Sign-inversion of magnetovortical charge from gauge invariant thermodynamics

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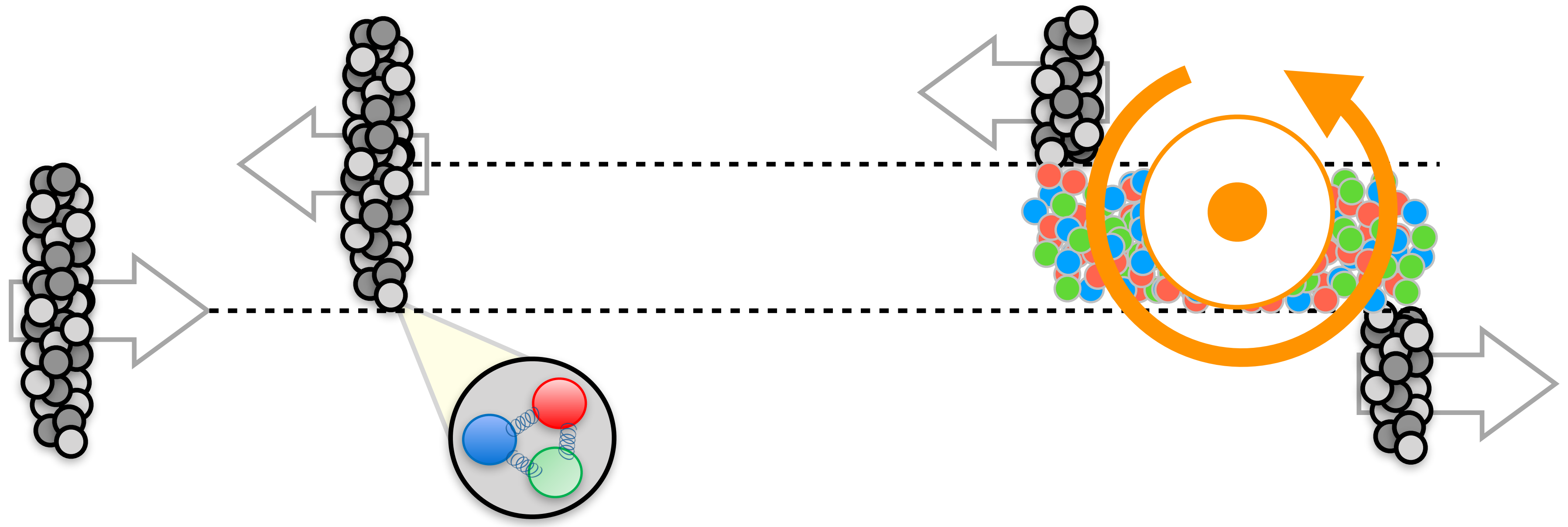
K. Fukushima, K. Hattori and KM, arXiv:2409.******* [hep-ph]

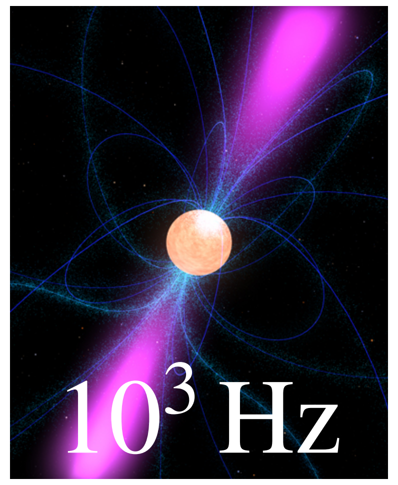
QCD matter under rotation

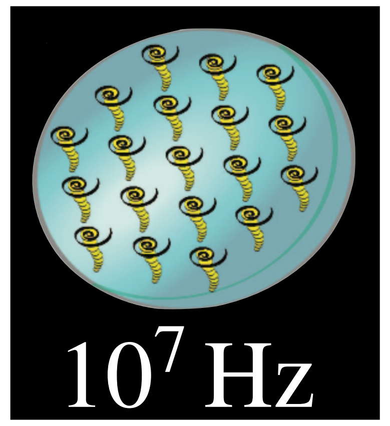
heavy-ion collision

$\vec{\Omega} \parallel \vec{B}$



10^{22} Hz $\ggg$

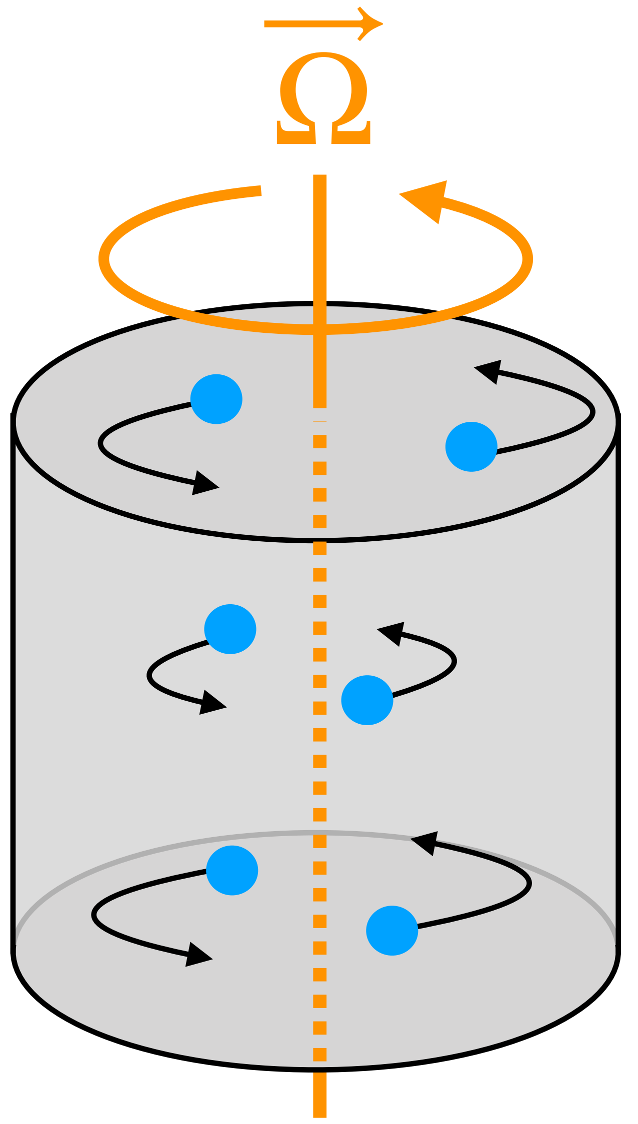




superfluid ^{4}He

- elementary particles affected by $\vec{\Omega}$ (= source of angular momentum)
- $\vec{\Omega} \parallel \vec{B}$ is more crucial than either $\vec{\Omega}$ or $\vec{B}$

Early attempt : Thermodynamics

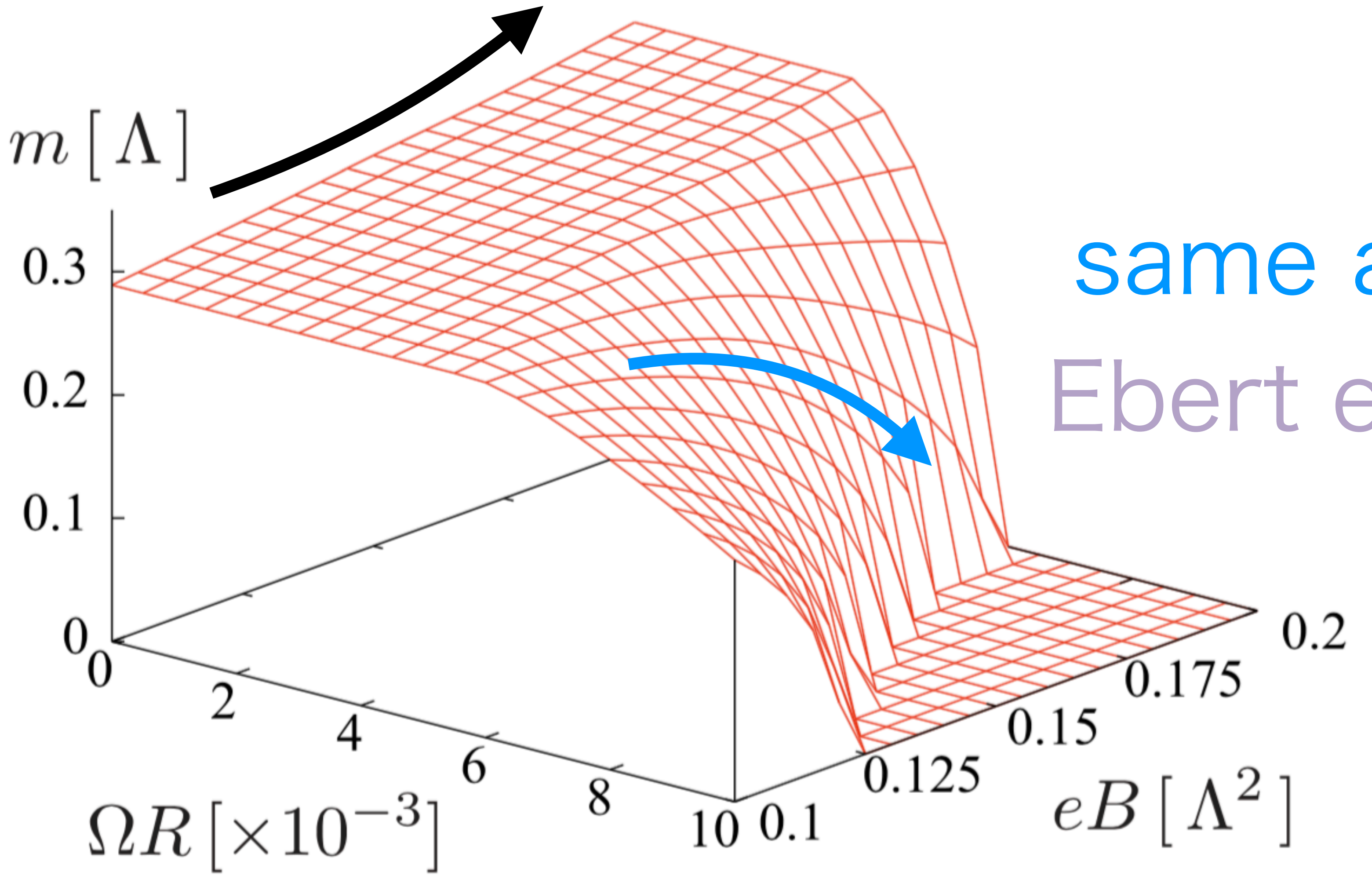


Landau-Lifshitz (1958)   Vilenkin (1979)

$$Z = \mathrm{tr}\exp[-\beta(H - \Omega\mathcal{J})] \longleftrightarrow H - \mu N$$

Chen-Fukushima-Huang-Mameda (2016)

NJL model under $\vec{\Omega} \parallel \vec{B}$



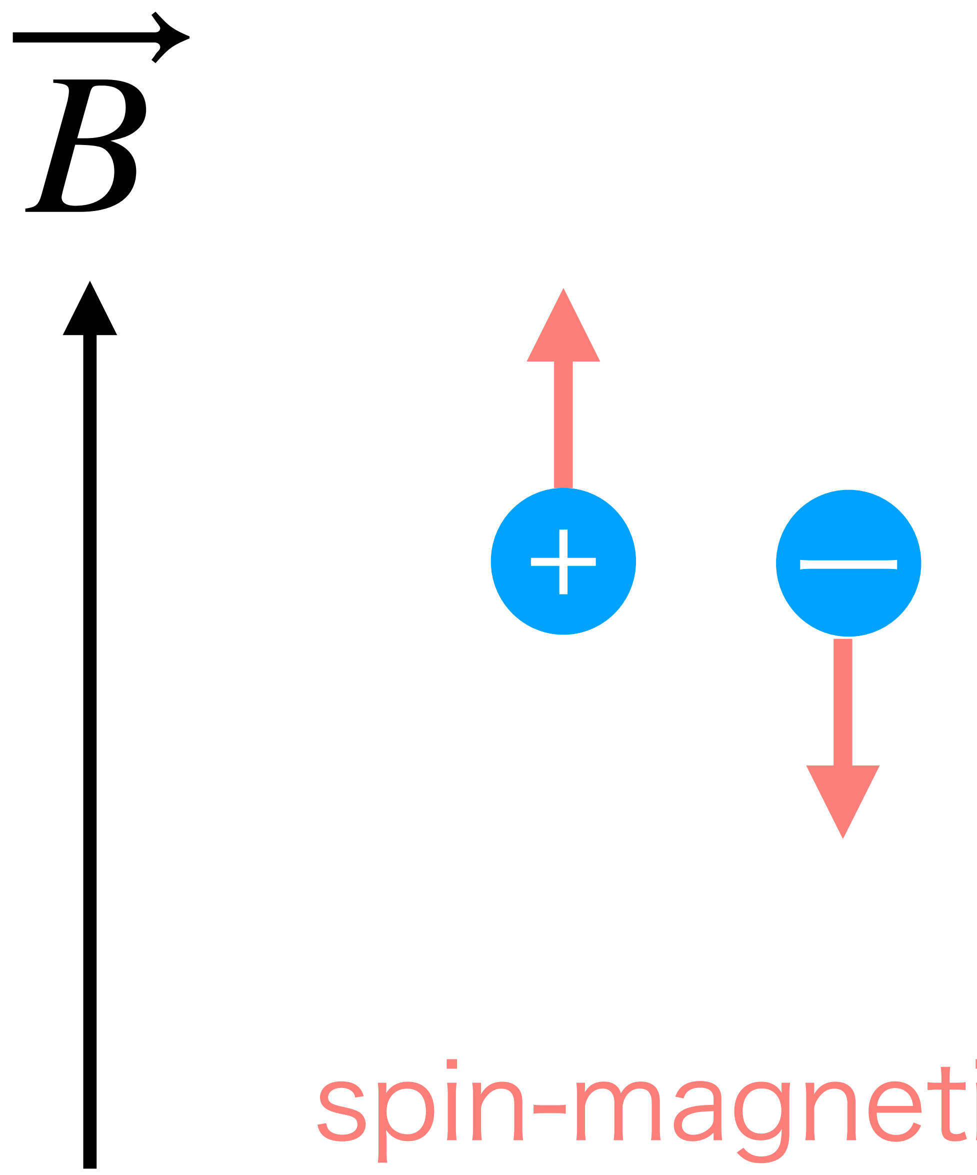
same as finite μ

Ebert et al. (2016)

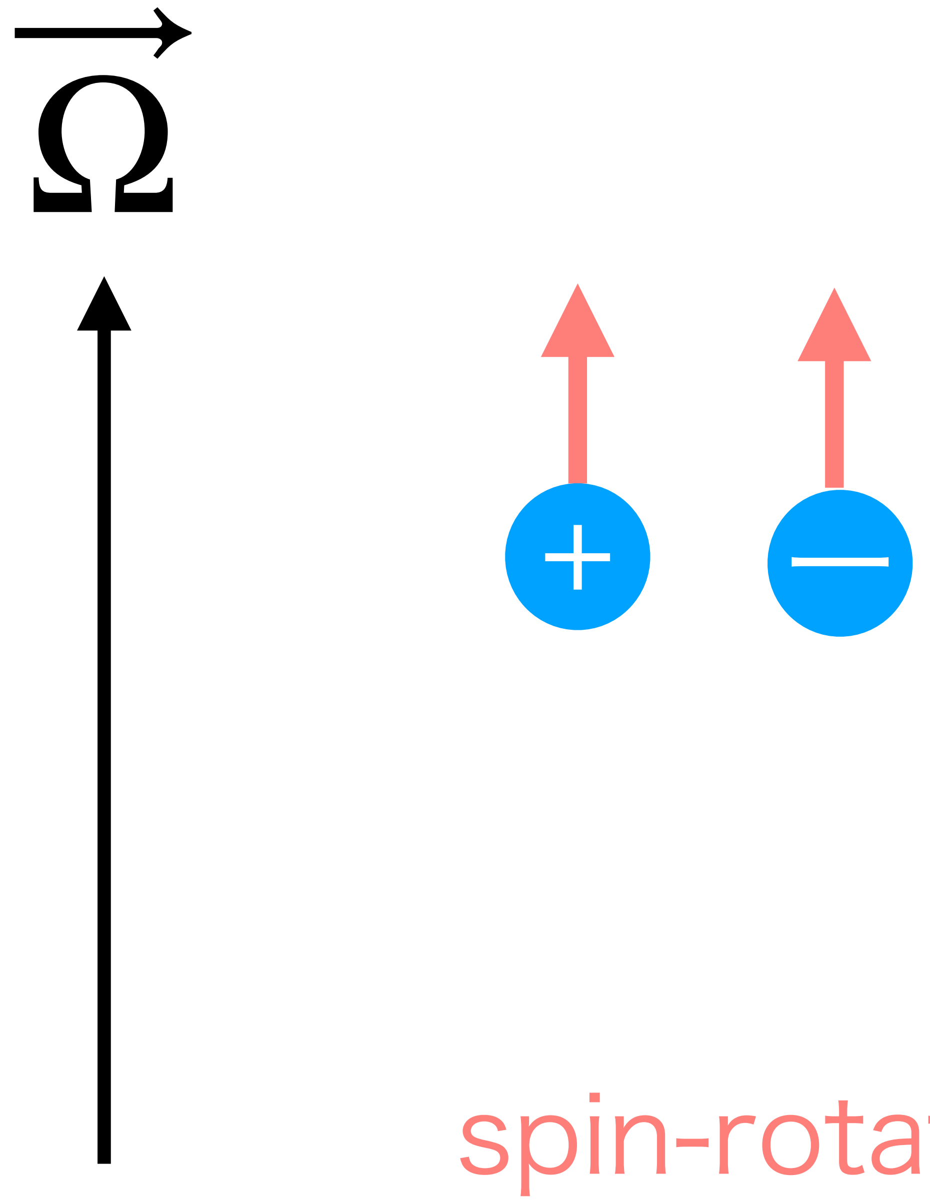
Early attempt : Transport

Hattori-Yin (2016)
Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$



spin-magnetization



spin-rotation

$$E = E_0 - \Omega(l_z + s_z)$$

Early attempt : Transport

Hattori-Yin (2016)
Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$

$$\vec{B} \parallel \vec{\Omega}$$

Puzzle on magneto-vortical charge

Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$ Hattori-Yin (2016)

free energy

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

Chen-Fukushima-Huang-Mameda (2016)
Ebihara-Fukushima-Mameda (2017)

Answer?

Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$ Hattori-Yin (2016)

free energy

$$\rho = \frac{eB\Omega}{4\pi^2} + \text{(divergence w.r.t. AM)}$$

Finial answer

Fukushima-Hattori-Mameda (in prep.)

correct Kubo formula

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

correct free energy

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

I will convince you!

Choice of angular momenta

$$Z = \mathrm{tr}\left[\mathrm{e}^{-\beta(H-\Omega\mathcal{J})}\right] \qquad \mathcal{J} = \int_{\boldsymbol{x}} \psi^\dagger (L + S)\psi$$

Chen-Fukushima-Huang-Mameda (2016)

$$L_{\mathrm{can}} = xp_y - yp_x$$

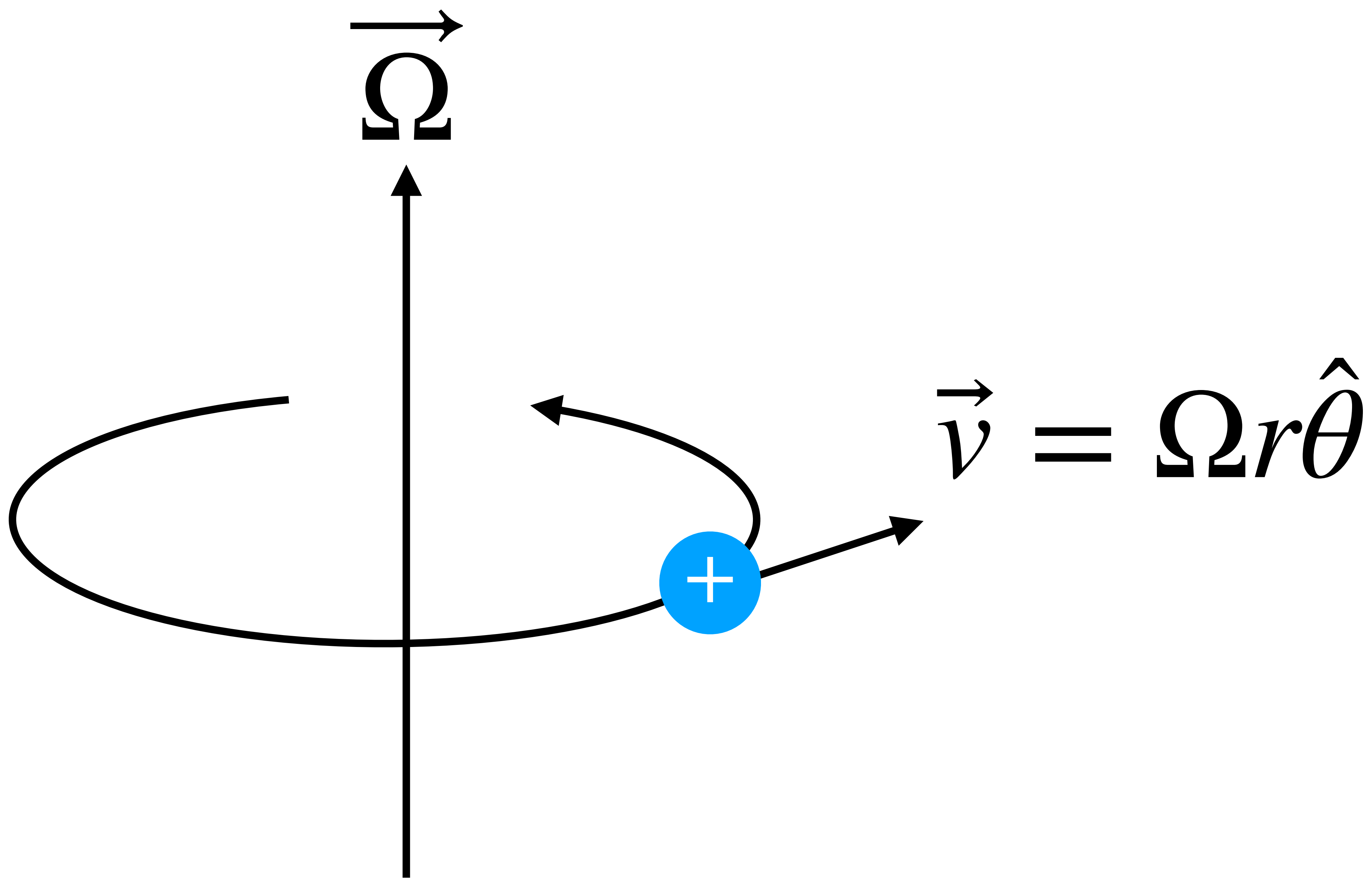
conserved AM

Fukushima-Hattori-Mameda (in prep.)

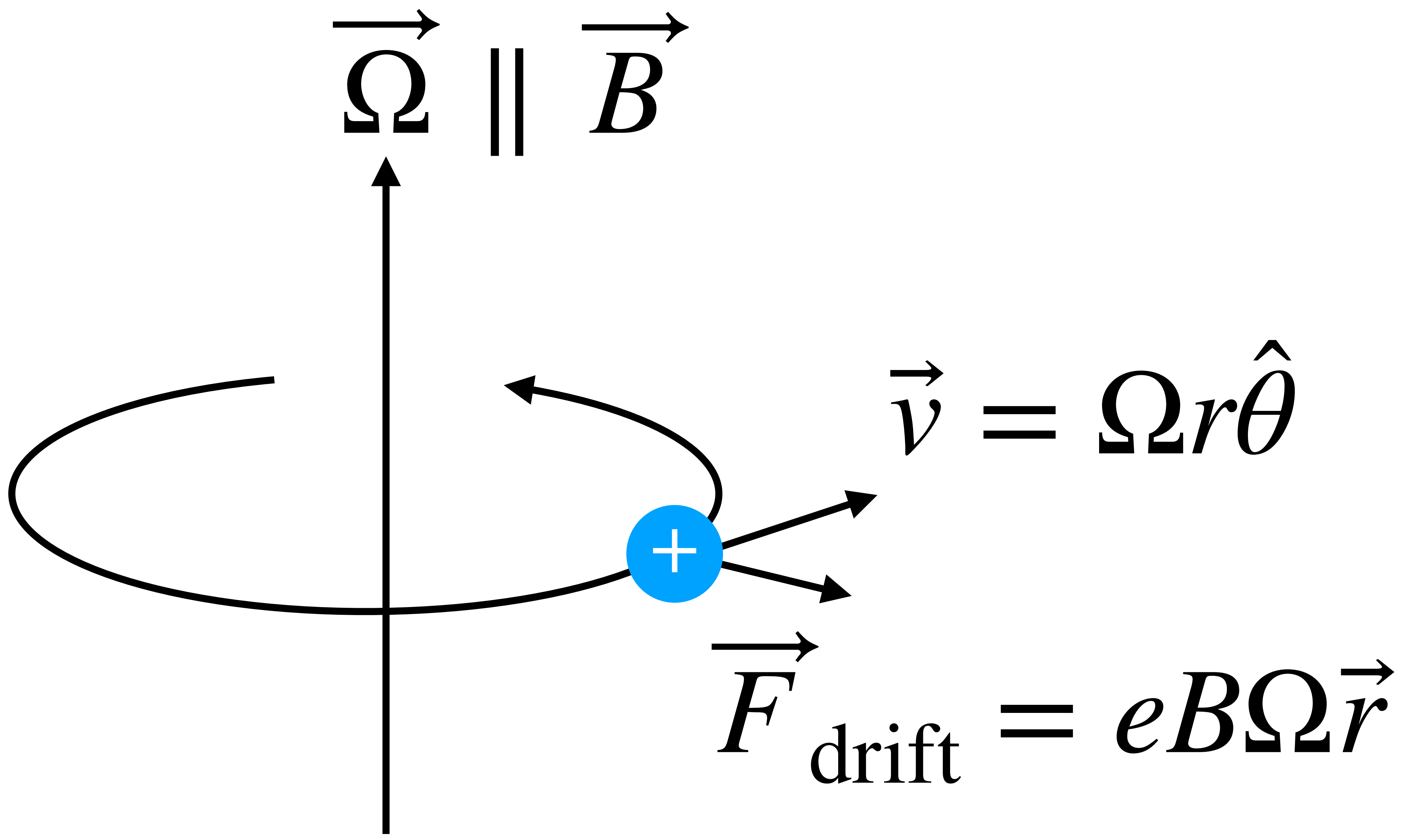
$$L_{\mathrm{kin}} = x\Pi_y - y\Pi_x$$
$$\Pi_i = p_i - eA_i$$

gauge invariant AM

Classical interpretation



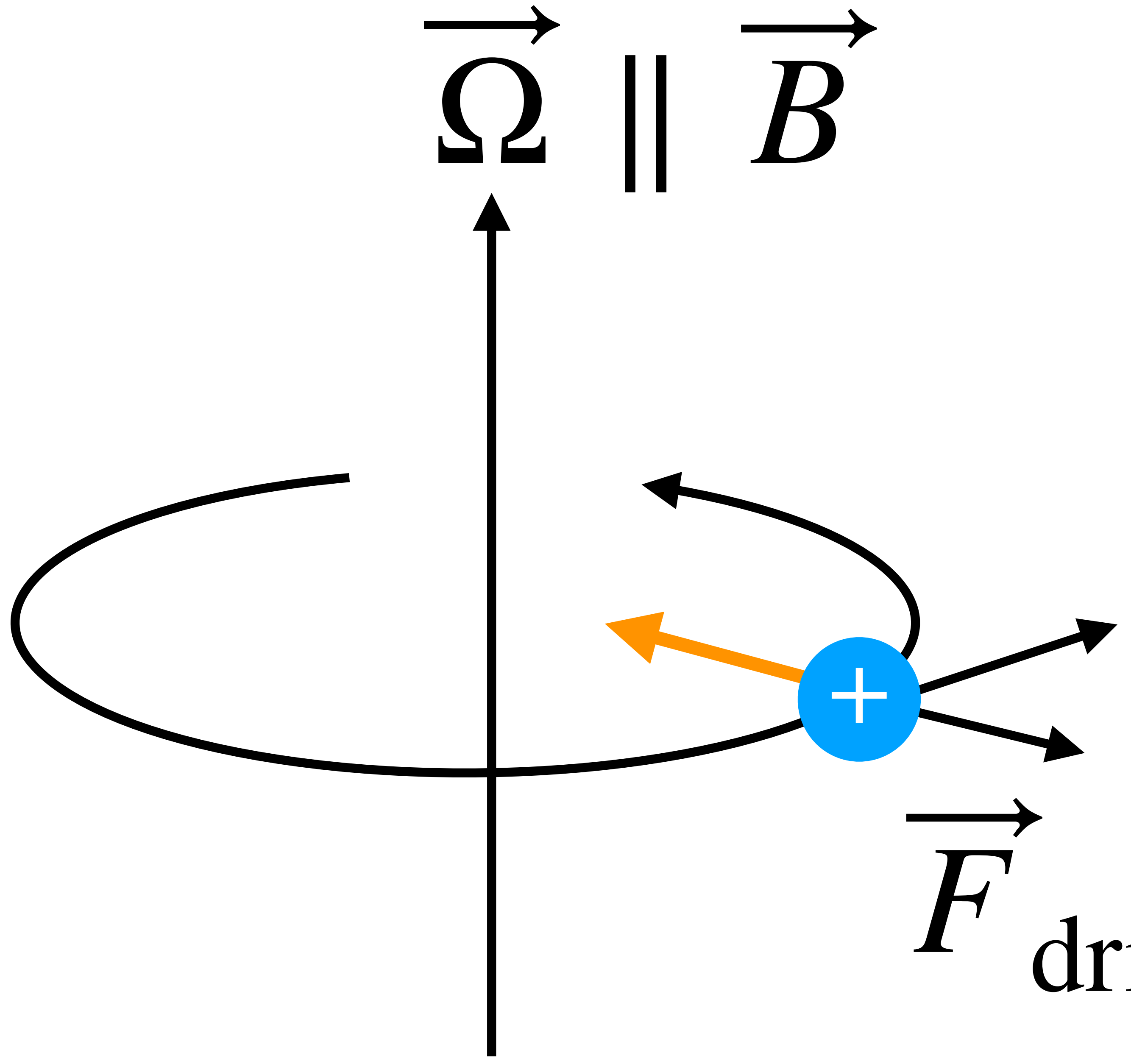
Classical interpretation



no longer circular

$H - \Omega L_{\mathrm{can}}$ unstable

Classical interpretation



$\vec{v} = \Omega r \hat{\theta}$

$\vec{F}_{\text{drift}} = eB\Omega\vec{r}$

$$e\vec{E} = -eB\Omega\vec{r}$$
$$= -\vec{\nabla}[\Omega(L_{\text{can}} - L_{\text{kin}})]$$

for symmetric gauge

$$H + \Omega(L_{\text{can}} - L_{\text{kin}}) - \Omega L_{\text{can}} = H - \Omega L_{\text{kin}}$$ stable

cf. Buzzegoli (2020)

gauge invariance ⟷ thermodynamic stability

Almost solved?

$$\mathcal{J} = \int_{\boldsymbol{x}} \psi^\dagger (L + S)\psi \qquad L_{\text{kin}} = x\Pi_y - y\Pi_x$$

gauge invariant AM

free Dirac fermion under B

$$Z = \mathrm{tr}\left[\mathrm{e}^{-\beta(H-\Omega\mathcal{J})}\right]$$

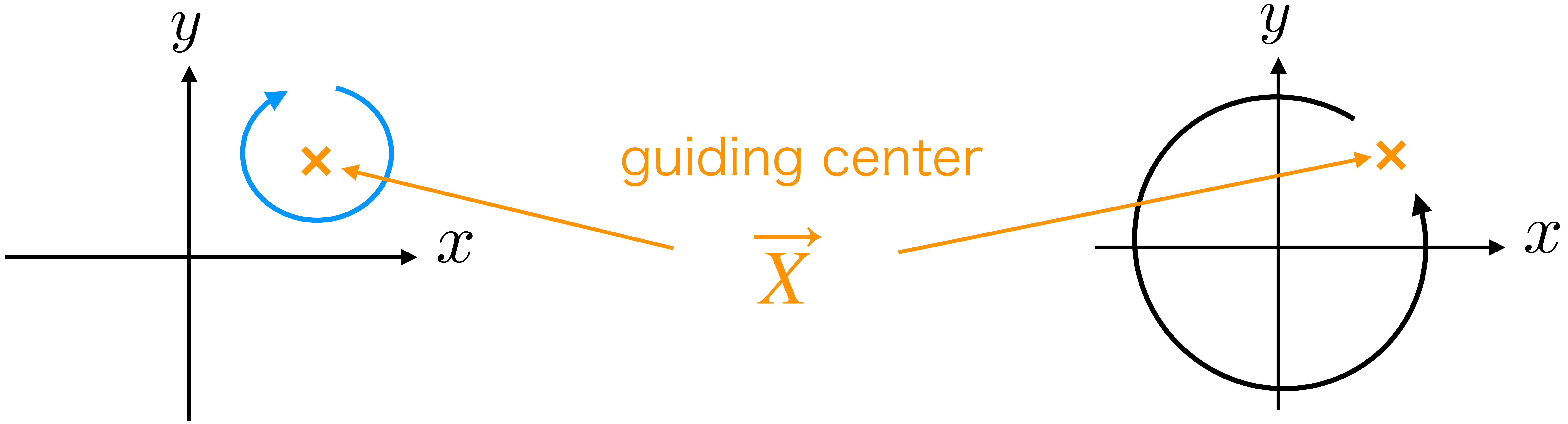
$$= \det\left[-\mathrm{i}\gamma^i D_i + M - \gamma^0\Omega(L_{\text{kin}} + S)\right]$$

How to diagonalize this?

Quantum mechanics

$$L_{\mathrm{kin}} = x\Pi_y - y\Pi_x = \Lambda + \Delta$$

$$\Lambda = (x - X)\Pi_y - (y - Y)\Pi_x \qquad \Delta = X\Pi_y - Y\Pi_x$$

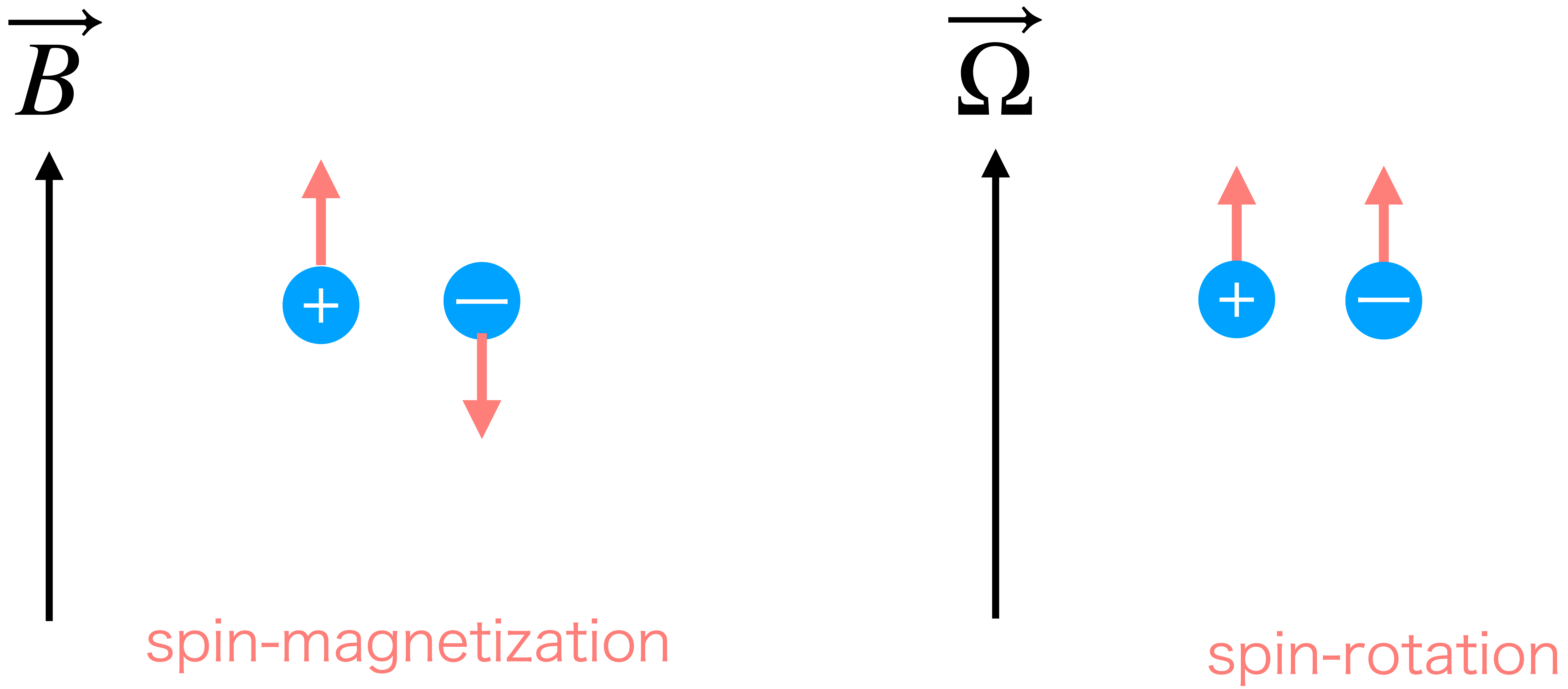


cyclotron motion — circular motion of guiding center

$$= -(2a^\dagger a + 1) \qquad = \mathrm{i}(a^\dagger b^\dagger - ab)$$

Lowest Landau Level (LLL)

$$\langle S \rangle_{\mathrm{LLL}} = +1/2$$

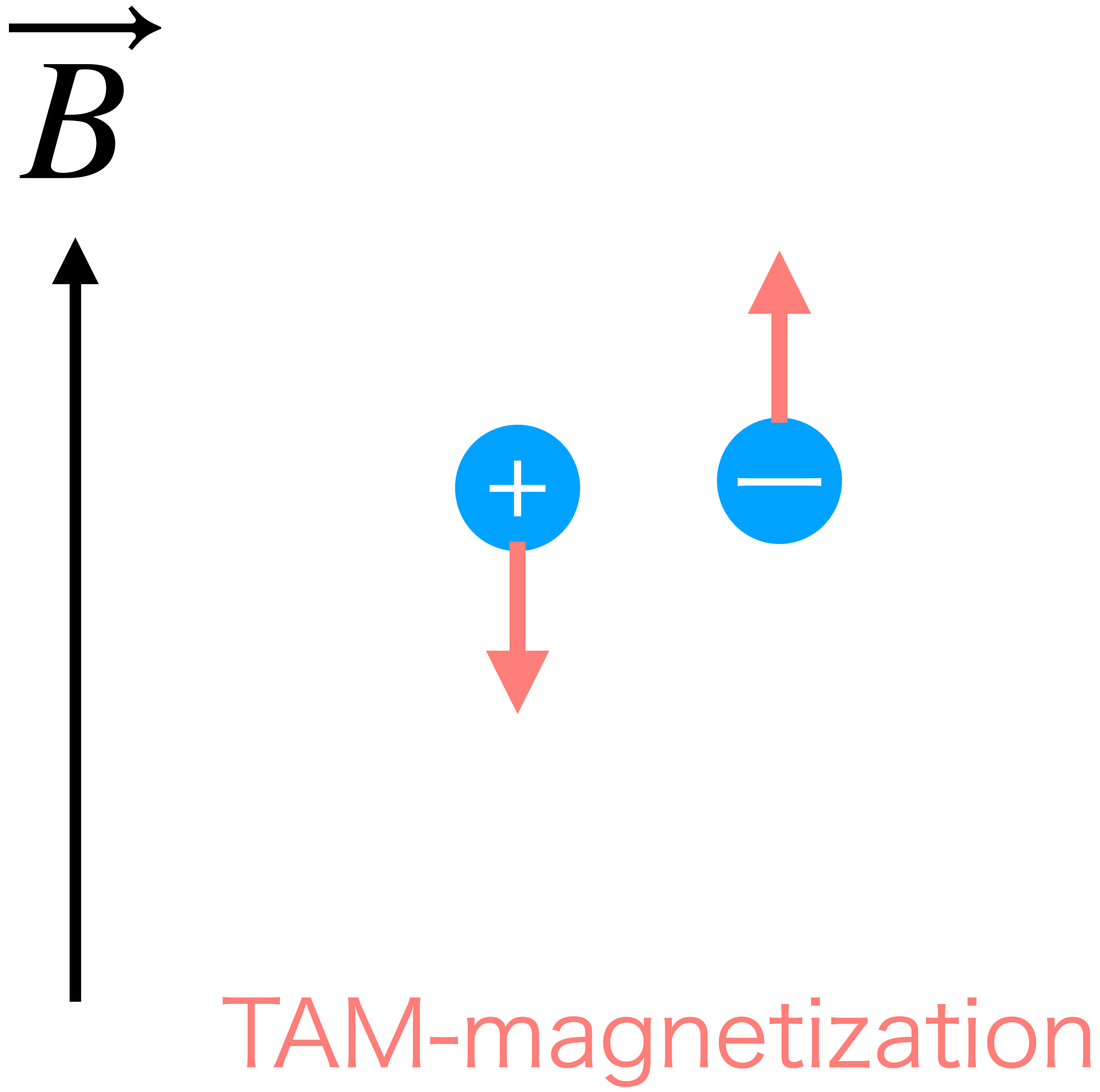


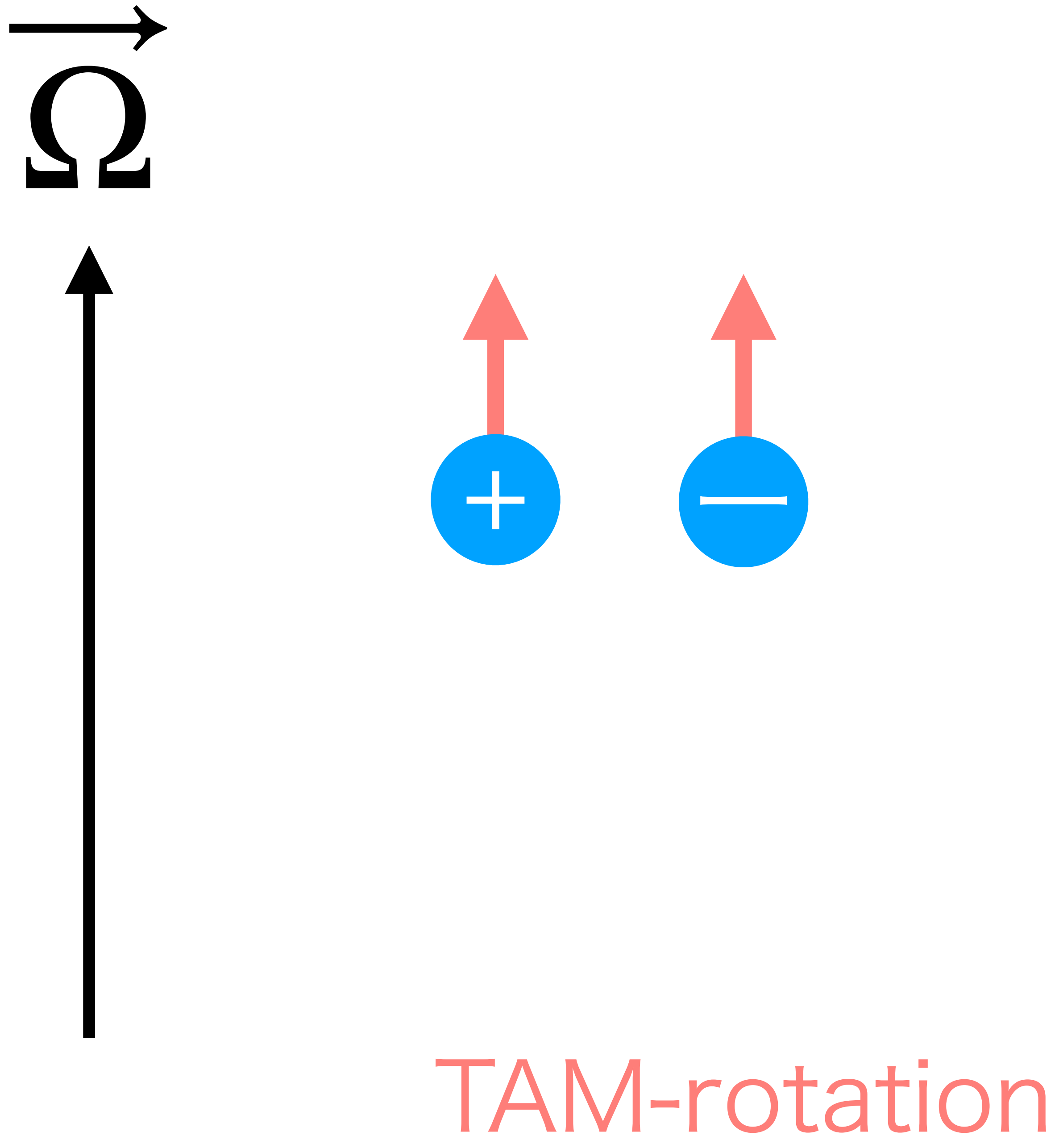
$$E = E_0 - \Omega(l_z + s_z)$$

Lowest Landau Level (LLL)

$$\langle L_{\mathrm{kin}} + S \rangle_{\mathrm{LLL}} = \langle \Lambda + \Delta + S \rangle_{\mathrm{LLL}} = -1/2$$

$-1 \quad 0 \quad +1/2$





$$E = E_0 - \Omega(l_z + s_z)$$

Lowest Landau Level (LLL)

$$\langle L_{\mathrm{kin}} + S \rangle_{\mathrm{LLL}} = \langle \Lambda + \Delta + S \rangle_{\mathrm{LLL}} = -1/2$$

$$\underset{-1}{\Lambda} \qquad \underset{0}{\Delta} \qquad \underset{+1/2}{S}$$

$\vec{B} \parallel \vec{\Omega}$

of $+$ $<$ # of $-$

This suggests $\rho = -\dfrac{eB\Omega}{4\pi^2}$

Thermodynamics

Fukushima-Hattori-Mameda (in prep.)

$$Z = \det\left[-\mathrm{i}\gamma^i D_i + M - \gamma^0 \underline{\Omega(\Lambda + \Delta + S)}\right]$$

$\nu = -\,\Omega/2$ (LLL)

$$P_{\mathrm{LLL}} = \frac{eB}{2\pi}\int\frac{\mathrm{d}p_z}{2\pi}\left[\ln\left(1+\mathrm{e}^{-\beta(\epsilon-\nu)}\right) + \ln\left(1+\mathrm{e}^{-\beta(\epsilon+\nu)}\right)\right]$$

massless limit $$\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial\nu} = -\frac{eB\Omega}{4\pi^2}$$ (T-independent)

Comparisons

Fukushima-Hattori-Mameda (in prep.)
free energy (LLL)

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}}$$

spin orbital

Ebihara-Fukushima-Mameda (2017)
free energy (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

due to $\vec{F}_{\text{drift}} = eB\Omega\vec{r}$

Hattori-Yin (2016)
linear responce (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2}$$

a mistake found

Yang et. al (2020) Mameda(2023)
chiral kinetic theory
correct

$$\rho = \frac{eB\Omega}{4\pi^2}$$

no Landau level formed by weak B

Relation to chiral anomaly

for any B only for strong B

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}}$$

Only the spin part is due to chiral anomaly?

Hattori-Yin (2016)	YES	This is T-independent
Yang-Yamamoto (2021)	YES	This is derived as a Chern-Simons current

Relation to chiral anomaly

charge

$$\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \mu} = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}} + \frac{eB\mu}{2\pi^2}$$

spin orbital

angular momentum

$$J = \frac{\partial P_{\mathrm{LLL}}}{\partial \Omega} = \underline{\frac{eB\mu}{4\pi^2}} - \underline{\frac{eB\mu}{2\pi^2}} + \frac{eB\Omega}{8\pi^2}$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\mathrm{LLL}}}{\partial \mu \, \partial \Omega}$$

→ same coefficients shared

Relation to chiral anomaly

charge $$\rho = \frac{\partial P_{\rm LLL}}{\partial \mu} = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}} + \frac{eB\mu}{2\pi^2}$$

angular momentum $$J = \frac{\partial P_{\rm LLL}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

$$= j^5_{\rm CSE}/2$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\rm LLL}}{\partial \mu\, \partial \Omega}$$ ⇒ Since $j^5_{\rm CSE}$ is anomaly-related, so is ρ

Summary

- ✓ reformulate gauge-invariant and stable thermodynamics
- ✓ Magnetovortical charge sign-inverted by cyclotron motion
- ✓ The charge is anomaly-related
- ✓ applicability to

  HIC : spin polarization under strong B

  cold atoms : quantum simulator

  (nonrelativistic Hamiltonian can be diagonalized)