Closed-channel parameters of Feshbach resonances

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Tohoku University

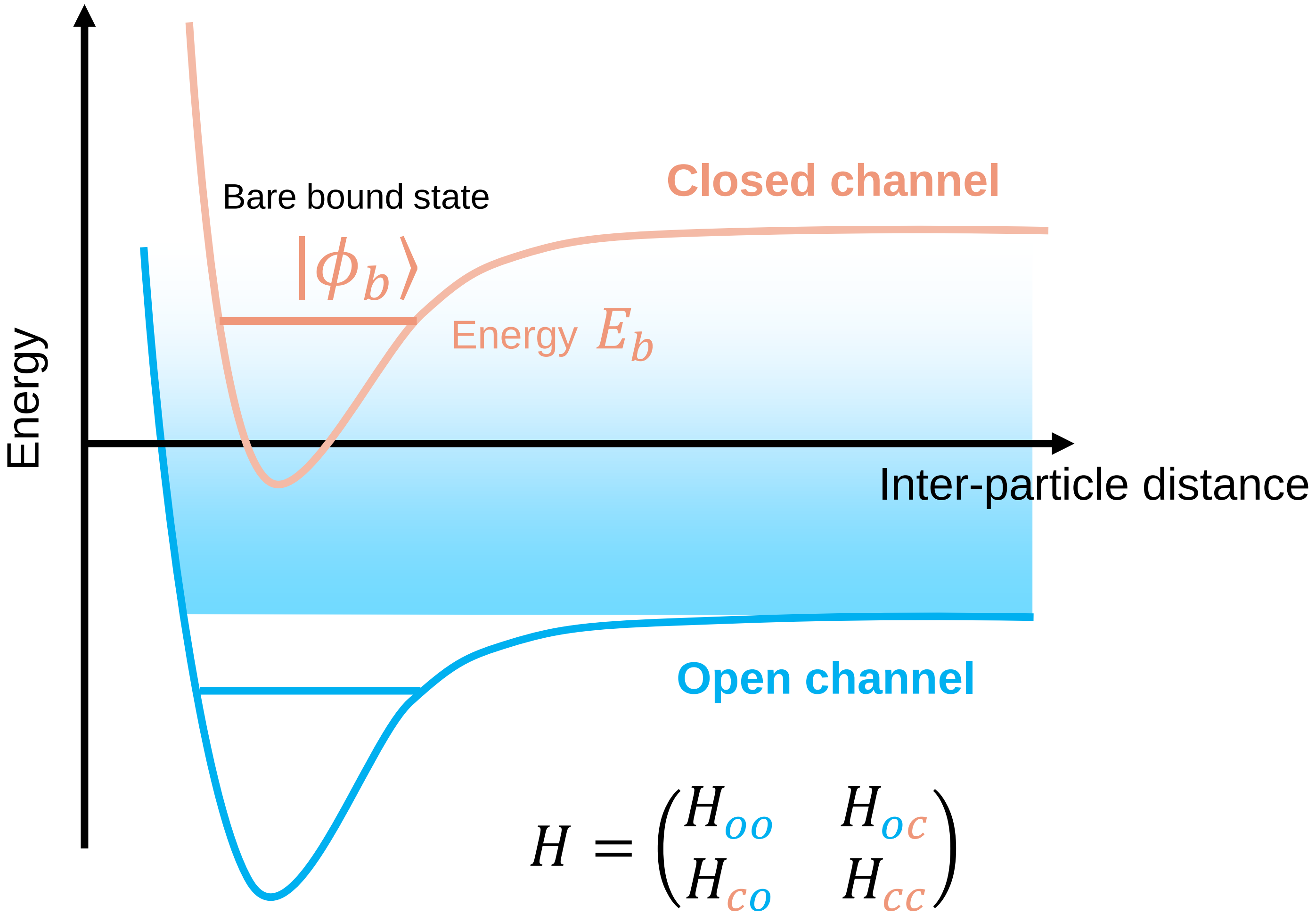
Motivation

- Feshbach resonances: resonance created by a bound state coupled to a continuum.
  - **Ultracold atoms**: very successful tool to control the interactions between ultra-cold atoms for more than 20 years.
  - **Hadron physics**: hadron resonances near thresholds

- We know how to make models of Feshbach resonances, but **what is physical** and **what is arbitrary** in these models?
- Can we learn about the bound state causing the resonance?

Fano-Feshbach resonances

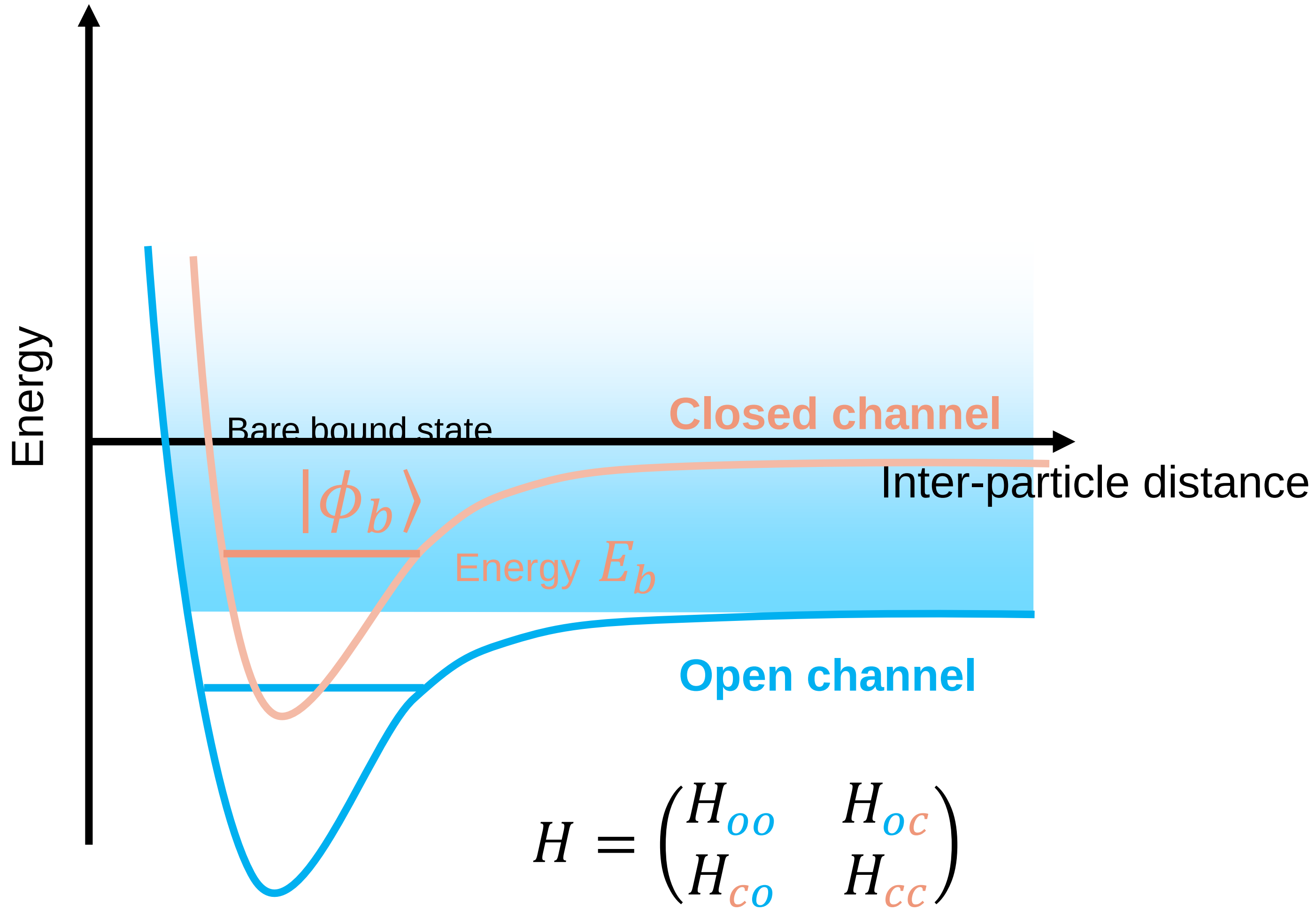
Principle: two particles can be in different internal states (channels) which are coupled to each other.



$$H = \begin{pmatrix} H_{oo} & H_{oc} \\ H_{co} & H_{cc} \end{pmatrix}$$

Fano-Feshbach resonances

Principle: two atoms can be in different electronic channels which are coupled to each other.



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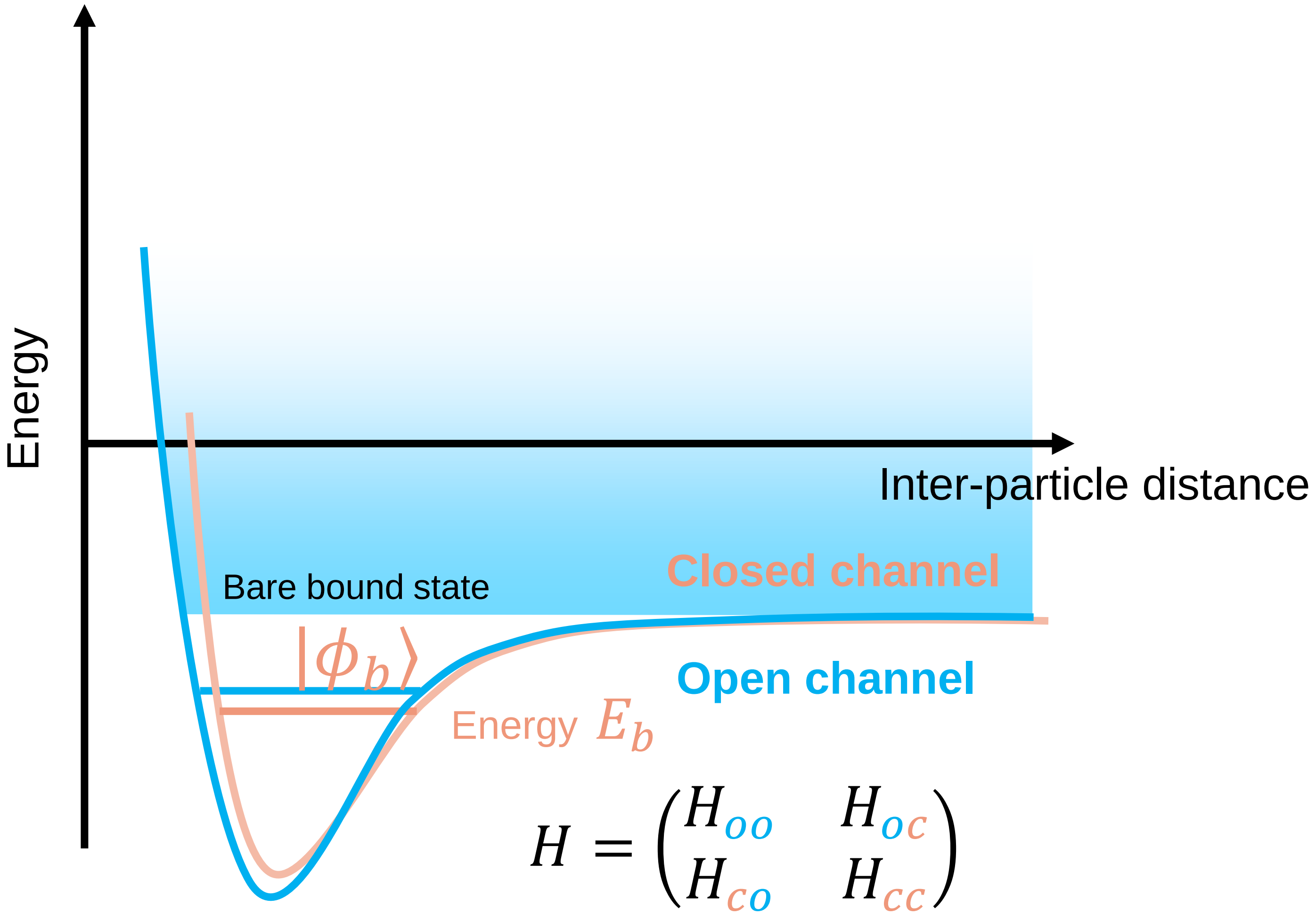
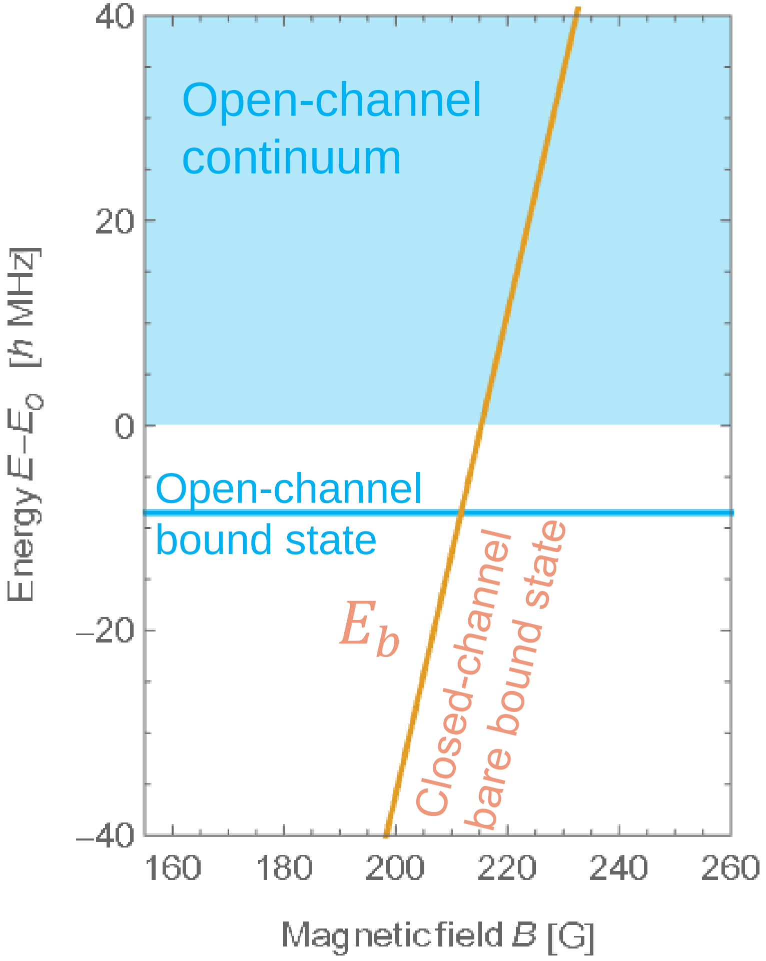


Two-body observables



Resonance of ^{40}K atoms near 202 G

$$\gamma = \lim_{k\to 0} \frac{\Gamma}{2k}$$
$$\Delta_0 = \lim_{k\to 0} \Delta$$

Two-body observables

1) Scattering phase shift:

$$\eta = \eta_o - \arctan \frac{\Gamma(E)/2}{E - E_b - \Delta(E)}$$

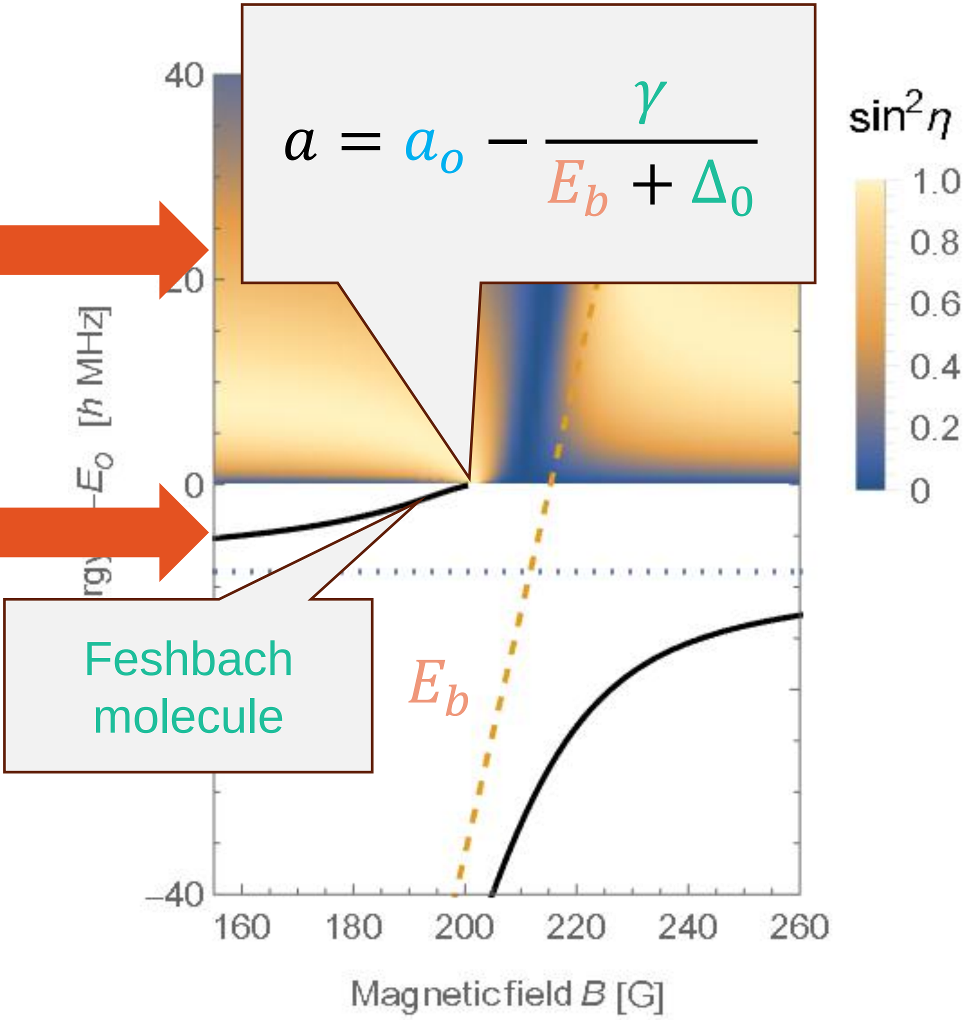
2) Dressed bound-state energy:

$$E = E_b + \Delta(E)$$

Resonance shift Δ and width Γ:

$$\Delta - i\Gamma/2 = \langle \phi_b | H_{co} \frac{1}{E - H_{oo}} H_{oc} | \phi_b \rangle$$

Should depend on the closed-channel details!



Resonance of ^{40}K atoms near 202 G

Important question

From these observables, can we determine the closed-channel details?

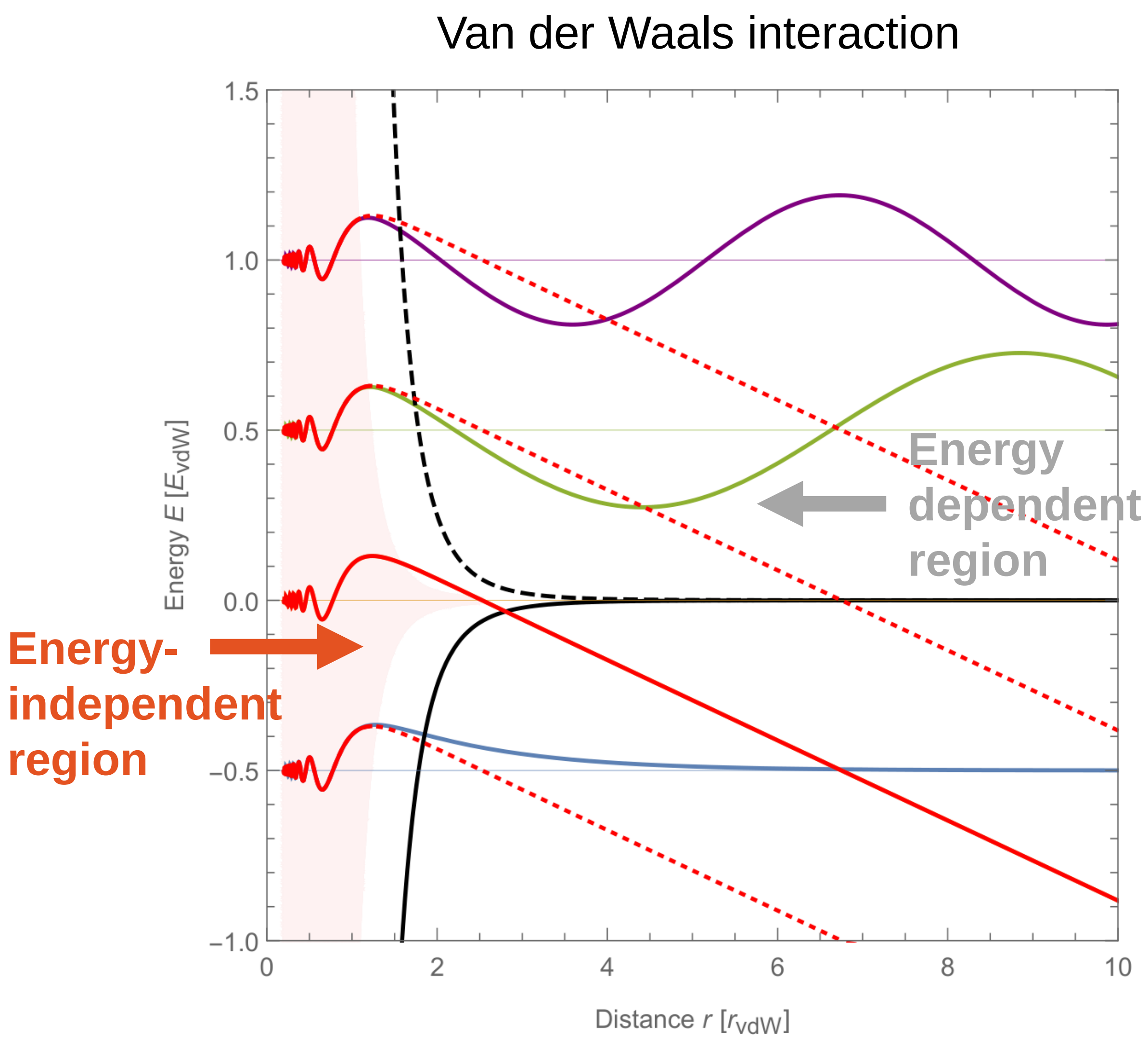
- Energy E_b of the bare bound state ?
- Scattering length a_c in the closed channel ?

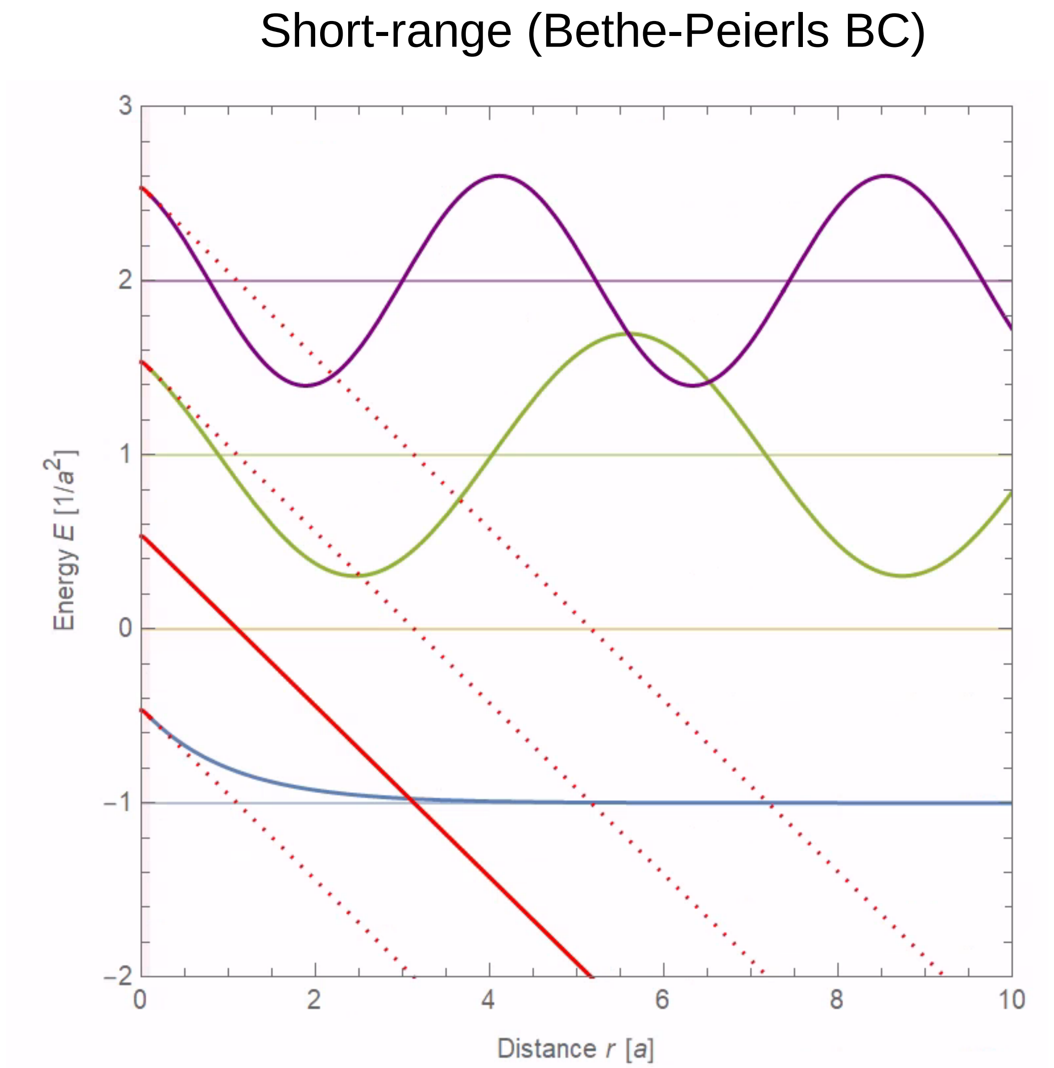
Answer: **no**.

For resonances described by a Quantum Defect Theory (QDT) such as

- All the atomic Feshbach resonances
- Hadron resonances very close to a threshold.

Quantum Defect Theory (QDT)





Quantum Defect Theory (QDT)

- The QDT gives explicit expressions for the shift and width:

$$\Delta(E) = \cdots$$

$$\frac{\Gamma(E)}{2} = \cdots$$

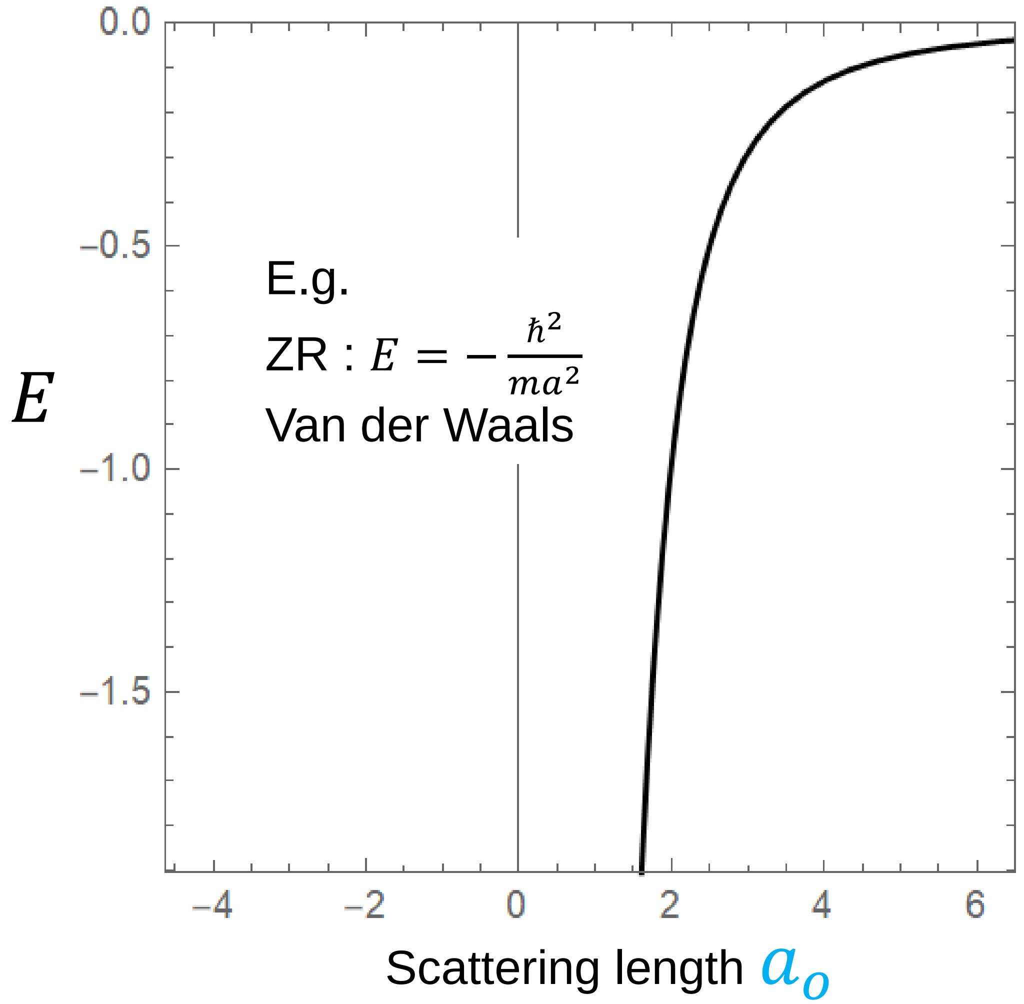
- The expressions should be ***renormalised*** in terms of observables

Renormalised Quantum Defect Theory

dressed bare shift

$$E = E_b + \underbrace{\Delta(E)}_{\Delta_0 + \frac{\gamma}{\lambda(E) - a_o}}$$

Energy of an open-channel bound state



Renormalised Quantum Defect Theory

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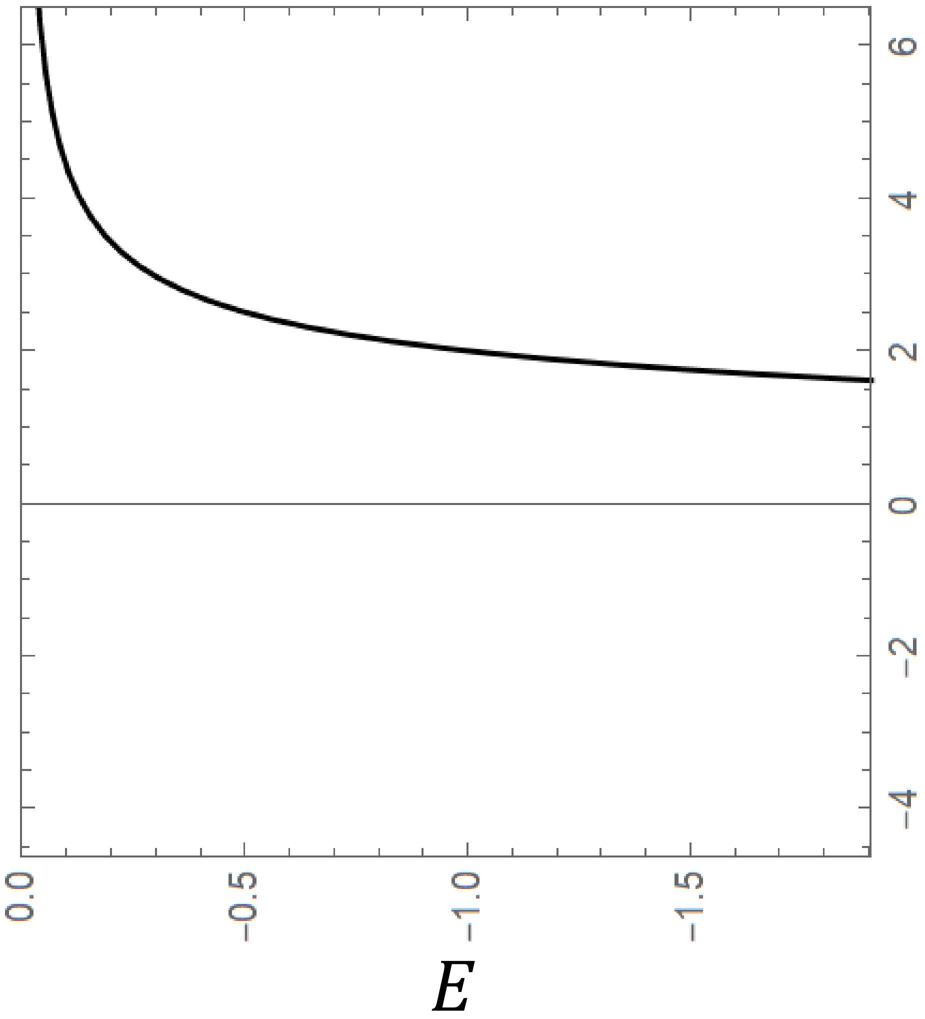
Energy of an open-channel bound state

Scattering length a_o

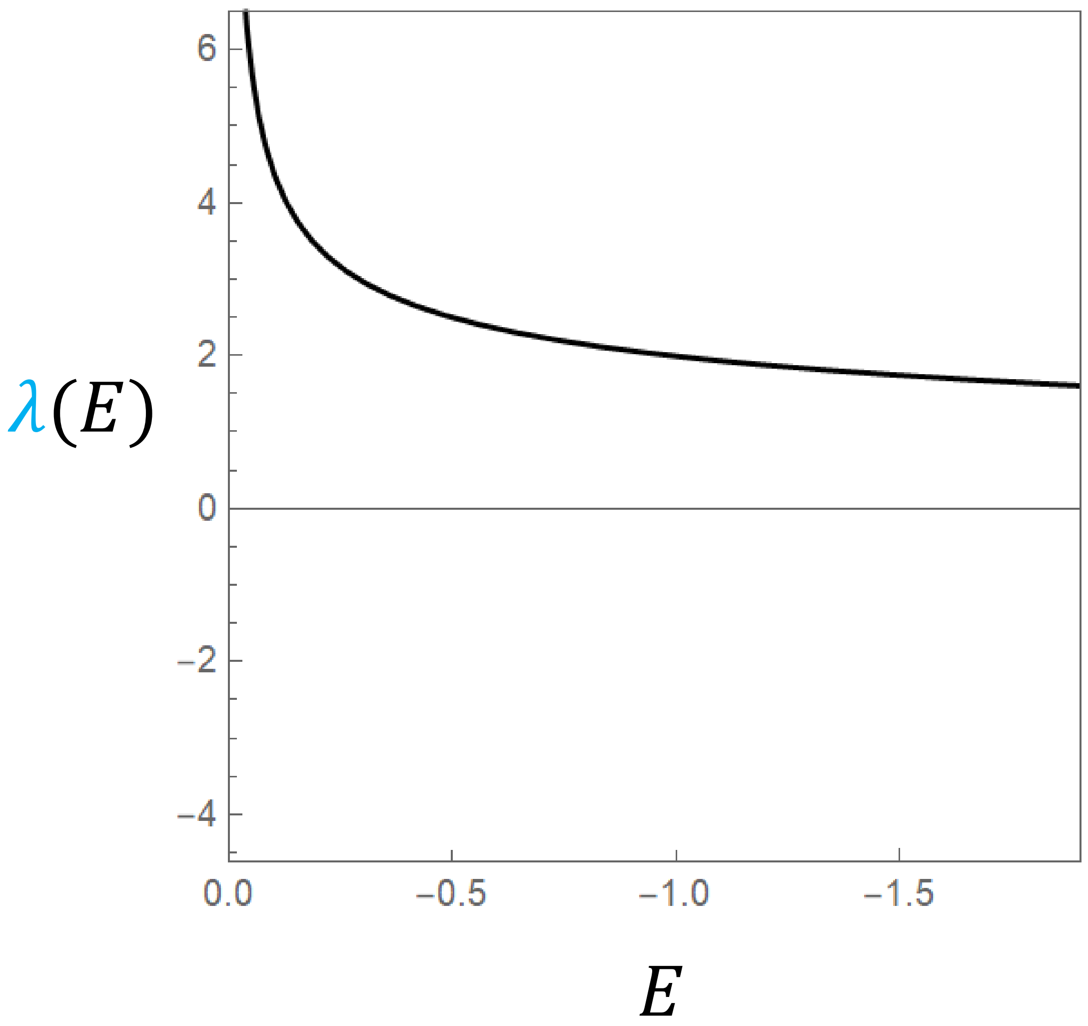


Renormalised Quantum Defect Theory

$$E = E_b + \underbrace{\Delta(E)}_{\Delta_0 + \frac{\gamma}{\lambda(E) - a_o}}$$

$$E - \frac{\gamma}{\lambda(E) - a_o} = E_b + \Delta_0$$

Energy of an open-channel bound state



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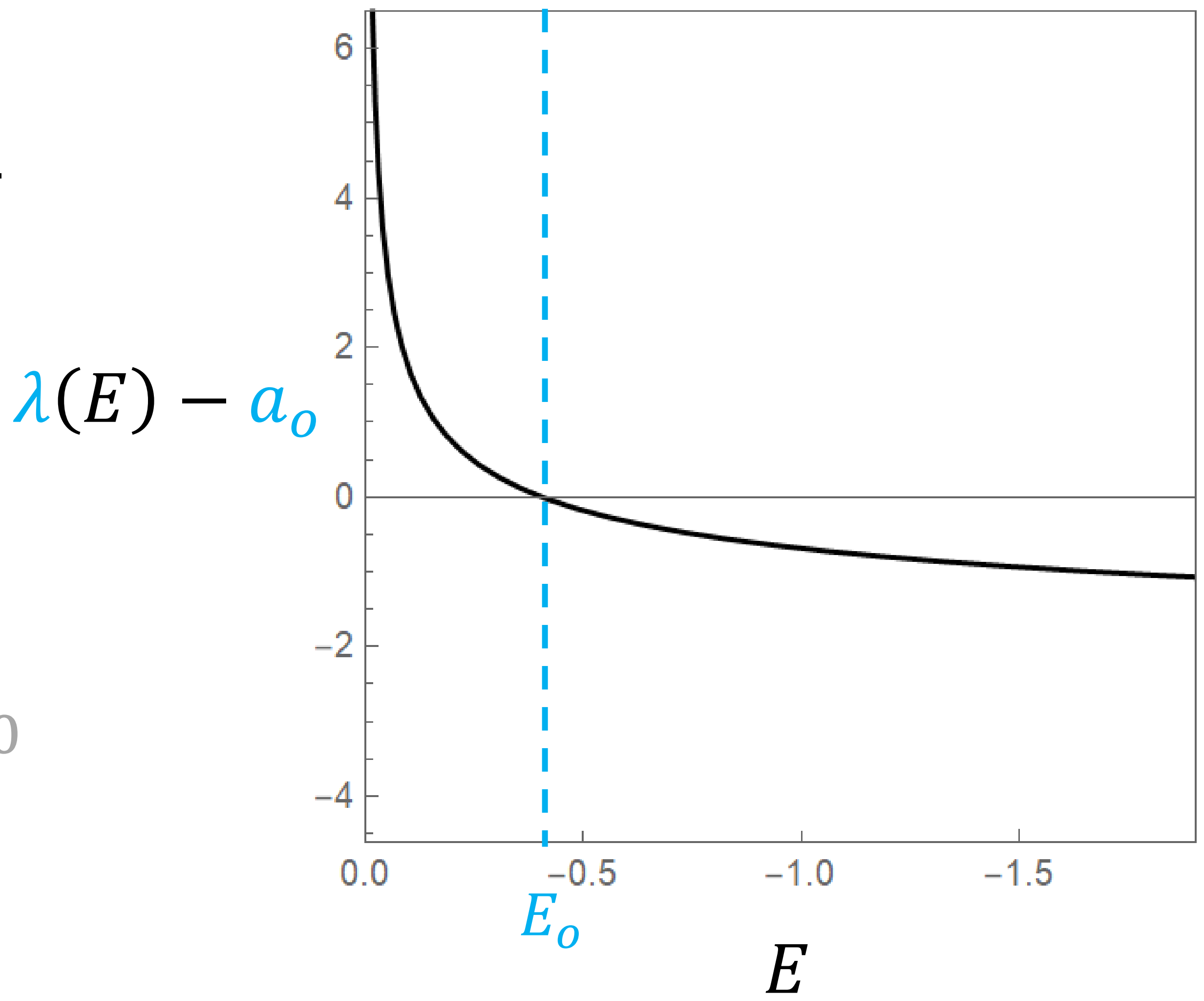

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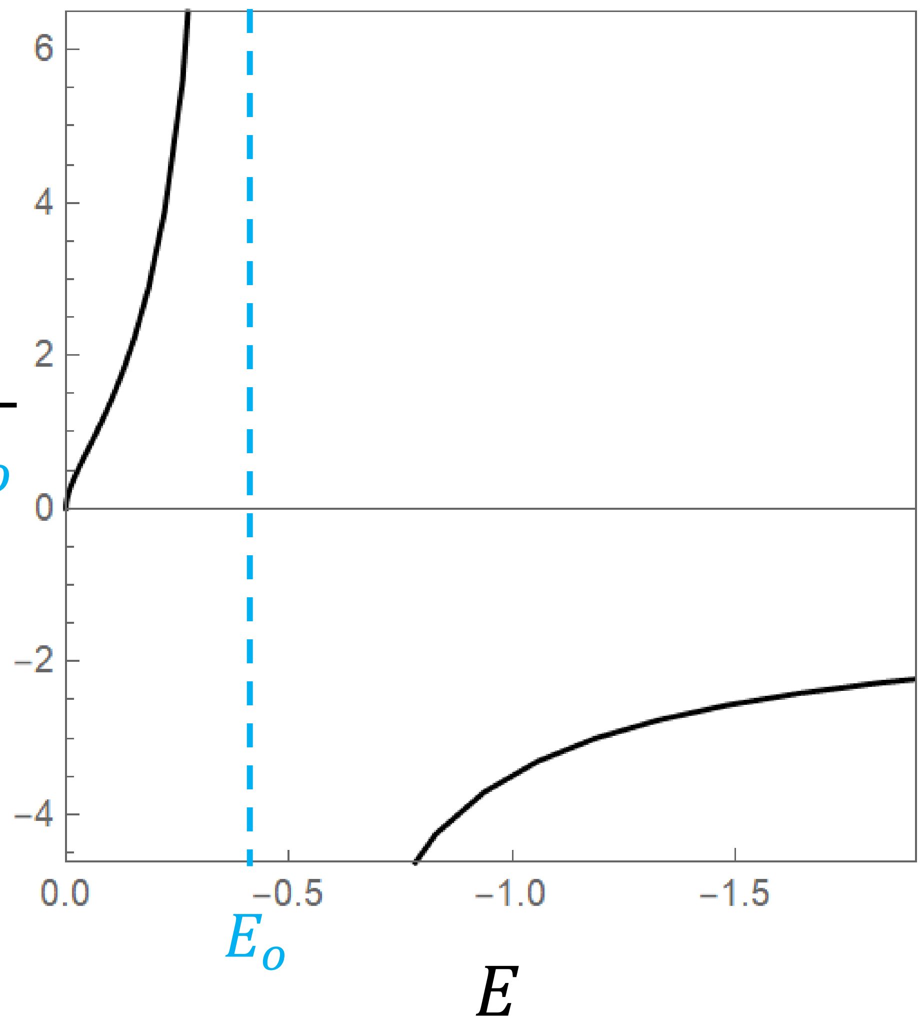
$$\frac{\gamma}{\lambda(E) - a_o}$$



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$$-\frac{\gamma}{\lambda(E) - a_o}$$

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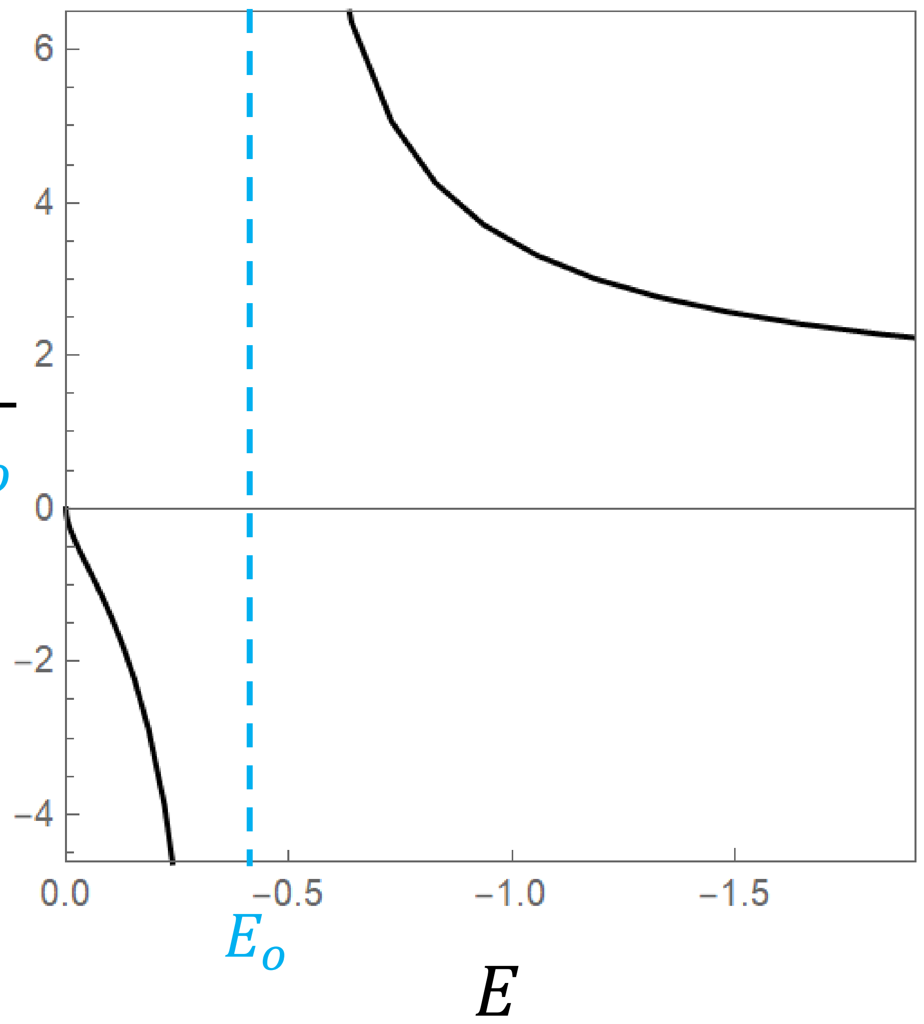

Renormalised Quantum Defect Theory

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$$\Downarrow$$

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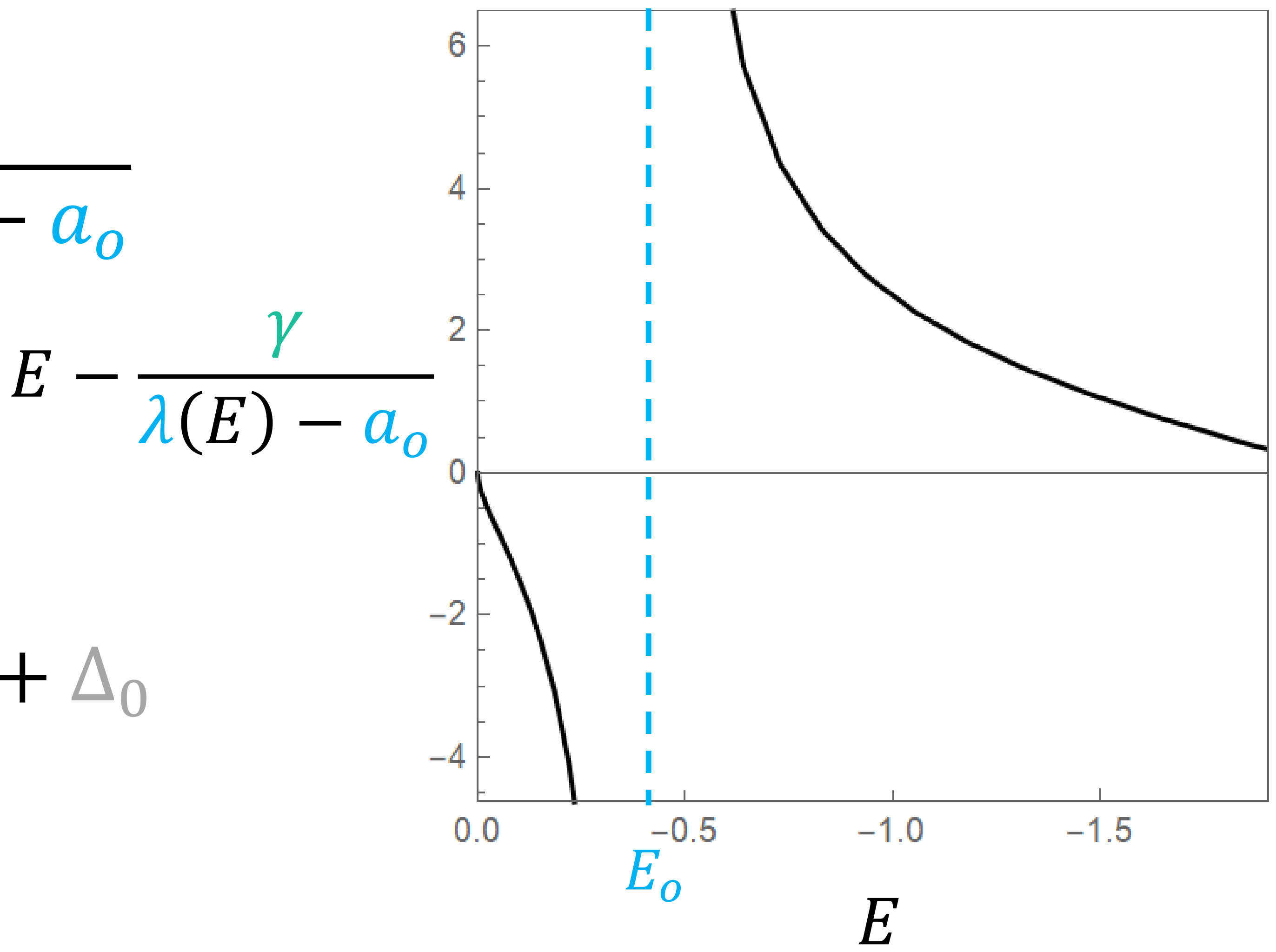

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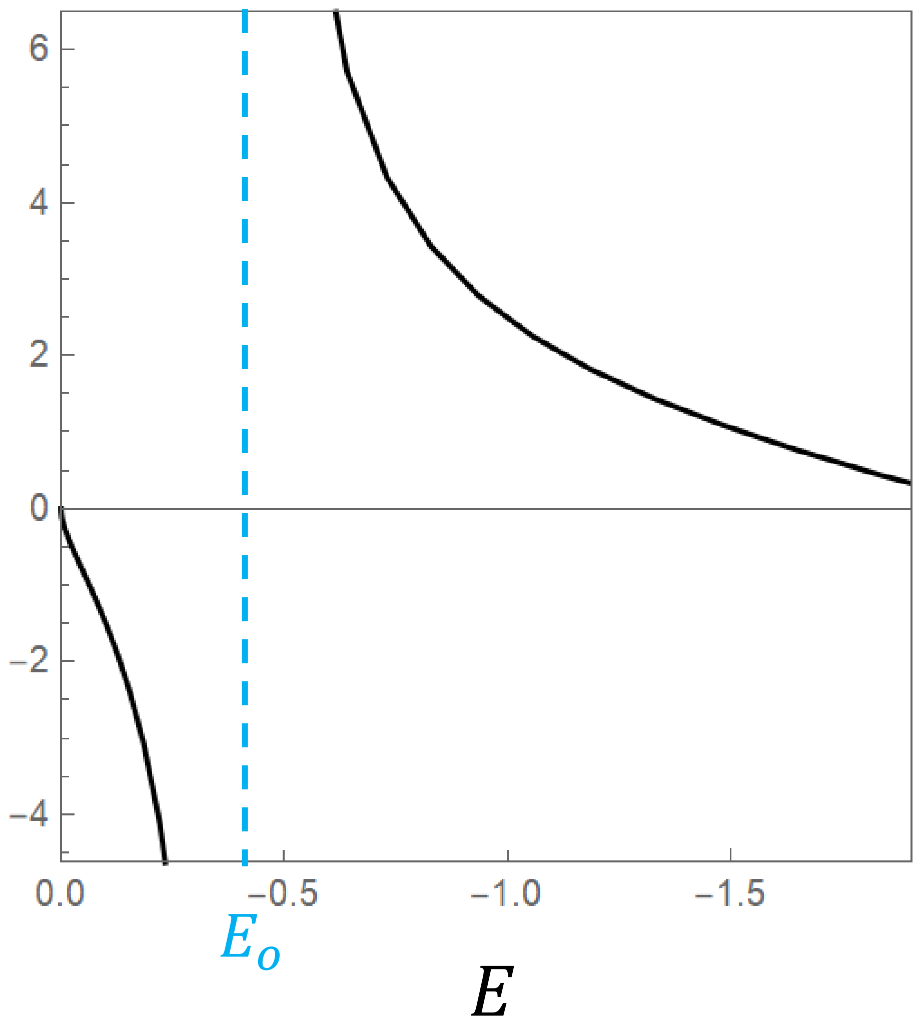
$E_b + \Delta_0$



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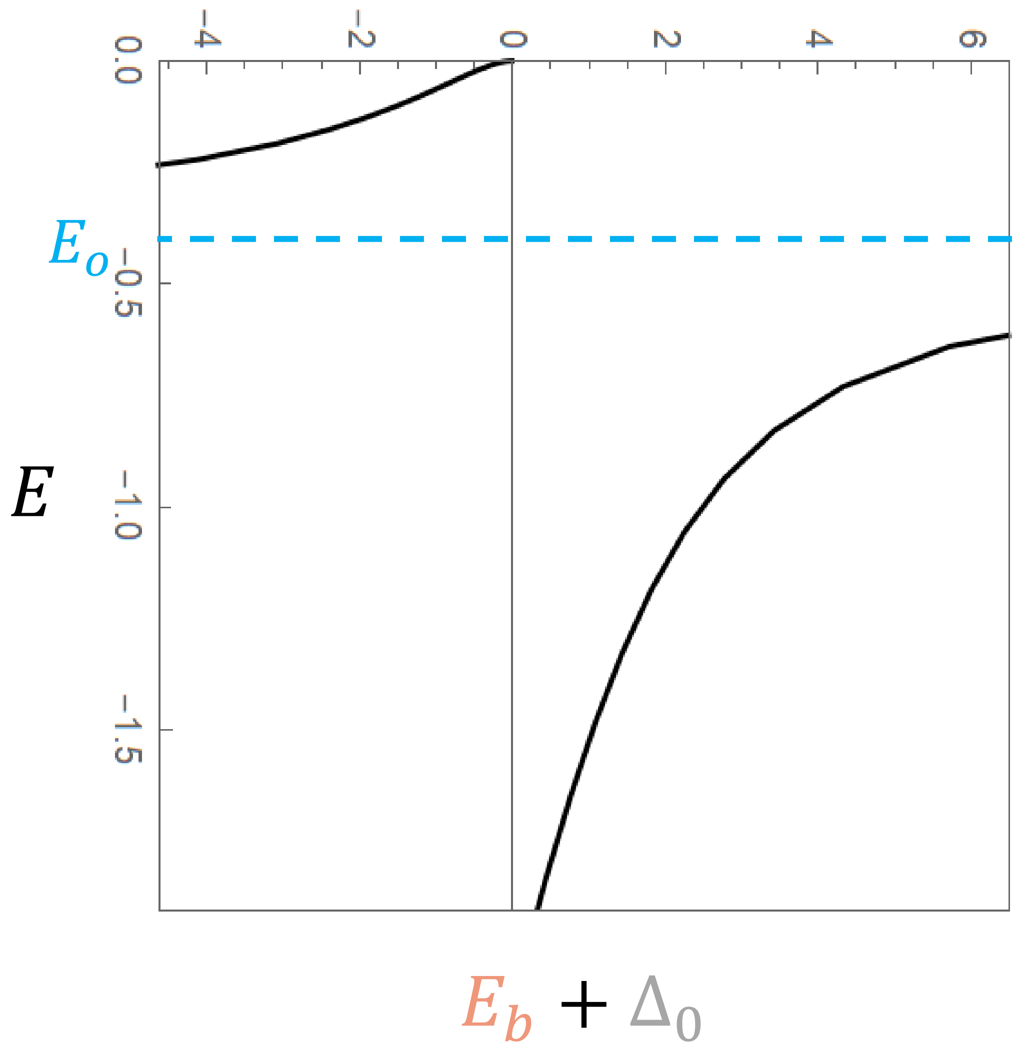

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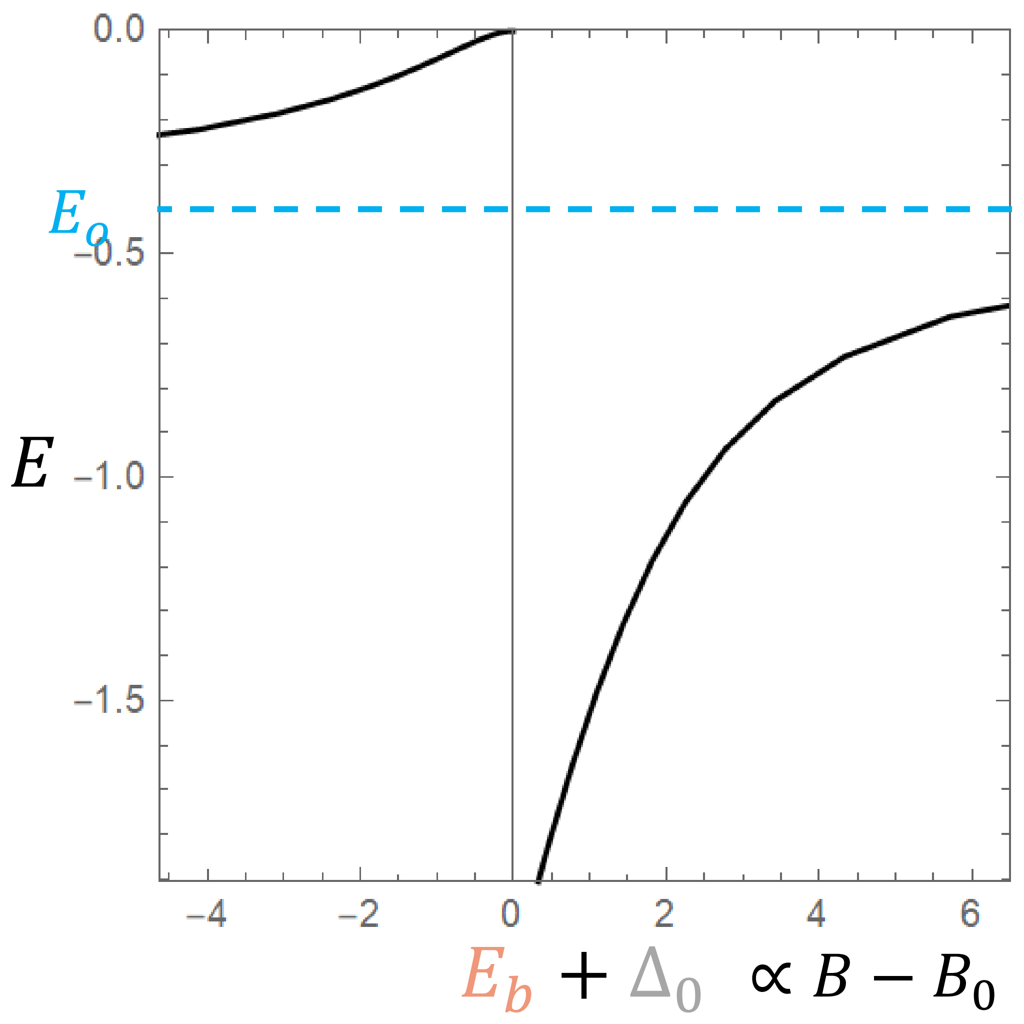

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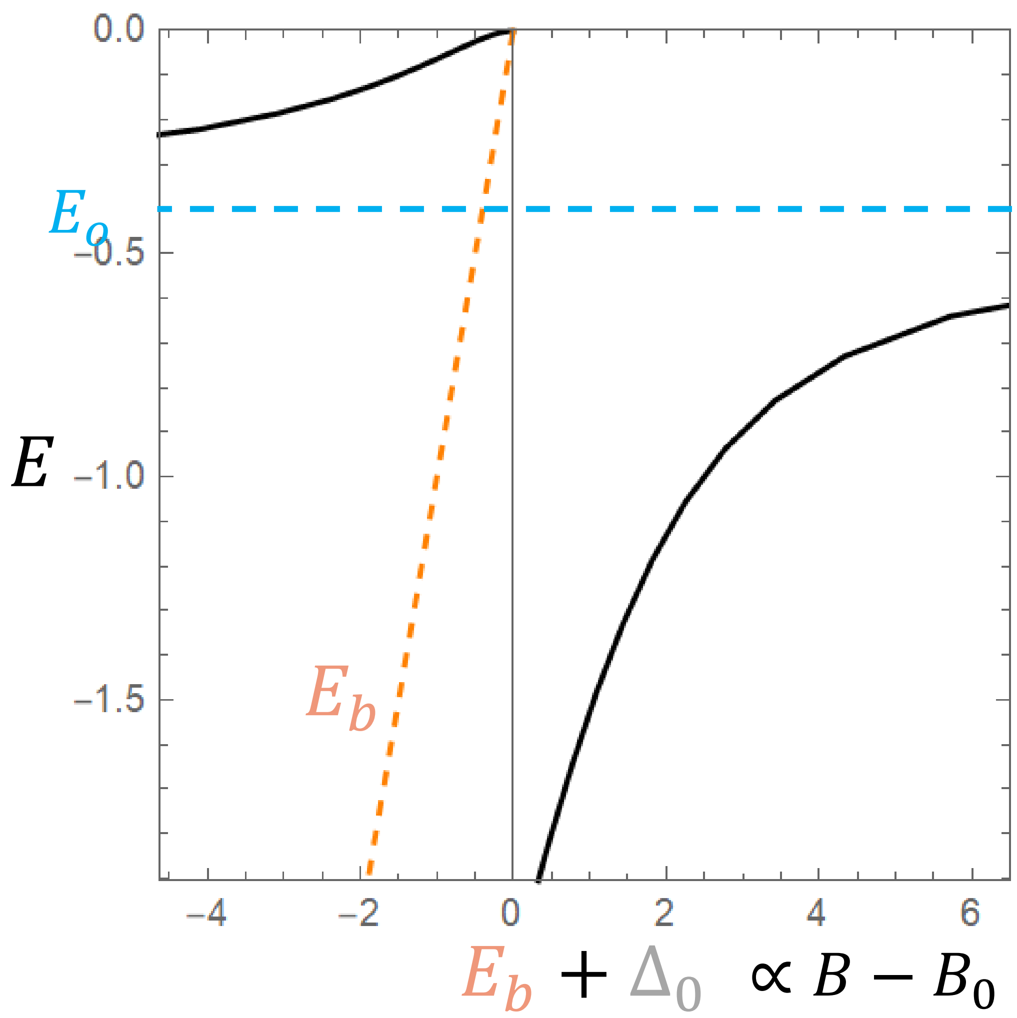

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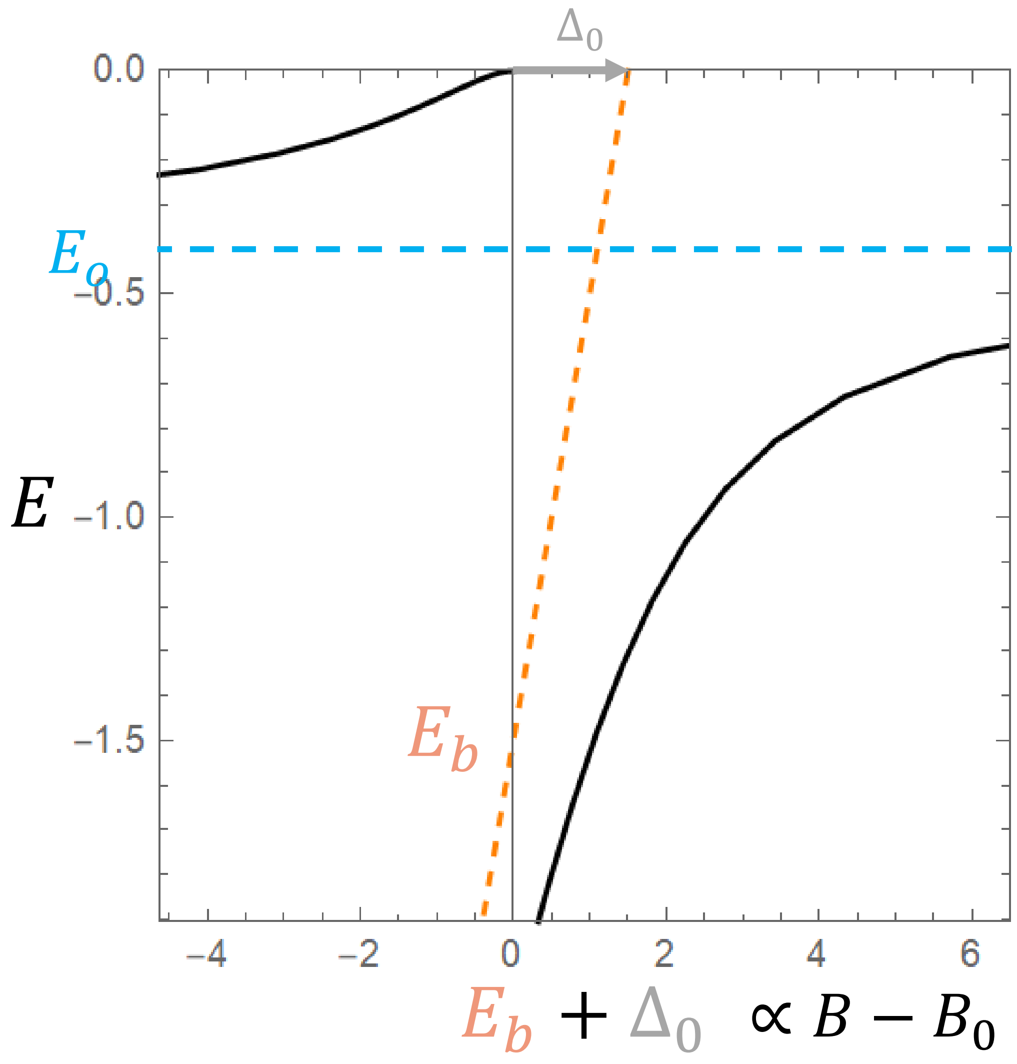

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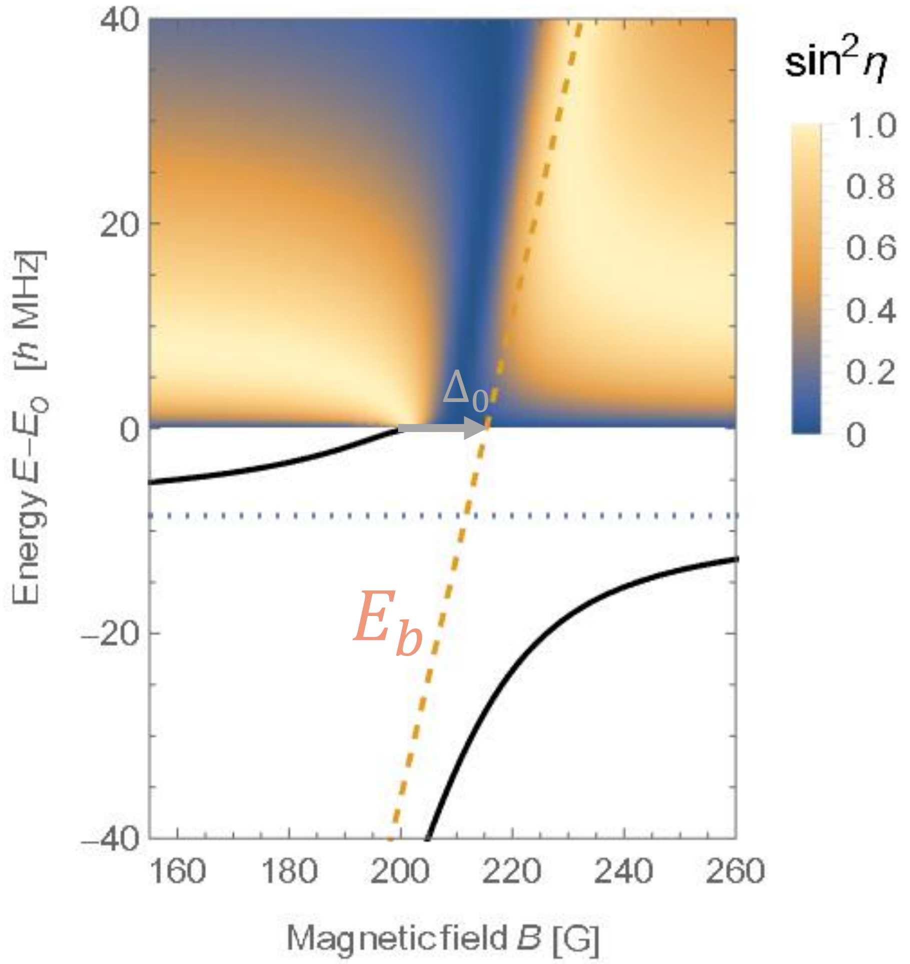


Resonance of ^{40}K atoms near 202 G

Renormalised Quantum Defect Theory

$$E = E_b + \underbrace{\Delta(E)}_{\Delta_0 + \frac{\gamma}{\lambda(E) - a_o}}$$

Expression for the shift Δ_0:

$$\Delta_0 = \gamma \frac{a_o - \bar{a}}{\bar{a}^2 + (a_o - \bar{a})^2}$$

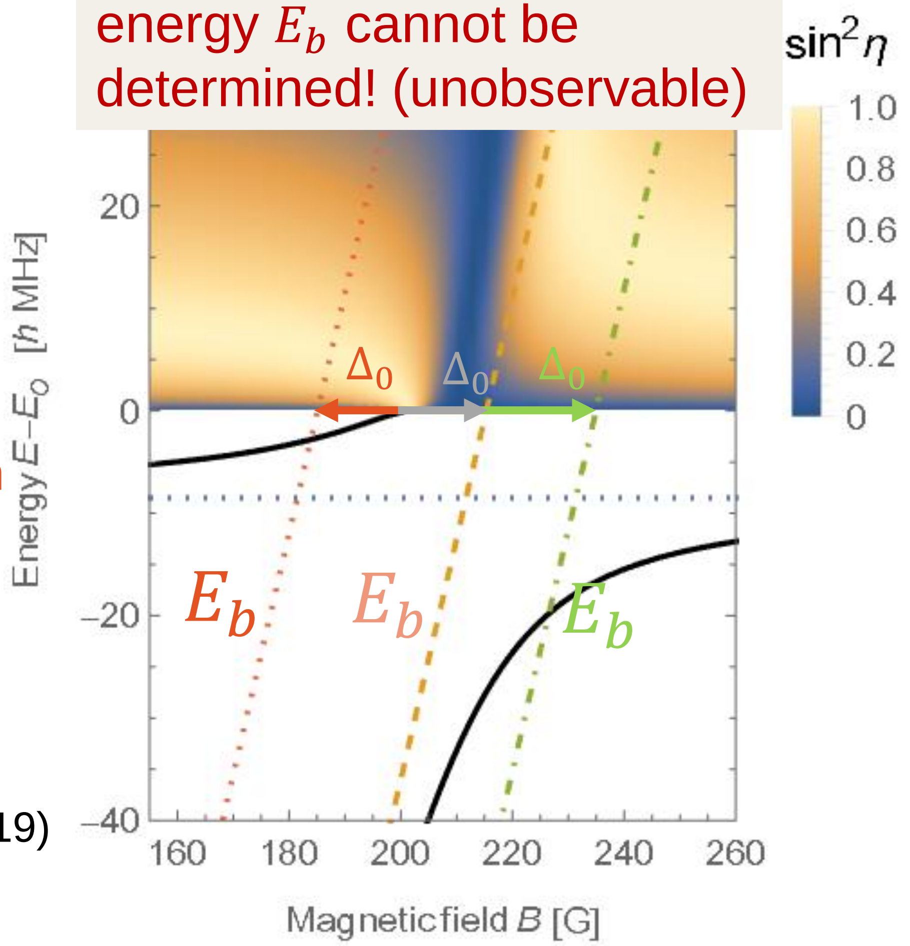
Chin, Grimm, Julienne, Tiesinga
Rev. Mod. Phys. 82, 1225 (2010)
incorrect

Closed-channel scattering length

$$\Delta_0 = \frac{\gamma}{a_o - a_c}$$

Naidon & Pricoupenko
Phys. Rev. A 100, 042710 (2019)

The shift Δ_0 and the bare energy E_b cannot be determined! (unobservable)



Resonance of ^{40}K atoms near 202 G

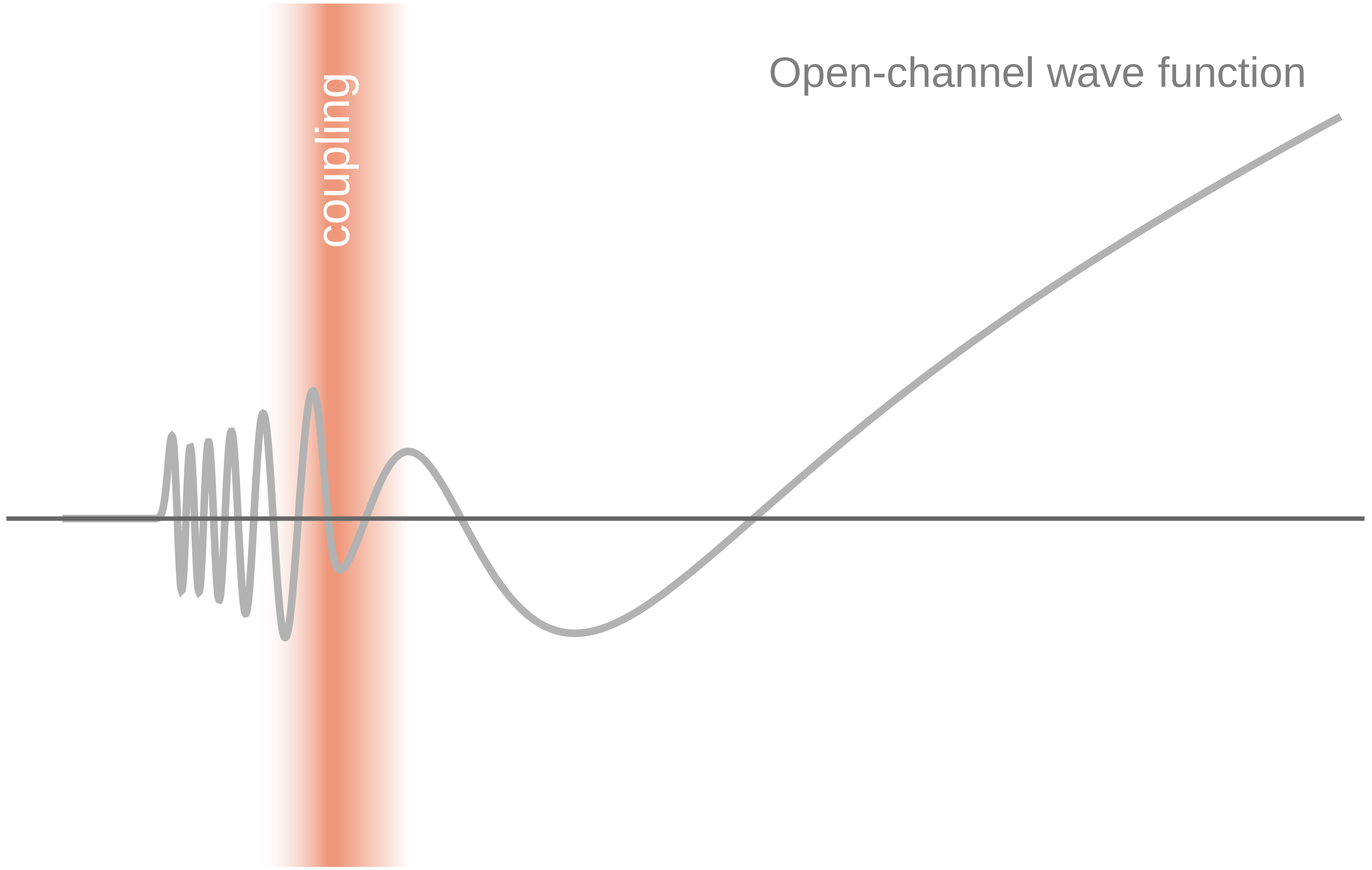
Preliminary conclusion

Closed-channel parameters (bare energy E_b, scattering length a_c) are **undetermined** by the standard two-body observables (scattering phase shifts, binding energy).

But one can also probe **short-range** two-body observables using a third particle :

- Three-body observables
- Photoassociation signal (probing with photon)

Short-range observables



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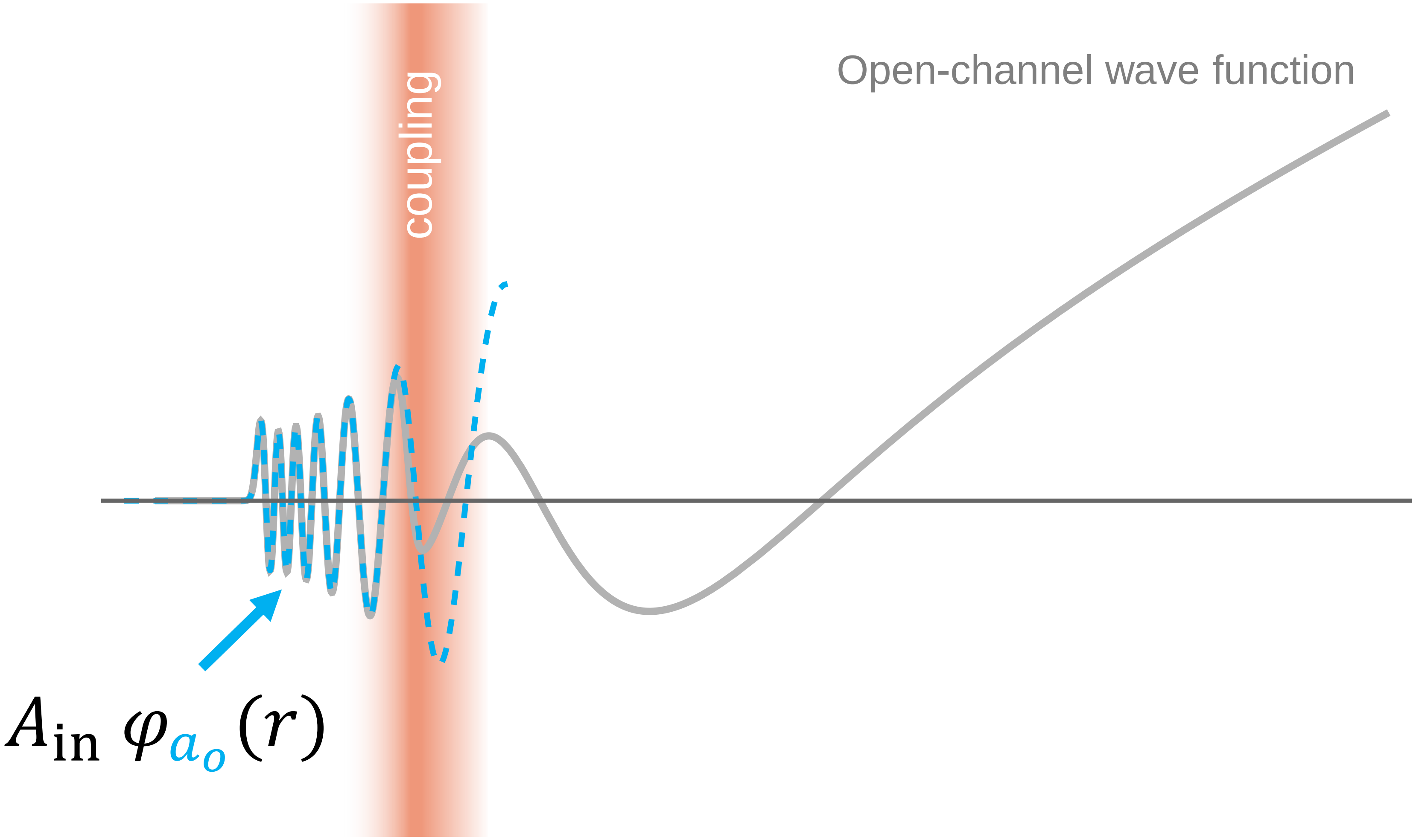

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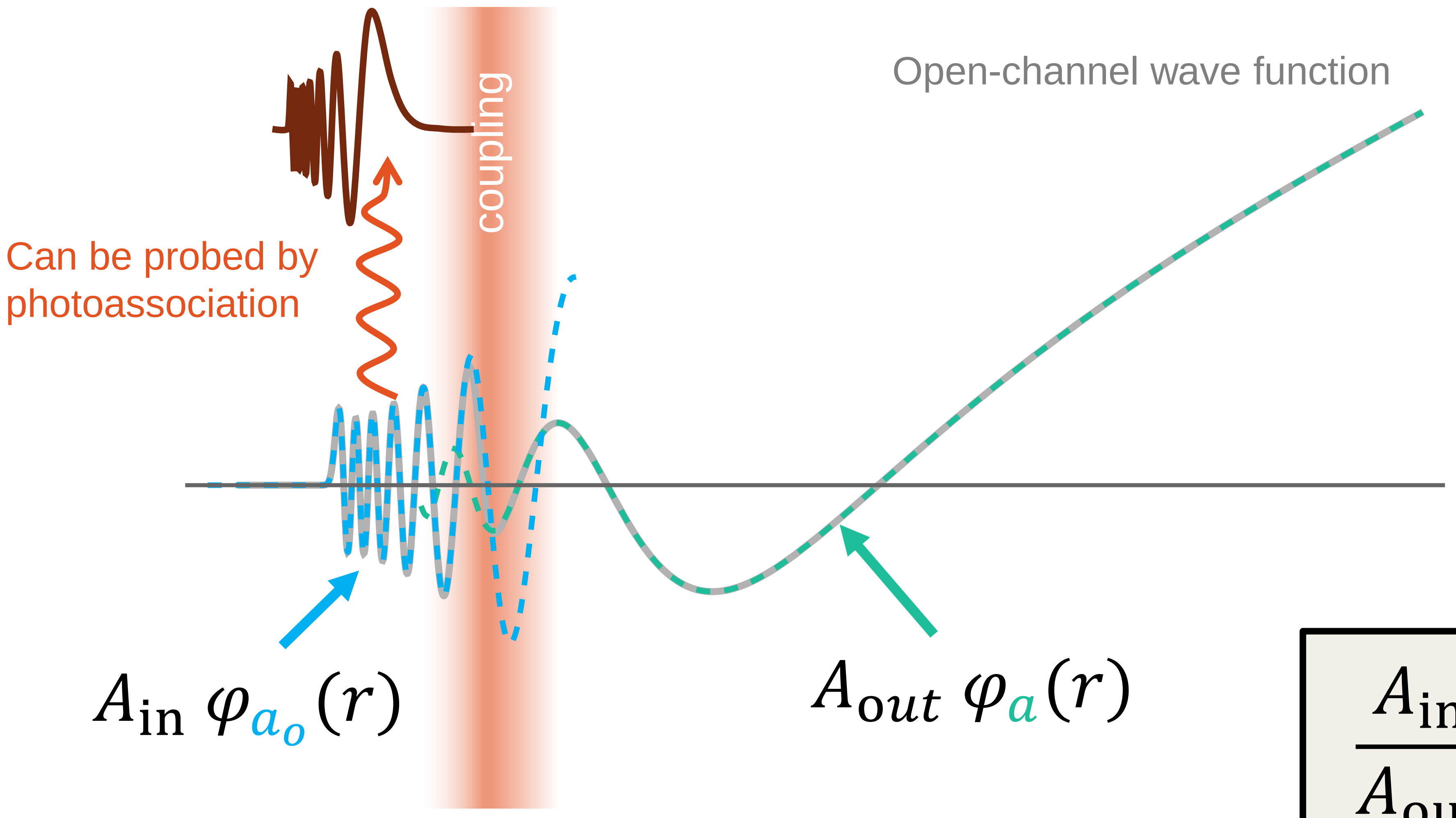
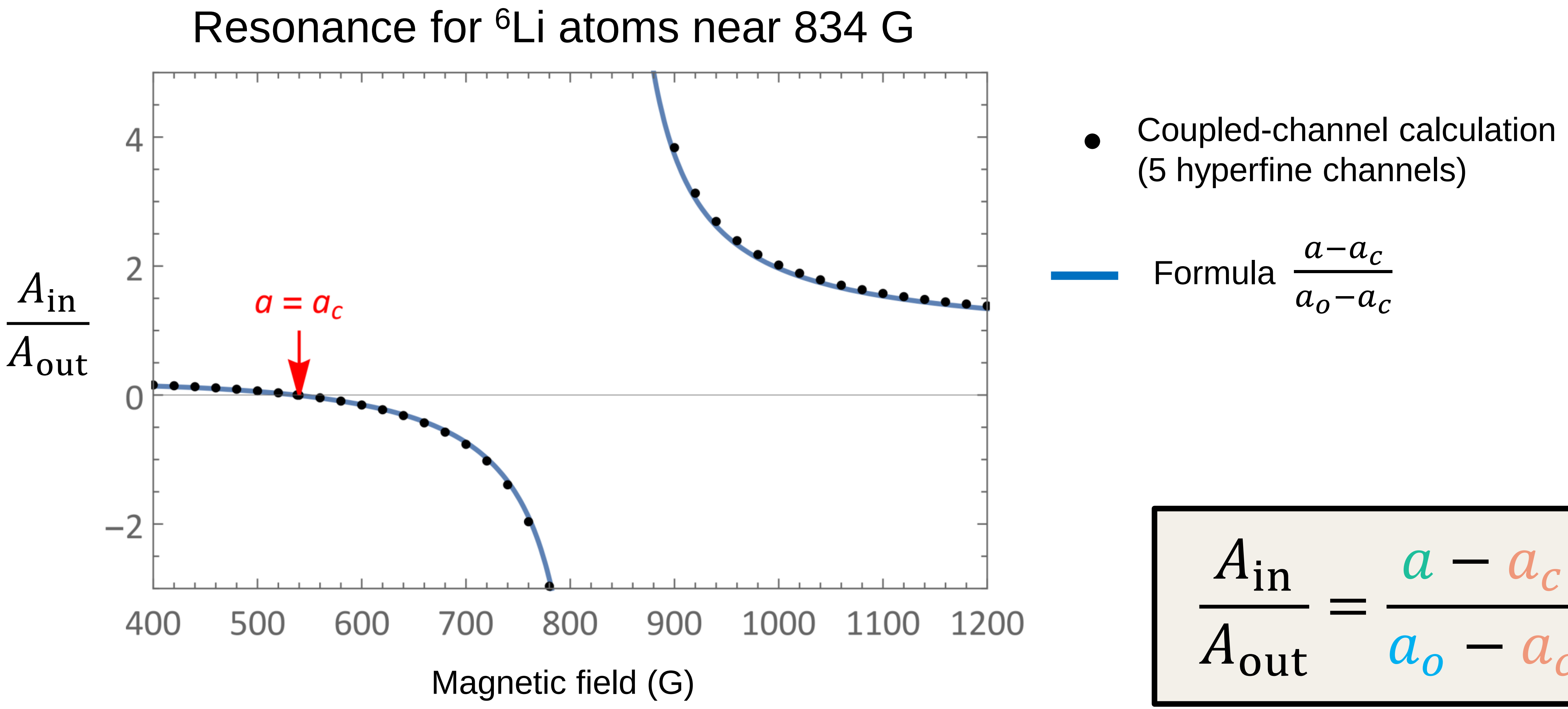


$$\frac{A_{\text{in}}}{A_{\text{out}}} = \frac{a - a_c}{a_o - a_c}$$

Short-range observables



$$\frac{A_{in}}{A_{out}} = \frac{a - a_c}{a_o - a_c}$$

Conclusion

The renormalised quantum defect theory gives a simple description of Feshbach resonances.

Standard two-body observables depend only the open channel!

Closed-channel parameters (bare energy E_b, scattering length a_c) can be revealed by short-range two-body observables. They may affect three-body observables!

arXiv:2403.14962