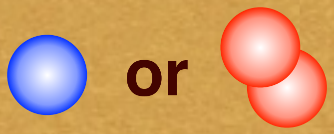
- X of hadrons, nuclei, and atoms
- X of deuteron is reasonable
- $X \geq 0.5$ in all cases studied

Bound state	Compositeness X
d	$0.74 \leqslant X \leqslant 1$
$X(3872)$	$0.53 \leqslant X \leqslant 1$
$D^*_{s0}(2317)$	$0.81 \leqslant X \leqslant 1$
$D_{s1}(2460)$	$0.55 \leqslant X \leqslant 1$
$N\Omega$ dibaryon	$0.80 \leqslant X \leqslant 1$
$\Omega\Omega$ dibaryon	$0.79 \leqslant X \leqslant 1$
${}^3_\Lambda$H	$0.74 \leqslant X \leqslant 1$
^{4}He dimer	$0.93 \leqslant X \leqslant 1$

Near-threshold bound states are mostly composite

Contents

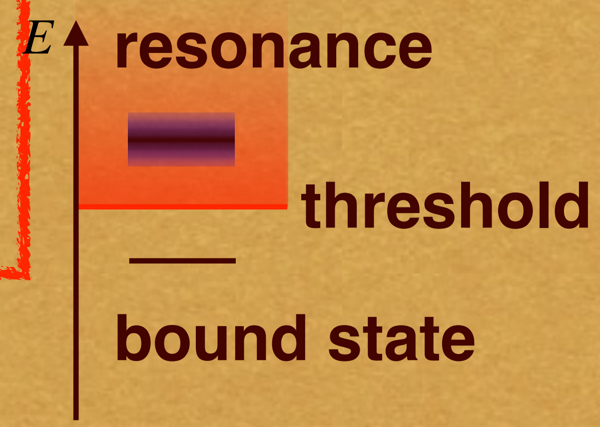
- Introduction — threshold rule?
- Compositeness



S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

- Near-threshold bound states

T. Hyodo, PRC90, 055208 (2014);
T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)



- Near-threshold resonances

T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

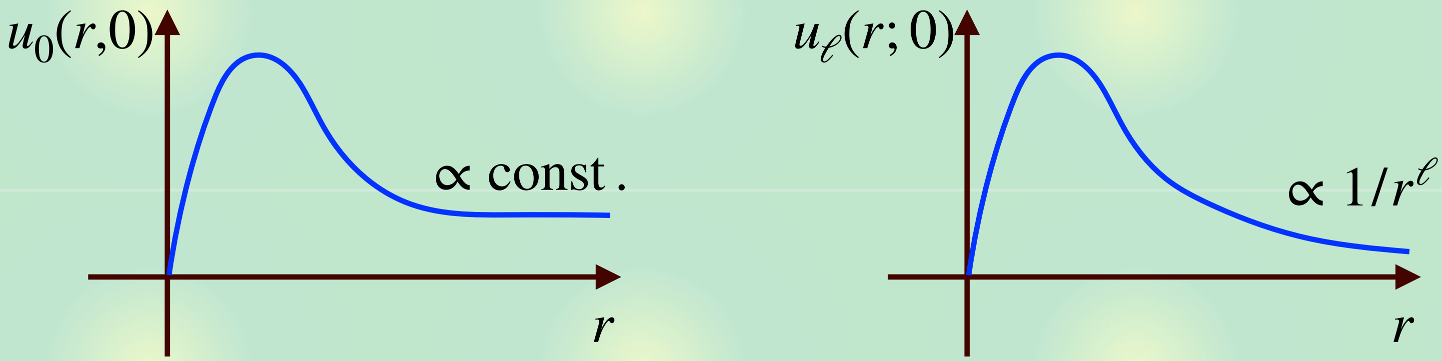
- Summary

compositeness theorem and intuitive picture

Compositeness theorem: $X = 1$ in $B \to 0$ limit

T. Hyodo, PRC90, 055208 (2014)

- wavefunction of $B = 0$ state is not normalizable ($|a_0| \to \infty$)



- compositeness in coordinate space

$$1 = |\langle B_0|B\rangle|^2 + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle \boldsymbol{p}|B\rangle|^2 = |\langle B_0|B\rangle|^2 + \underbrace{\int d\boldsymbol{r}\,|\Psi(\boldsymbol{r})|^2}_{\to\ \infty}$$

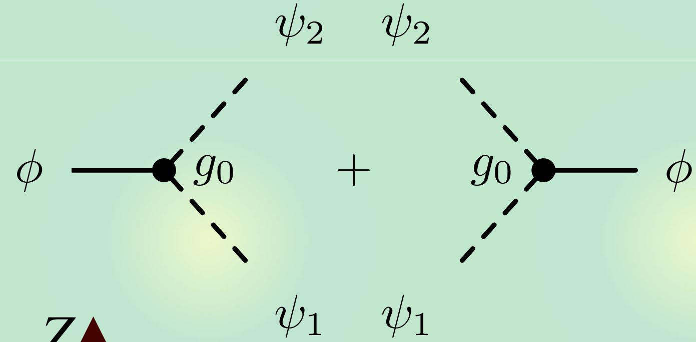
$$\frac{|\langle B_0|B\rangle|^2}{\int d\boldsymbol{r}\,|\Psi(\boldsymbol{r})|^2} \to 0 \quad \Rightarrow \quad X = 1$$

Finite binding case

Elementarity of bound state with small but finite B

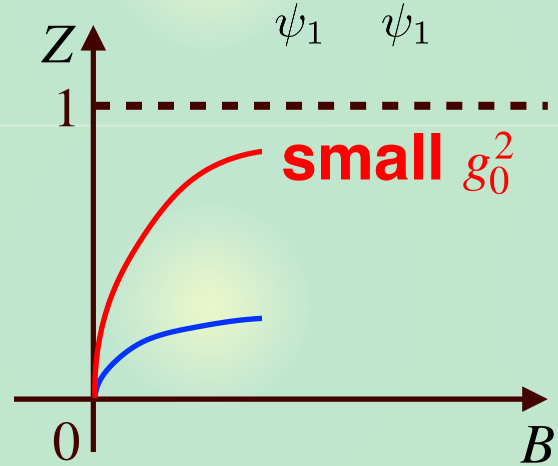
$$Z = \frac{1}{1 - \Sigma'(-B)}$$

$$\sim \frac{1}{1 + Cg_0^2/\sqrt{B}} \sim \frac{\sqrt{B}}{Cg_0^2} + \cdots \neq 0$$



B **dependence of** Z

- $\lim_{B\to 0} Z = 0$ **is fixed**
- $Z \ll 1$ **for small** B **(composite)**



For sufficiently small g_0^2, $\sqrt{B}/g_0^2 \sim \mathcal{O}(1)$ for small B

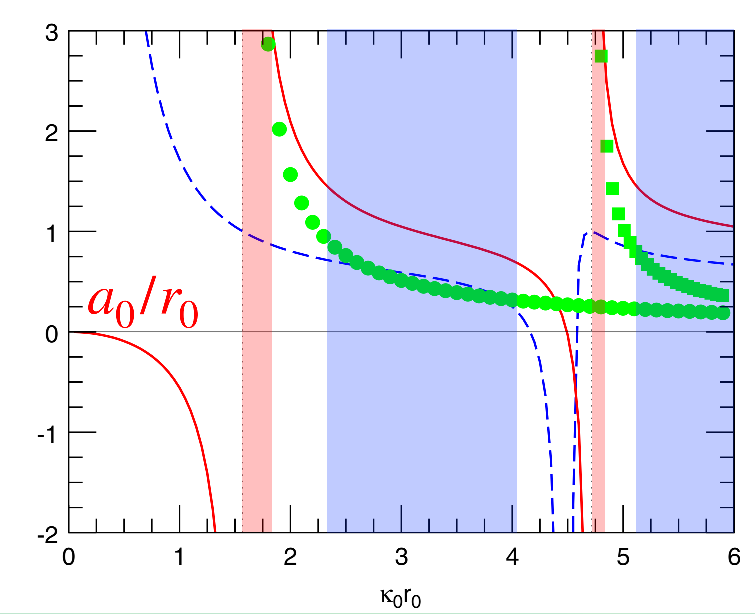
—> sizable Z with small B by fine tuning of parameter g_0^2

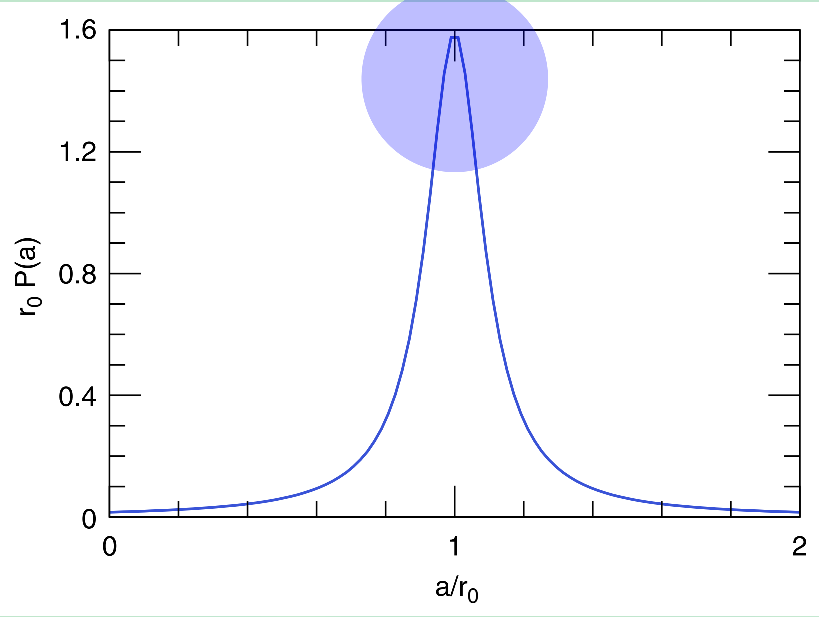
How probable is such fine tuning?

Quantifying fine tuning

Probability distribution of a_0 of square-well potential

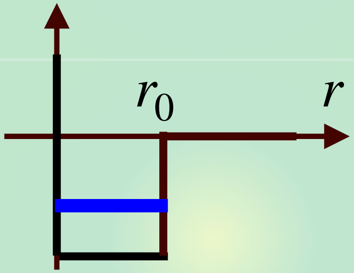
E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)





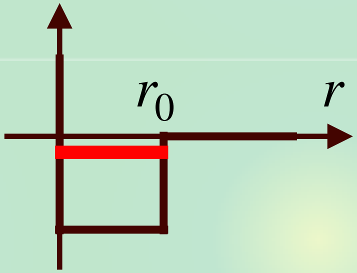
typical

: $a_0/r_0 \sim 1$



shallow

: $a_0/r_0 \gg 1$

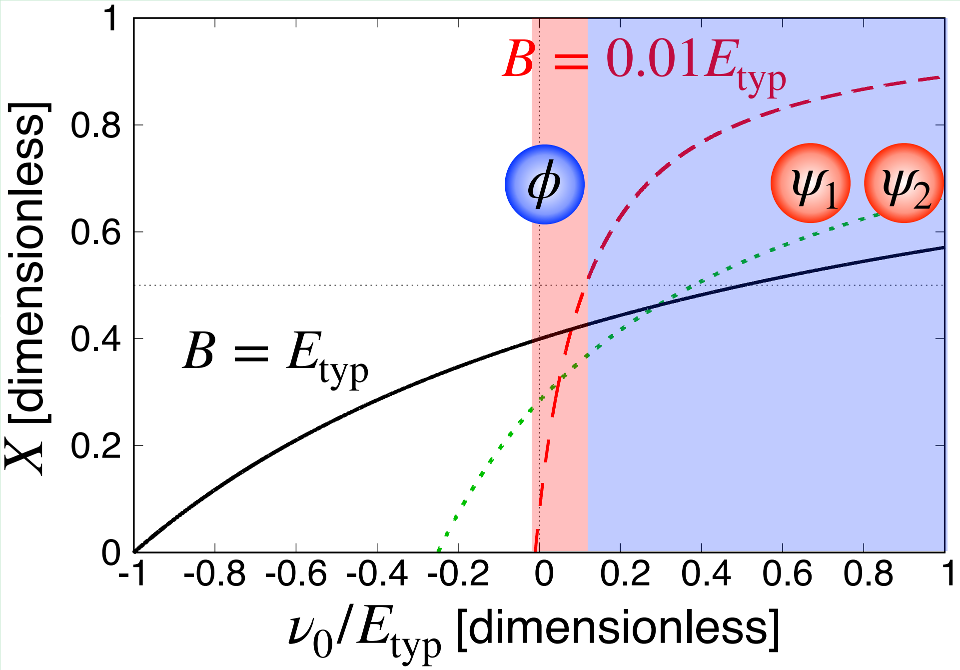


Fine-tuning can be quantified by parameter dependence

Structure of bound state

Compositeness X in the allowed ν_0 region

T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)



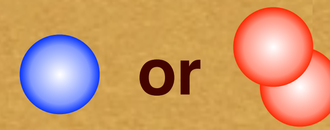
- Typical bound state $B = E_{\rm typ}$: mostly elementary
- Shallow bound state $B = 0.01E_{\rm typ}$: mostly composite

Shallow elementary state <— only with fine tuning = unlikely

Contents

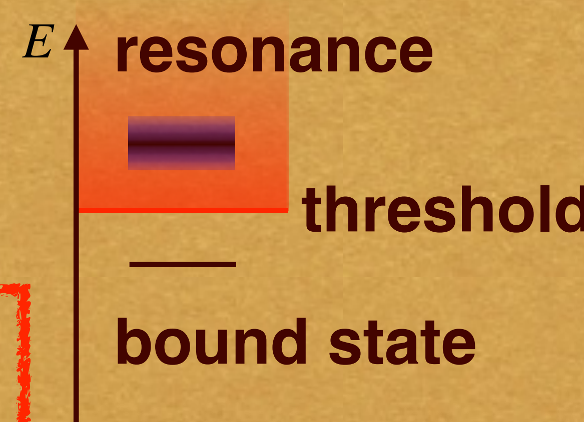
Introduction — threshold rule?

Compositeness



S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

Near-threshold bound states



T. Hyodo, PRC90, 055208 (2014);
T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)

Near-threshold resonances

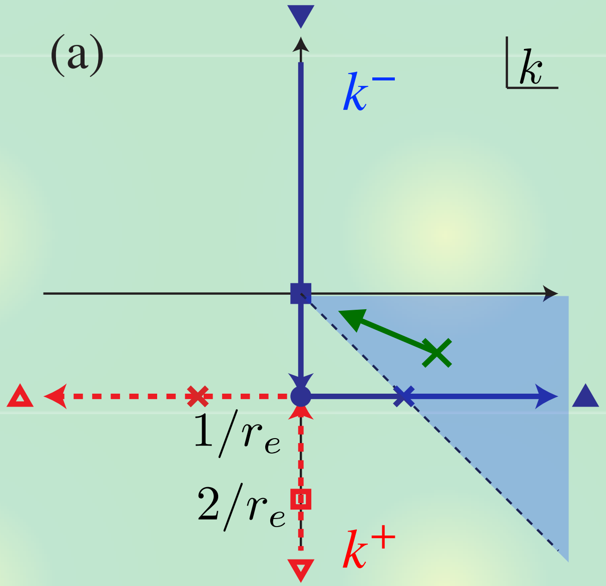
T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

Summary

Resonances in effective range expansion

Two poles in effective range expansion (ERE)

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006);
T. Hyodo, PRL111, 132002 (2013);
T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]



$$k^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a_0} - 1 + i0^+}$$

- **pole positions** $k^{\pm}$ **<—>** (a_0, r_e)

Resonance solution ($r_e < 0$)

$$\frac{1}{|r_e|}\sqrt{\frac{2r_e}{a_0} - 1} \geq \frac{1}{|r_e|}, \quad \Rightarrow \quad \frac{r_e}{a_0} \geq 1, \quad \Rightarrow \quad |a_0| \leq |r_e|$$

- **resonance with** $|k^-| \to 0$ **: not only** $|a_0| \to \infty$ **but also** $|r_e| \to \infty$
- **energy** $E_R = M_R - i\frac{\Gamma_R}{2}$ **<—>** (a_0, r_e)

Compositeness of resonances

Compositeness: pure imaginary <— weak-binding relation

$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i\tan(\theta_k), \quad k^- = |k^-| e^{i\theta_k}$$

Resonance state: complex eigenenergy

$$H|R\rangle = E_R|R\rangle, \quad E_R = M_R - i\frac{\Gamma_R}{2} \in \mathbb{C}$$

- normalization by Gamow vector

$$\langle R|H = \langle R|E_R^*, \quad \langle\tilde{R}|H = \langle\tilde{R}|E_R$$

$$\langle R|R\rangle \to \infty, \quad \langle\tilde{R}|R\rangle = 1$$

- complex Z and X: probability?

$$Z \equiv \langle\tilde{R}|B_0\rangle\langle B_0|R\rangle \in \mathbb{C}, \quad X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}\langle\tilde{R}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|R\rangle \in \mathbb{C}$$

New interpretation scheme

Complex matrix element <—> uncertain nature of resonances

T. Berggren, PLB33, 547 (1970)

Introduce three probabilities $\mathscr{X} + \mathscr{Y} + \mathscr{Z} = 1$

T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

$$\mathscr{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$ **certainly finding composite**

$$\mathscr{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$ **uncertain**

$$\mathscr{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$ **certainly finding elementary**

- **Choice of parameter α: exclude large width resonance**

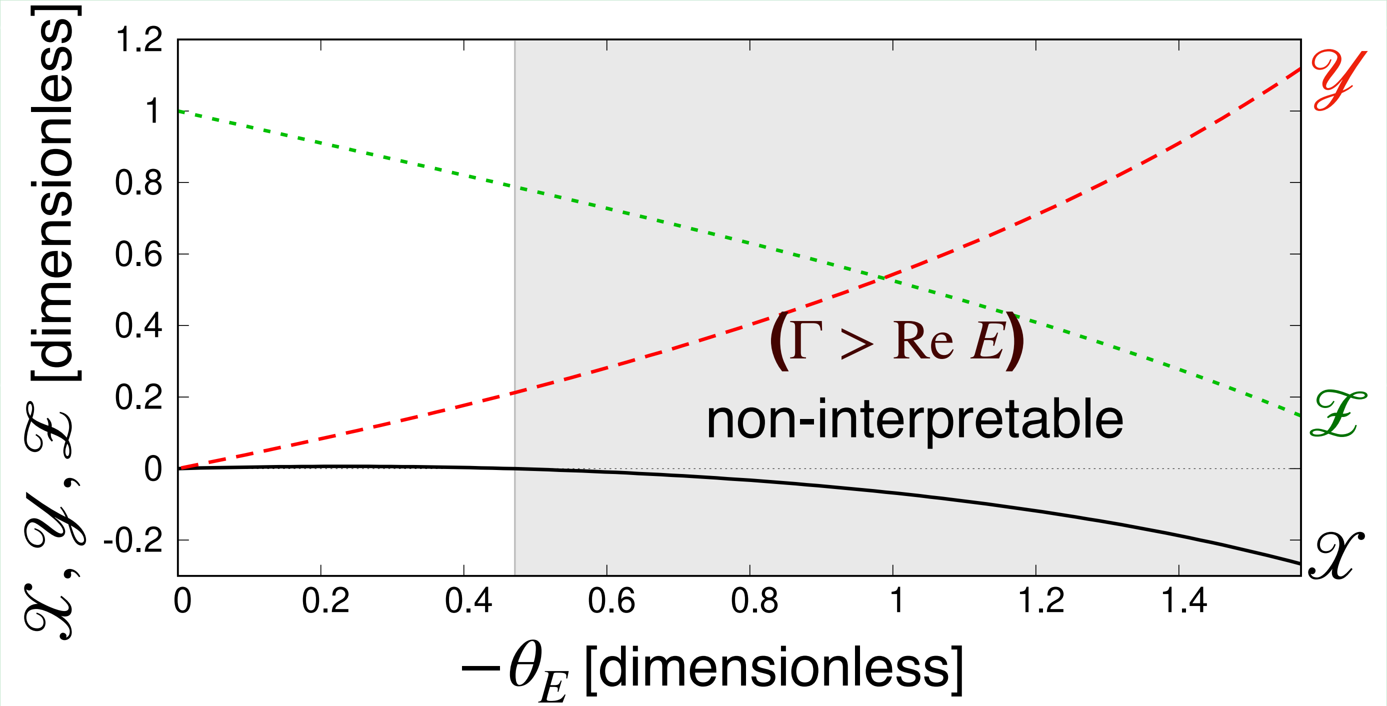
$$\alpha = \frac{\sqrt{5} - 1 + \sqrt{10 - 4\sqrt{5}}}{2} \approx 1.1318$$

- **If $\Gamma > \mathrm{Re}\, E$ —> $\mathscr{X} < 0$: non-interpretable state**

Compositeness of resonances

$\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ as functions of argument of eigenenergy

$$E_R = |E_R| e^{i\theta_E}$$



- Resonances are **not composite dominant** ($\mathcal{Z} \gtrsim 0.8$)

<— consistent with large negative r_e

Near-threshold resonances are not composite dominant

Summary

- **Compositeness X: probability of finding**

- **Bound state exactly at threshold**

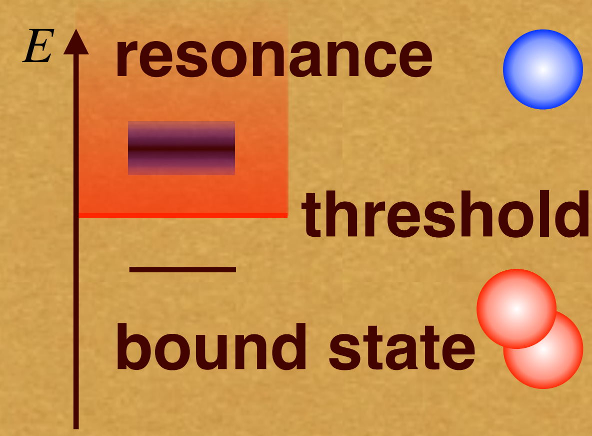
  T. Hyodo, PRC90, 055208 (2014);

  - completely **composite** $X = 1$

- **Near threshold bound states**

  T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)

  - in general, **composite** $X \sim 1$



- **Near-threshold resonances**

  T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

  - **non-composite**, $\mathcal{X} \lesssim 0.2$