Universality of Efimov states in cold atoms with dipole interaction

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UQS
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Cold atom

Cold atom   T ≈ 10^{-7} K

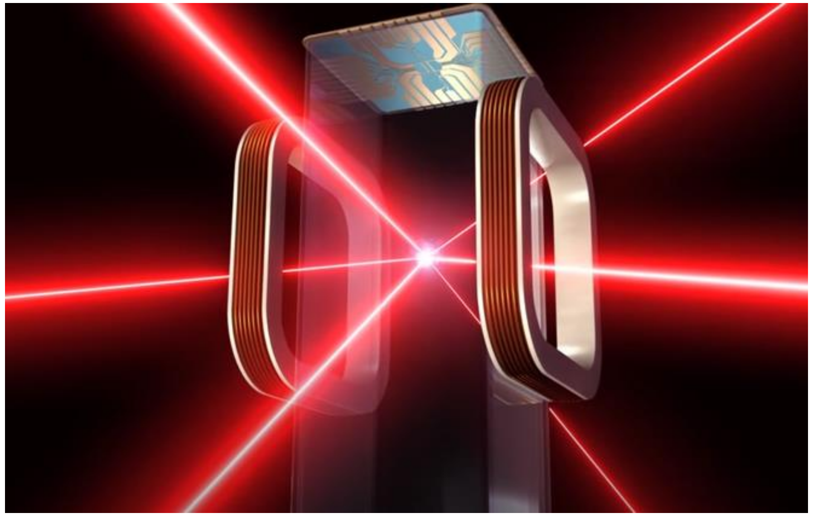
→Quantum phenomena at low T

Highly controllable

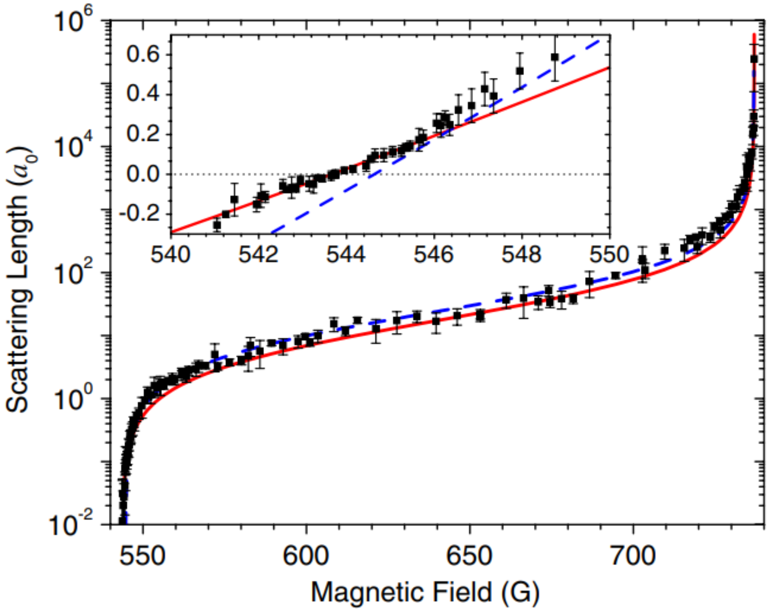
Interaction, dimension, …

Realize various systems

- Superfluid
- Unitary fermi gas
- Topological phases
- Efimov state



Nasa



Pollack et al, Phys. Rev. Lett. (2009)

Feshbach resonance

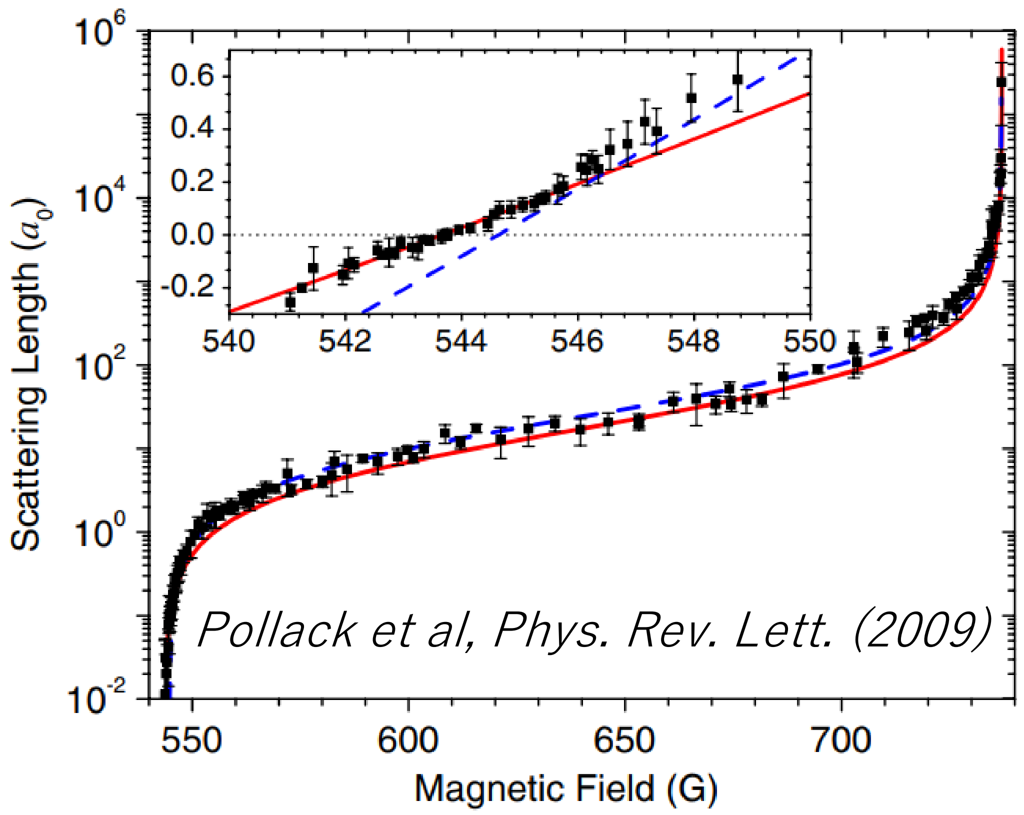
a : s-wave scattering length $\simeq$ strength of interaction

Feshbach resonance

Tuning of scattering length a

Realise large a

Reproduce strongly correlated system



Quantum simulation of strongly correlated system

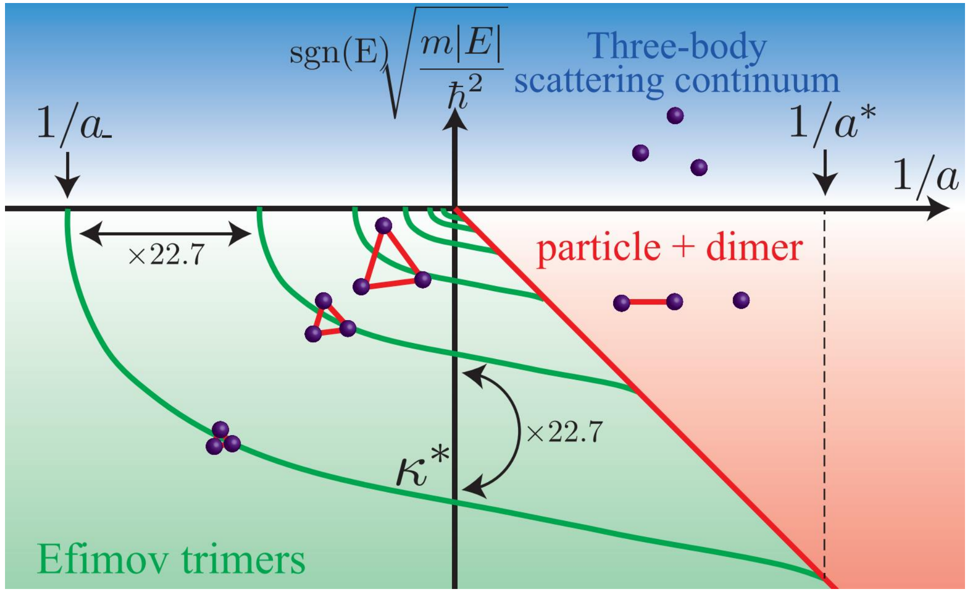
Efimov state

3-identical boson with $a = \pm\infty$

Infinite number of bound state

Discrete scale invariance

Sequence of 3-body bound state connected with scale transformation



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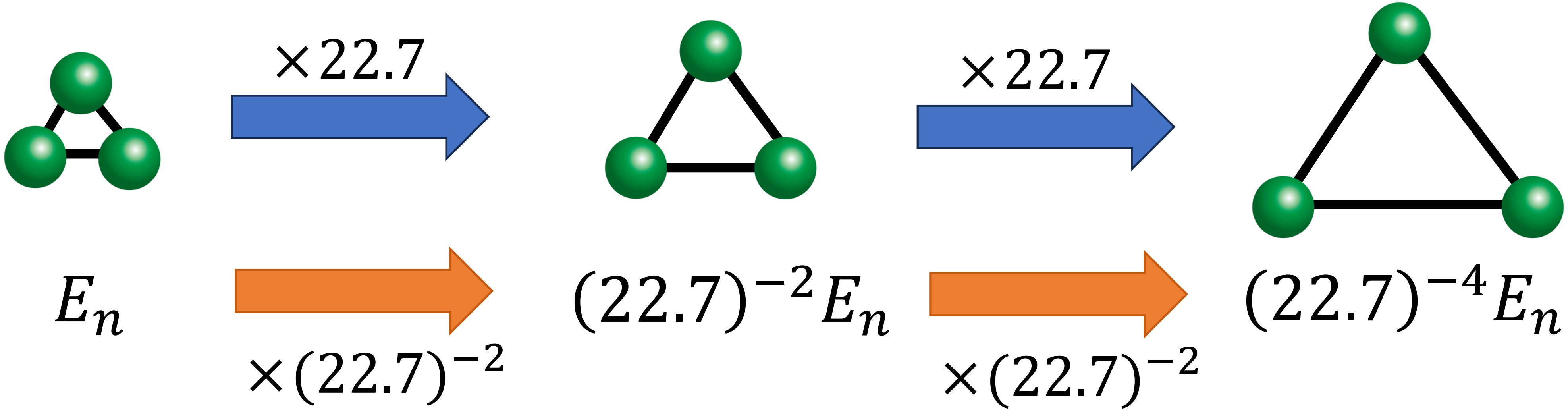
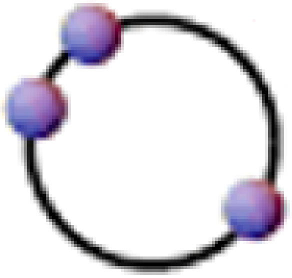
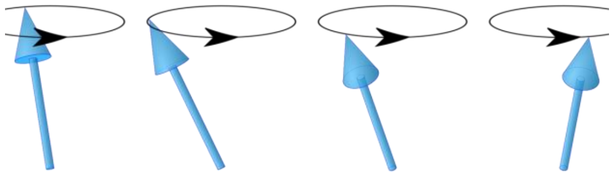


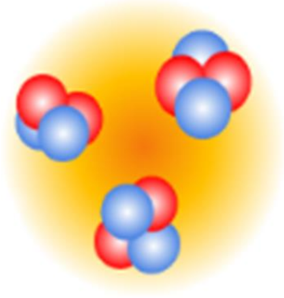
Universality of Efimov state



cold atom



magnon



nuclear system

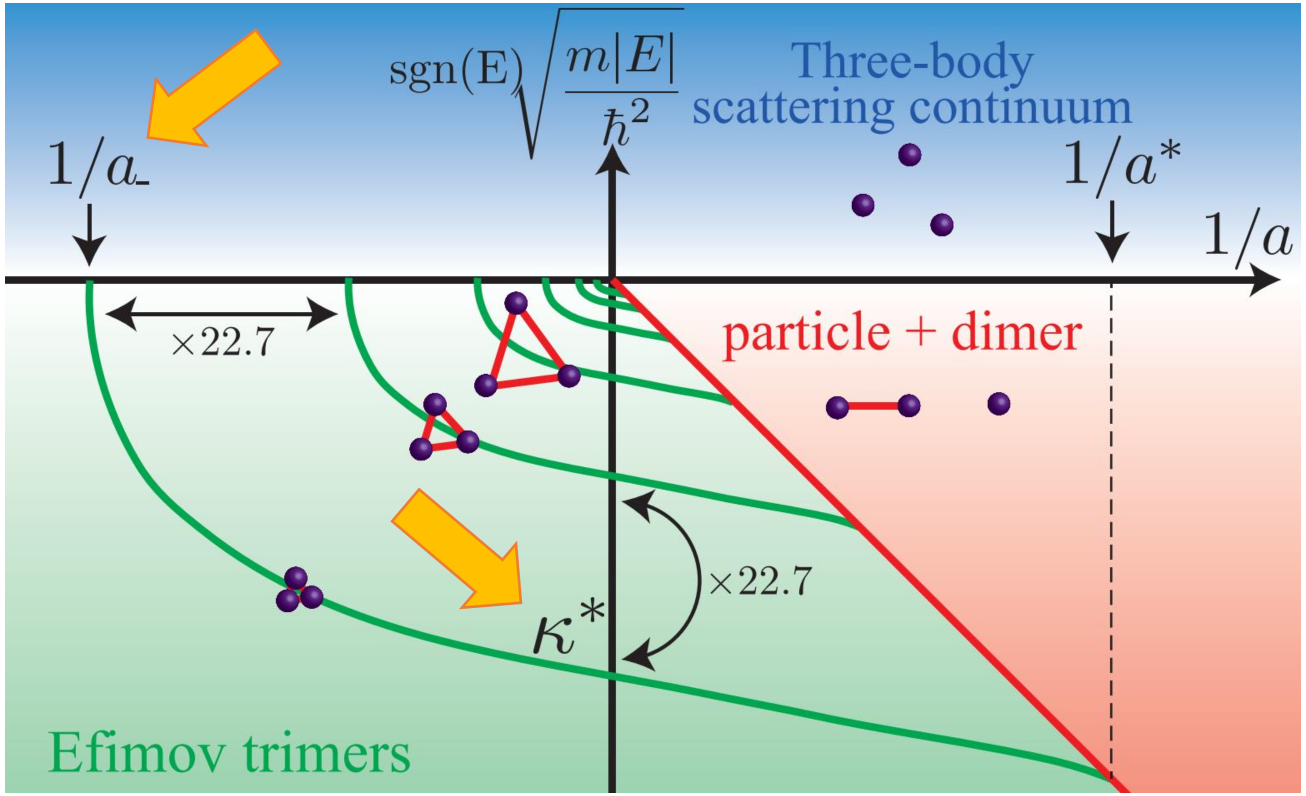
Interaction, scale are completely different

exhibit Efimov state

Universal phenomenon in 3-body system

3-body parameter

3-body parameter { 3-body binding energy κ^* / 3-body loss rate peak a_- }



$$\kappa^* \propto 1/a_-$$

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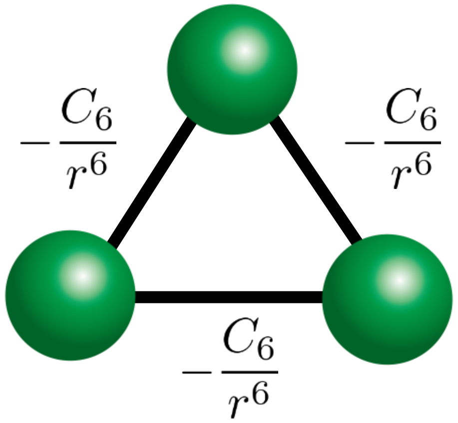
Universality of 3-body parameter

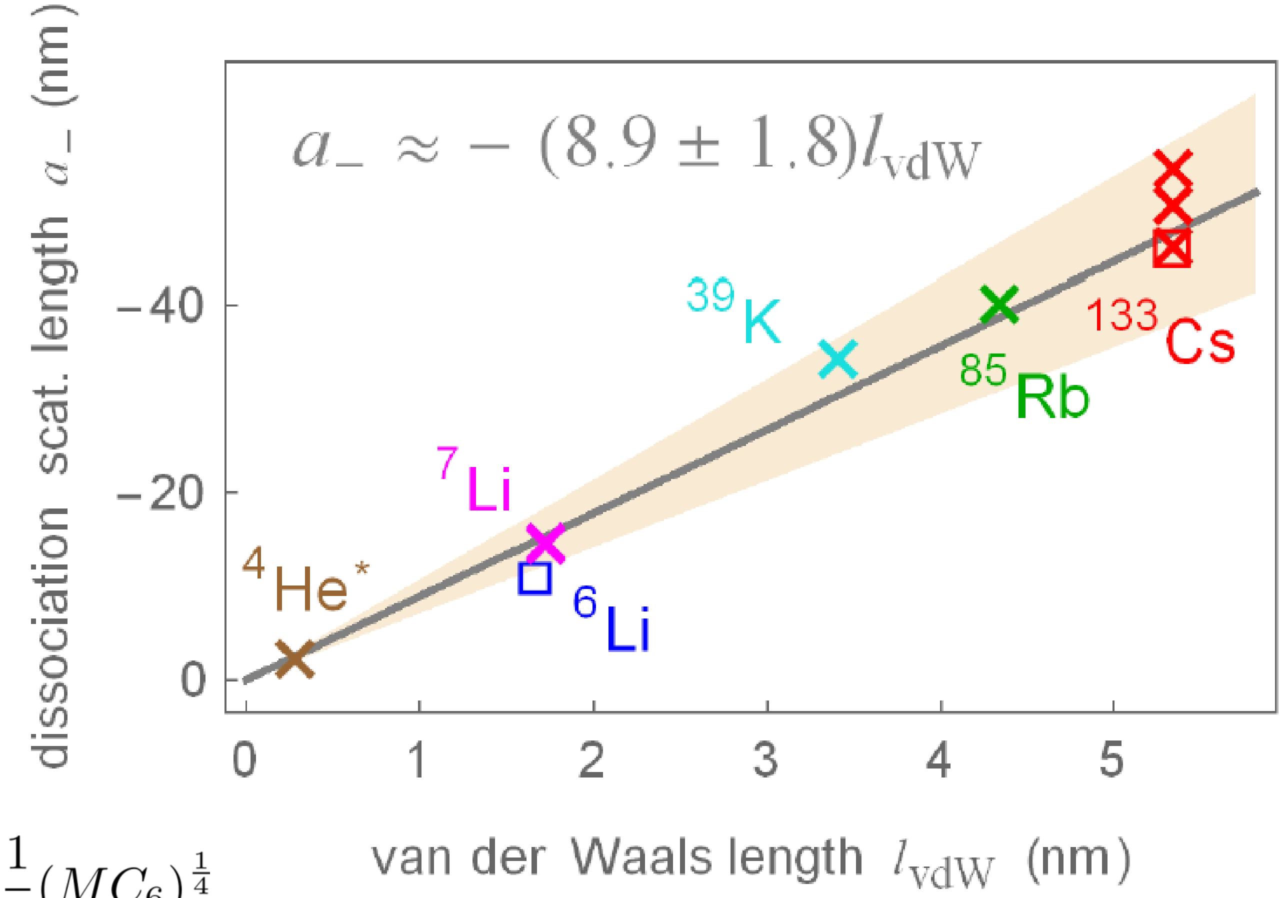
3-identical boson + vdw



Universal relation
$a_-/r_{\rm vdw} \simeq -9.1$

$$r_{\rm vdw} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}}$$

J. Wang et al, Phys. Rev. Lett. 108, 263001 (2012)
P. Naidon et al, Phys. Rev. A 104, 059903 (2021)



P. Naidon, S. Endo, Rep. Prog. Phys. 80, 056001 (2017)

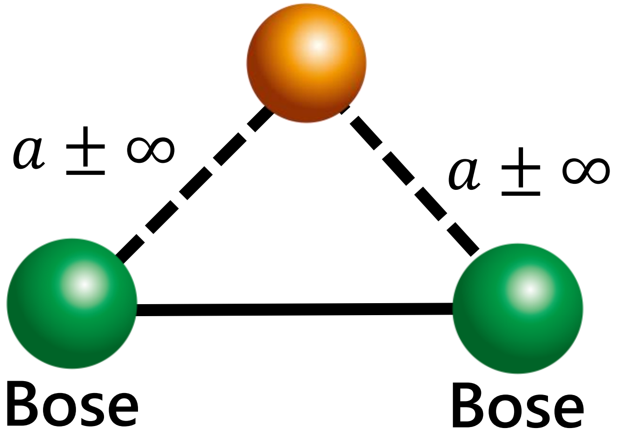
M. Berninger, et al, Phys. Rev. Lett. 107, 120401 (2011).
S. Roy,et al., Phys. Rev. Lett. 111, 053202 (2013).
J. Johansen, et al., Nature Phys. 13, 731 (2017).
R. Chapurin, et al., Phys. Rev. Lett. 123, 233402 (2019).
X. Xie, et al., Phys. Rev. Lett. 125, 243401 (2020).

Efimov state in mixture system

Bose mixture system

heavy-light scattering length $= \pm\infty$

➡ Efimov state appear

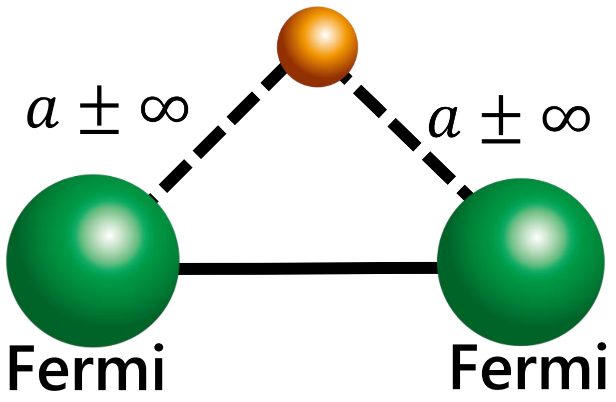


Fermi mixture system

heavy-light scattering length $= \pm\infty$

Mass ratio $\frac{M}{m} > 13.6069 \ldots$

➡ Efimov state appear, not observed



V. Efimov, Nucl. Phys. A 210, 157 (1973).
D. S. Petrov, Phys. Rev. A 67,010703 (2003).

New candidate for Efimov state in fermi system

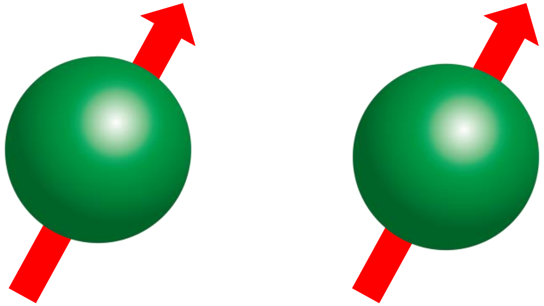
3-body parameter under vdw + dipole interaction

Feshbach resonance Er-Li mixture

Er-Li : Schäfer, Mizukami, Takahashi Phys.Rev.A:105.012816(2022)

Er mass ≫ Li mass ⟹ Fermi Efimov state !

Er atoms : Strong magnetic moment
Dipole interaction is important



Efimov state with van der Waals + dipole int.
⟹ 3-body parameter universal?

(3-body parameter ⟺ a_- ⟺ Binding energy at unitary limit)

Set up

Set up

Particle

2-heavy (spin polarized) + 1-light

Interaction

Heavy and Heavy

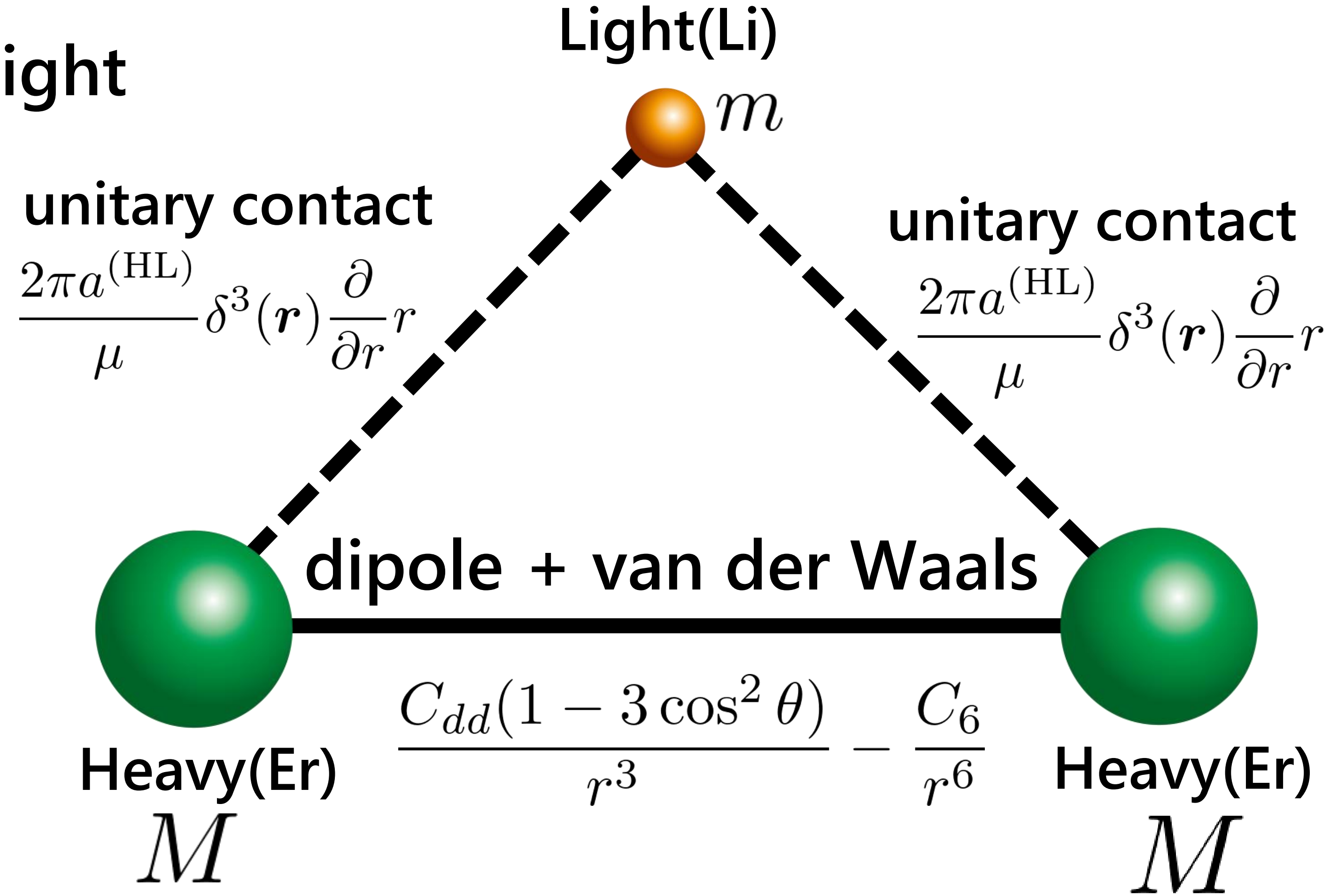
Dipole + van der Waals

$$\frac{C_{dd}(1-3\cos^2\theta)}{r^3} - \frac{C_6}{r^6}$$

Heavy and Light

unitary contact interaction

$$\frac{2\pi a^{(\mathrm{HL})}}{\mu}\delta^3(\boldsymbol{r})\frac{\partial}{\partial r}r \qquad (a^{(HL)} \pm \infty)$$



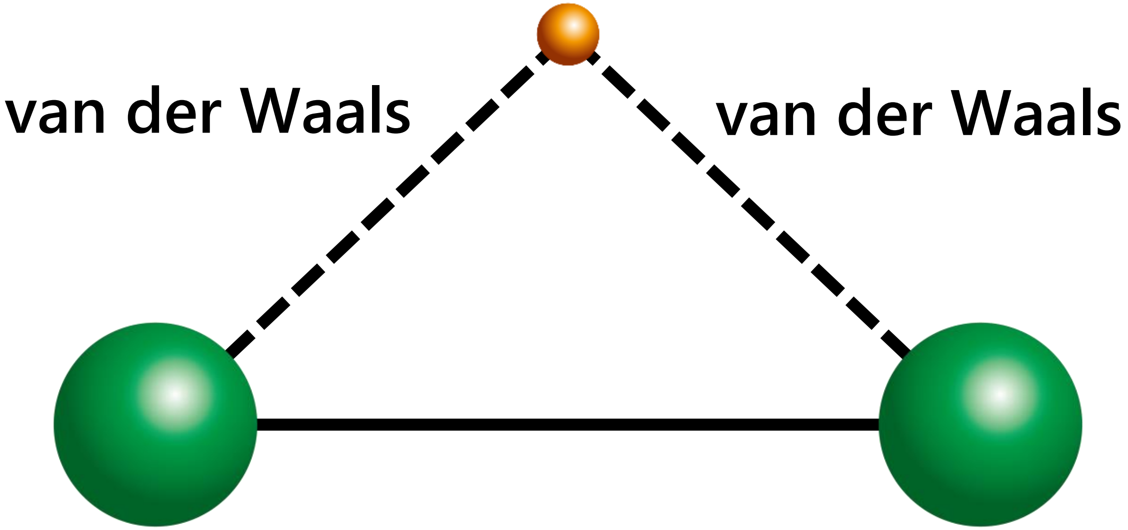
Method

Born-Oppenheimer approximation

M/m is large

→ BO approximation is appropriate

2-heavy + 1-light → **2-heavy**



$$\frac{C_{dd}(1-3\cos^2\theta)}{r^3}-\frac{C_6}{r^6}$$

$$\frac{C_{dd}(1-3\cos^2\theta)}{r^3}-\frac{C_6}{r^6}-\frac{\Omega^2}{2mr^2}$$

$$\Omega = 0.5671\ldots$$

Effect by light particle

QDT & Coupled channel calculation

(i)Quantum Defect Theory (Non-dipole)

Gao, Phys.Rev.A:58.1728(1998)
Gao, et.al. Phys.Rev.A:72.042719(2005)

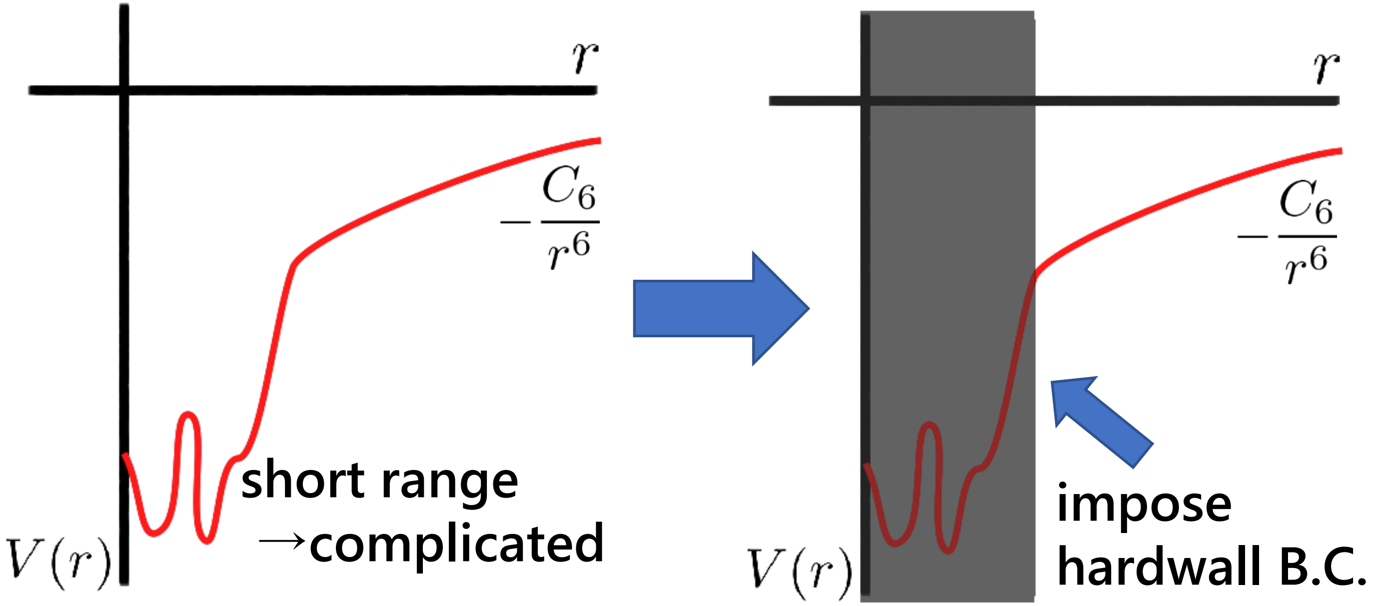
Interatomic potential at short range : Complicated

➡ **Impose hardwall potential $r < R_{\min}$** $(R_{\min} \ll r_{\mathrm{vdw}})$

$R_{\min}$ **determined**

⬇

Binding energy,
$a^{(\mathrm{HH})}$, $v_p^{(\mathrm{HH})}$ **determined**



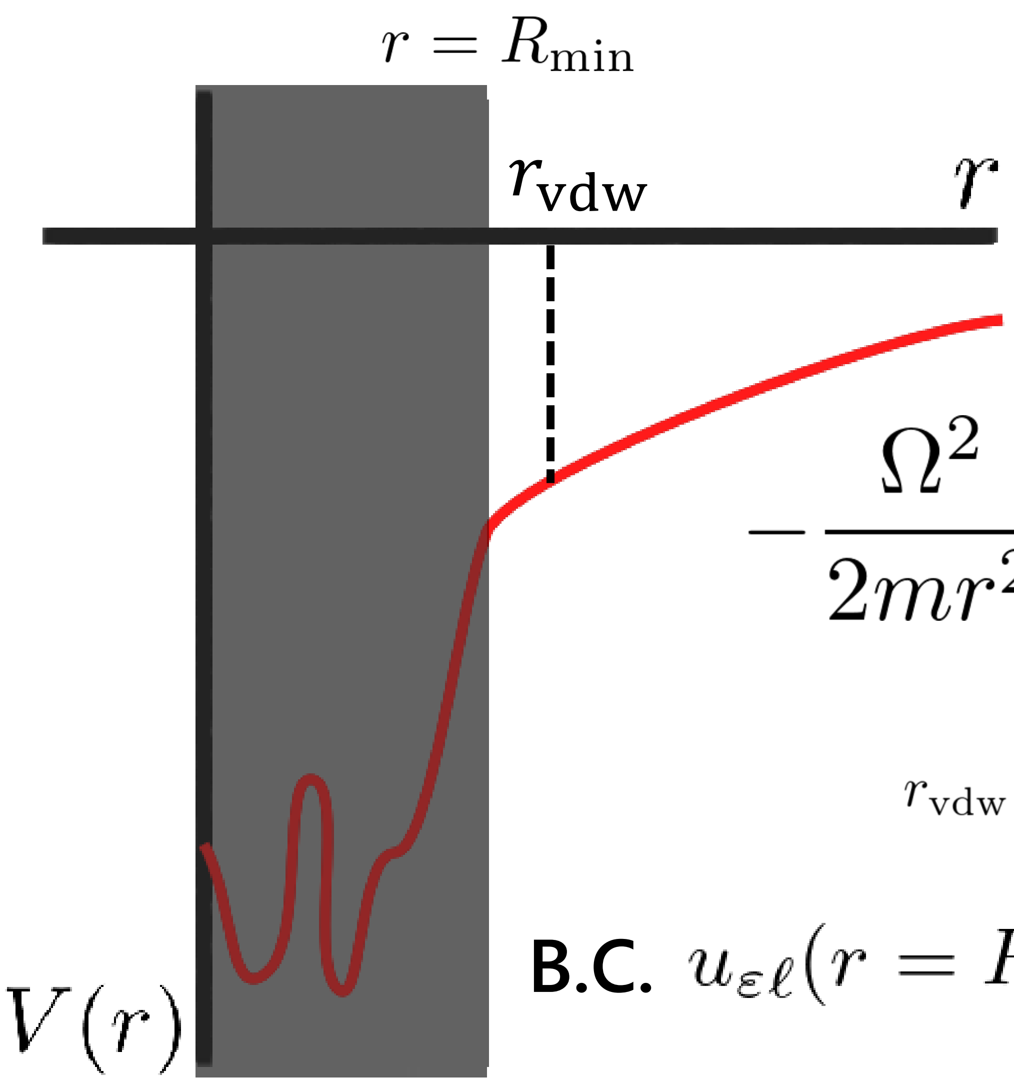
(ii)Numerical : Coupled channel calculation(dipole)

Dipole interaction : anisotropic ➡ **Mixture of different ℓ states**

Non-dipole

Analytical calculation

Analysis in non-dipole system



$$-\frac{\Omega^2}{2mr^2} - \frac{C_6}{r^6}$$

$$r_{\text{vdw}} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}}$$

B.C. $u_{\varepsilon\ell}(r = R_{\text{min}}) = 0$

Non dipole analysis($C_{dd} = 0$)

Analytical calculation(low energy expansion) with Quantum Defect Theory

・2-Heavy Boson + 1-light

$a^{(\mathrm{HH})}$: Scattering length between heavy bosons

$$|E| = \frac{4}{M r_{\mathrm{vdw}}^2} \exp\left[\frac{2}{|s_0|}\left\{\arctan\left(\frac{1}{\tanh\frac{\pi|s_0|}{4}} \frac{\frac{a^{(\mathrm{HH})}}{r_{\mathrm{vdw}}}\tan\frac{\pi}{8} + \frac{4\pi}{\Gamma^2(\frac{1}{4})}\left(1-\tan\frac{\pi}{8}\right)}{\frac{a^{(\mathrm{HH})}}{r_{\mathrm{vdw}}} - \frac{4\pi}{\Gamma^2(\frac{1}{4})}\left(1+\tan\frac{\pi}{8}\right)}\right) + \xi_0\right\}\right] e^{-\frac{2n\pi}{|s_0|}}$$

3-body parameter : Universally determined by r_{vdw} , $a^{(\mathrm{HH})}$

・2-Heavy Fermion + 1-light

$v_p^{(\mathrm{HH})}$: Scattering volume between heavy fermions

$$|E| = \frac{4}{M r_{\mathrm{vdw}}^2} \exp\left[\frac{2}{|s_1|}\left\{\arctan\left(\frac{1}{\tanh\frac{\pi|s_1|}{4}} \frac{\frac{v_p^{(\mathrm{HH})}}{r_{\mathrm{vdw}}^3}\tan\frac{3}{8}\pi + \frac{1}{3\sqrt{2}}\frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{7}{4})}\left(1+\tan\frac{3}{8}\pi\right)}{\frac{v_p^{(\mathrm{HH})}}{r_{\mathrm{vdw}}^3} + \frac{1}{3\sqrt{2}}\frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{7}{4})}\left(1-\tan\frac{3}{8}\pi\right)}\right) + \xi_1\right\}\right] e^{-\frac{2n\pi}{|s_1|}}$$

3-body parameter : Universally determined by r_{vdw}, $v_p^{(\mathrm{HH})}$

Numerical analysis for Boson

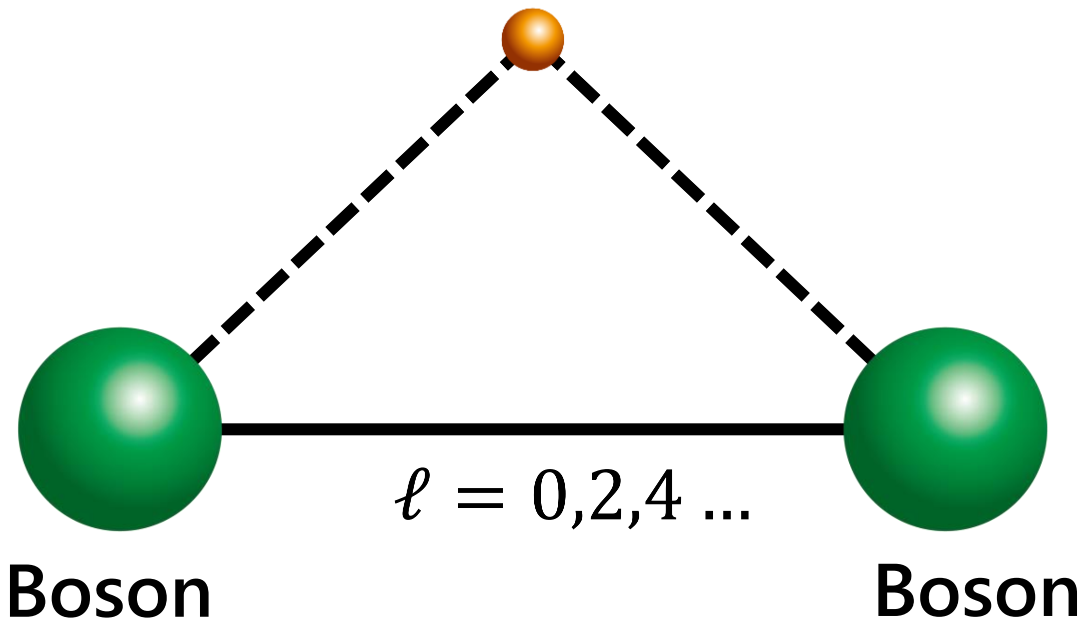
$$r_{\mathrm{vdw}} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}}$$

$$a_{dd} \equiv \frac{MC_{dd}}{3}$$

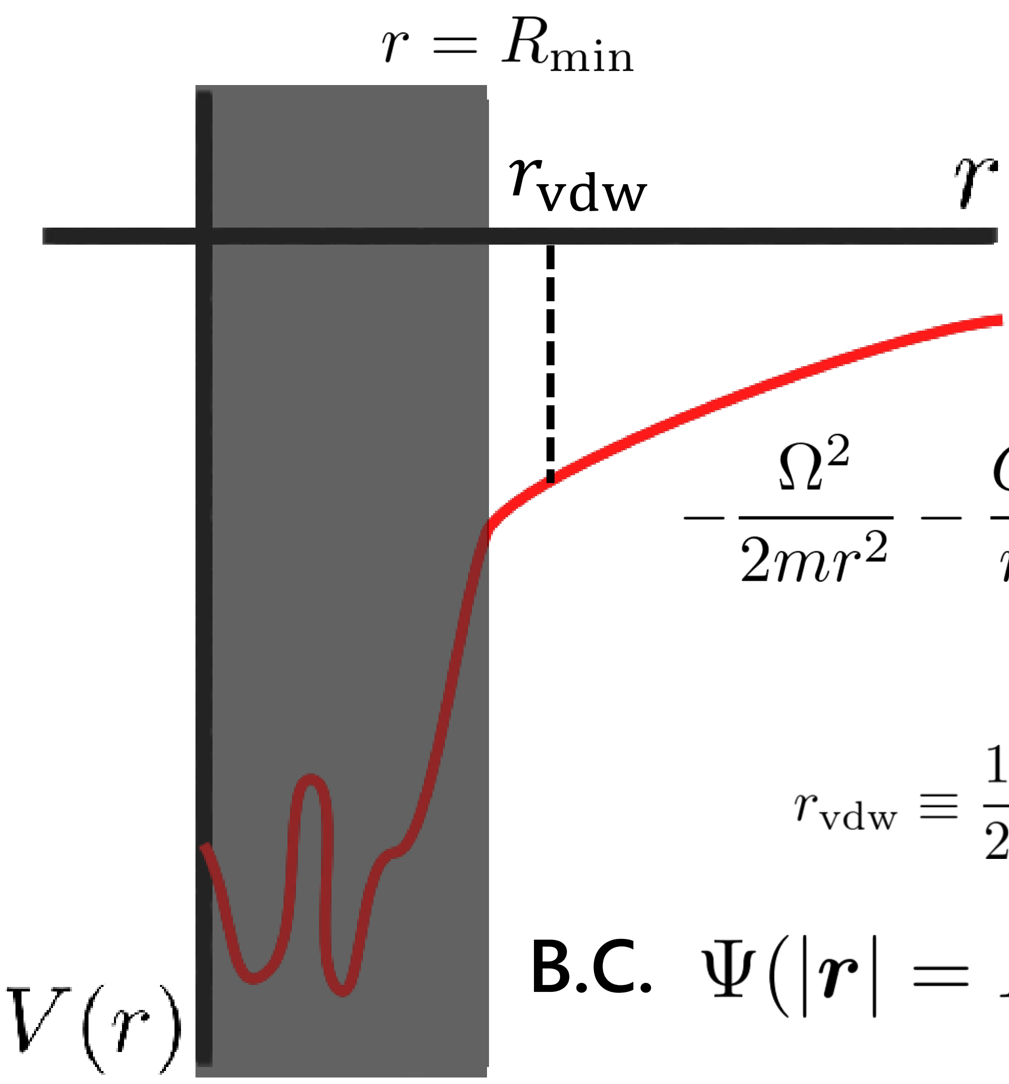
Non dipole($a_{dd} = 0$)

～Intermediate dipole region

($a_{dd} \sim r_{\mathrm{vdw}}$)



Analysis in dipole system(Boson)



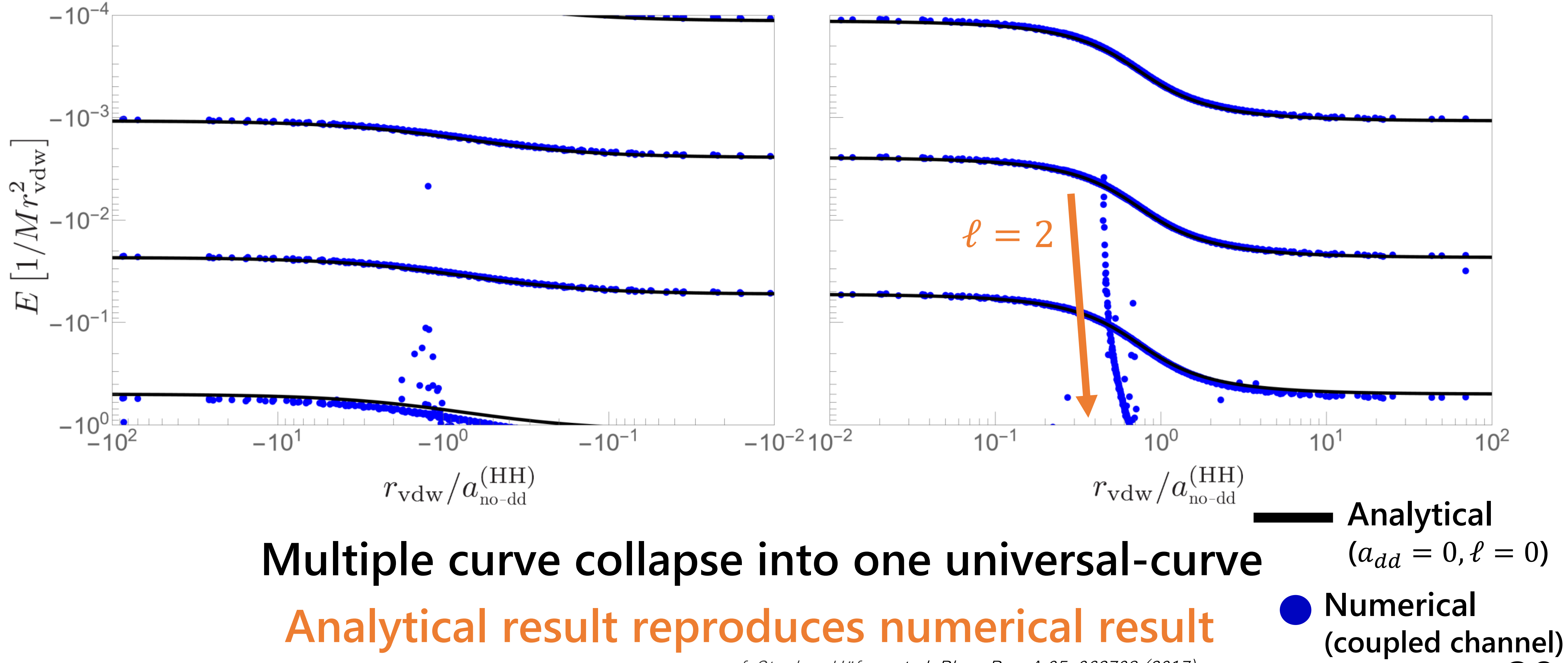
$$-\frac{\Omega^2}{2mr^2} - \frac{C_6}{r^6} + \frac{C_{dd}(1 - 3\cos^2\theta)}{r^3}$$

$$r_{\text{vdw}} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}} \qquad a_{dd} \equiv \frac{MC_{dd}}{3}$$

B.C. $\Psi(|\boldsymbol{r}| = R_{\min}) = 0$

$a_{\text{no-dd}}^{(\text{HH})}$ vs Energy

$a_{dd}/r_{\text{vdw}} = 0.0$ $\quad r_{\text{vdw}} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}} \quad a_{dd} \equiv \frac{MC_{dd}}{3}$



Multiple curve collapse into one universal-curve

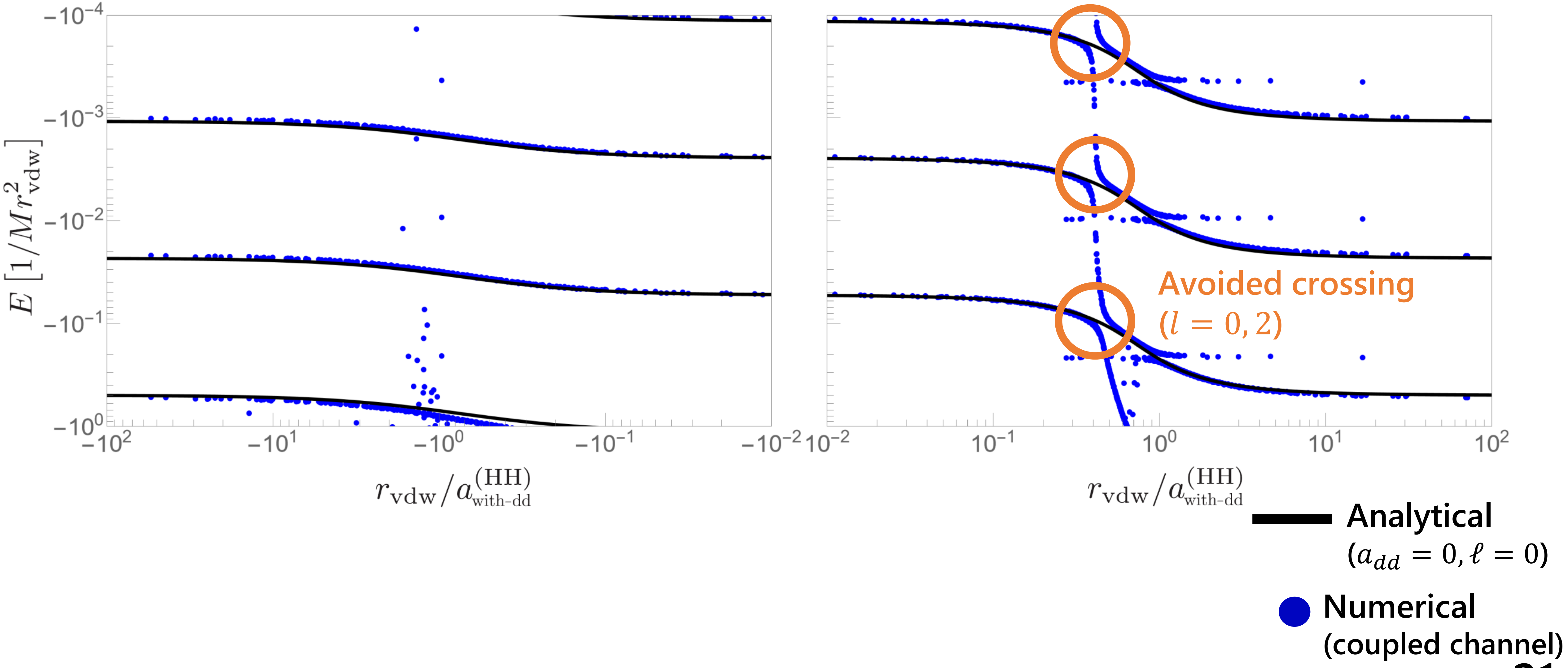
Analytical result reproduces numerical result

c.f. Stephan Häfner et al, Phys. Rev. A 95, 062708 (2017)

$a^{(\mathrm{HH})}_{\mathrm{with-dd}}$ vs Energy

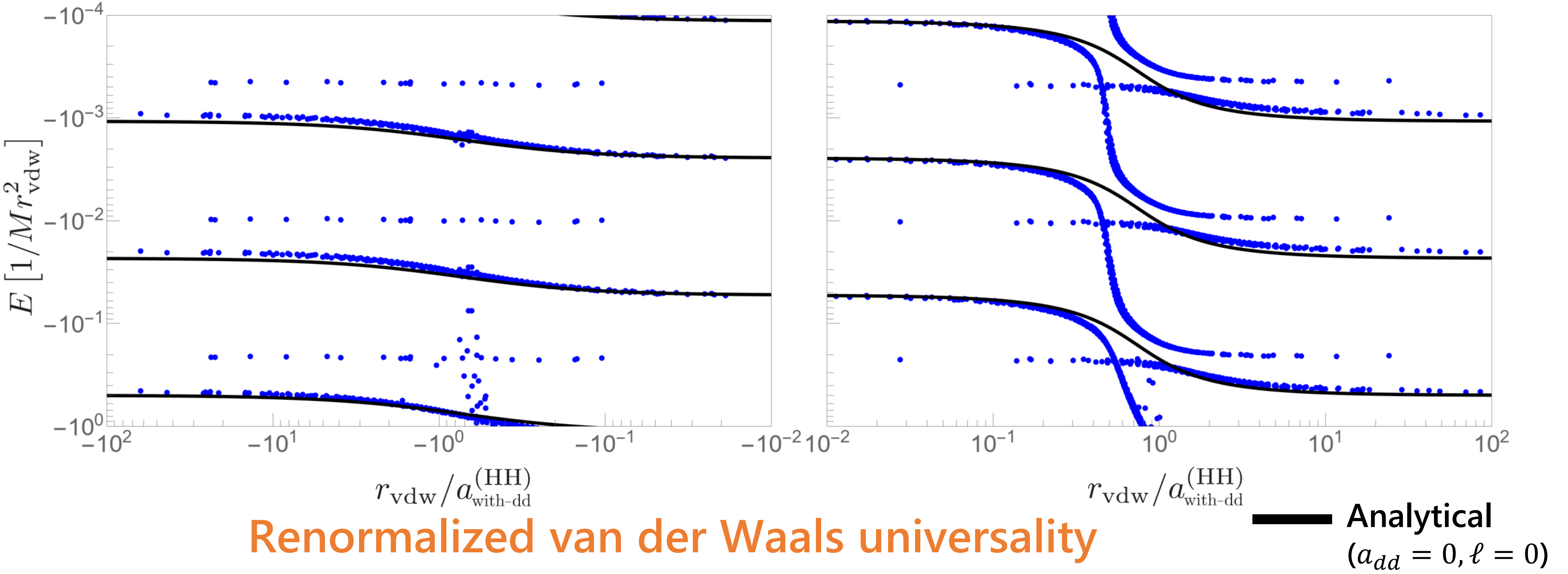
$a_{dd}/r_{\mathrm{vdw}} = 0.4$

$r_{\mathrm{vdw}} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}} \quad a_{dd} \equiv \frac{MC_{dd}}{3}$



$a^{(\mathrm{HH})}_{\text{with}-\mathrm{dd}}$ vs Energy

$a_{dd}/r_{\mathrm{vdw}} = 0.86755$ (^{166}Er- ^{166}Er) $\quad r_{\mathrm{vdw}} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}} \quad a_{dd} \equiv \frac{MC_{dd}}{3}$

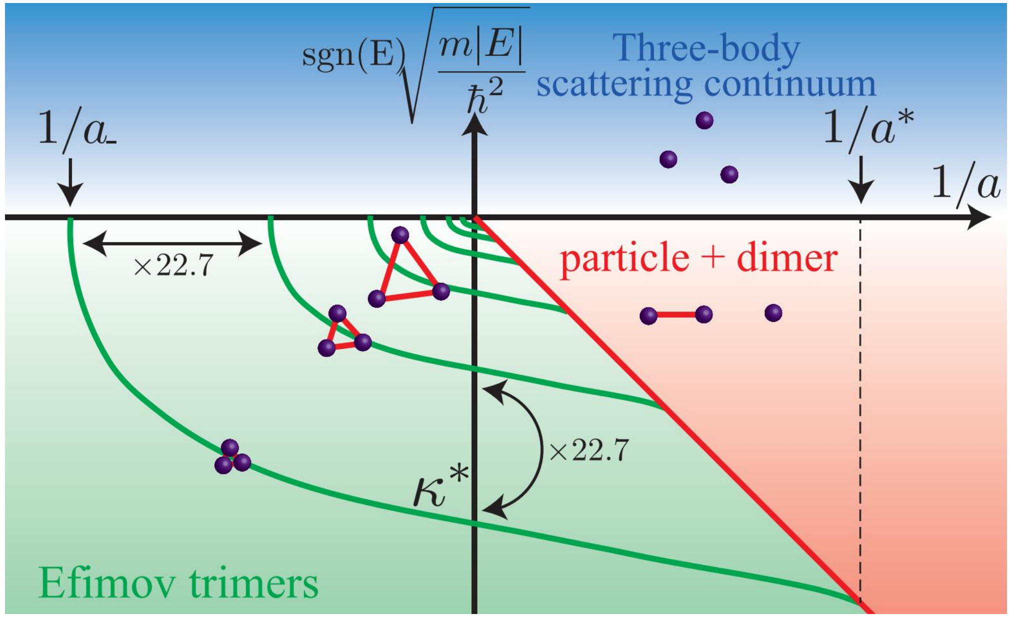


Renormalized van der Waals universality

- Non dipole curve reproduces dipole behavior
- All effects of dipole interaction incorporated into $a^{(\mathrm{HH})}$

Estimation of 3-body parameter for Er-Li

Species	$r_{\mathrm{vdw}}[a_0]$	$a_{dd}[a_0]$	$a^{(\mathrm{HH})}$ $[a_0]$	$\kappa_* r_{\mathrm{vdw}}$	$\frac{a_-^{(\mathrm{HL})}}{r_{\mathrm{vdw}}}$
^{166}Er-^{6}Li	75.5	65.5	68	0.495	-10.1
				0.107	-46.7
				2.29×10^{-2}	-217
				4.94×10^{-3}	-1.01×10^{3}
^{168}Er-^{6}Li	75.8	66.3	137	0.352	-14.2
				7.65×10^{-2}	-65.3
				1.66×10^{-2}	-300
				3.62×10^{-3}	-1.38×10^{3}
^{170}Er-^{6}Li	76.0	67	221	0.298	-16.8
				6.55×10^{-2}	-76.6
				1.44×10^{-2}	-349
				3.16×10^{-3}	-1.59×10^{3}



3-body binding energy ($\kappa^* r_{\mathrm{vdw}}$)

Estimated from non-dipole analytical relation $a^{(\mathrm{HH})}$ and energy

3-body loss rate peak ($a_-^{(\mathrm{HL})}$)

Calculated from above κ^* + zero-range universal relation κ^*, a_-

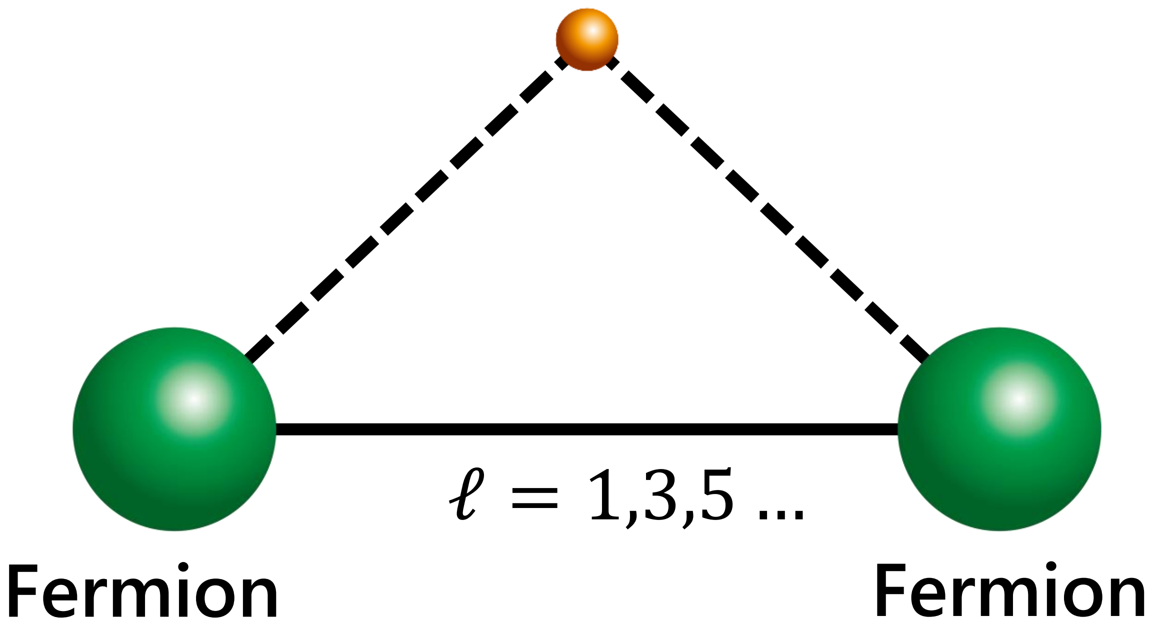
Numerical analysis for Fermion

$$r_{\rm vdw} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}}$$

$$a_{dd} \equiv \frac{MC_{dd}}{3}$$

Intermediate dipole region

$(a_{dd} \sim r_{\rm vdw})$



$v_p^{(\mathrm{HH})}$ under dipole int & alternative parameter

Fermi non-dipole system

Obtain relation $v_p^{(\mathrm{HH})}$ & 3-body parameter(E : binding energy)

However…

$v_p^{(\mathrm{HH})}$: ill-defined under dipole interaction !

J. L. Bohn, et.al. New J. Phys. ***11****, 055039(2009).*

Alternative parameter

Asymptotic form of wavefunction (p-wave)

$$u_{\ell=1}(r\to\infty)=\sqrt{r}\left[J_3\left(\sqrt{\frac{48}{5}\frac{a_{dd}}{r}}\right)-\underline{K_{c,F}^{(3D)}}Y_3\left(\sqrt{\frac{48}{5}\frac{a_{dd}}{r}}\right)\right]$$

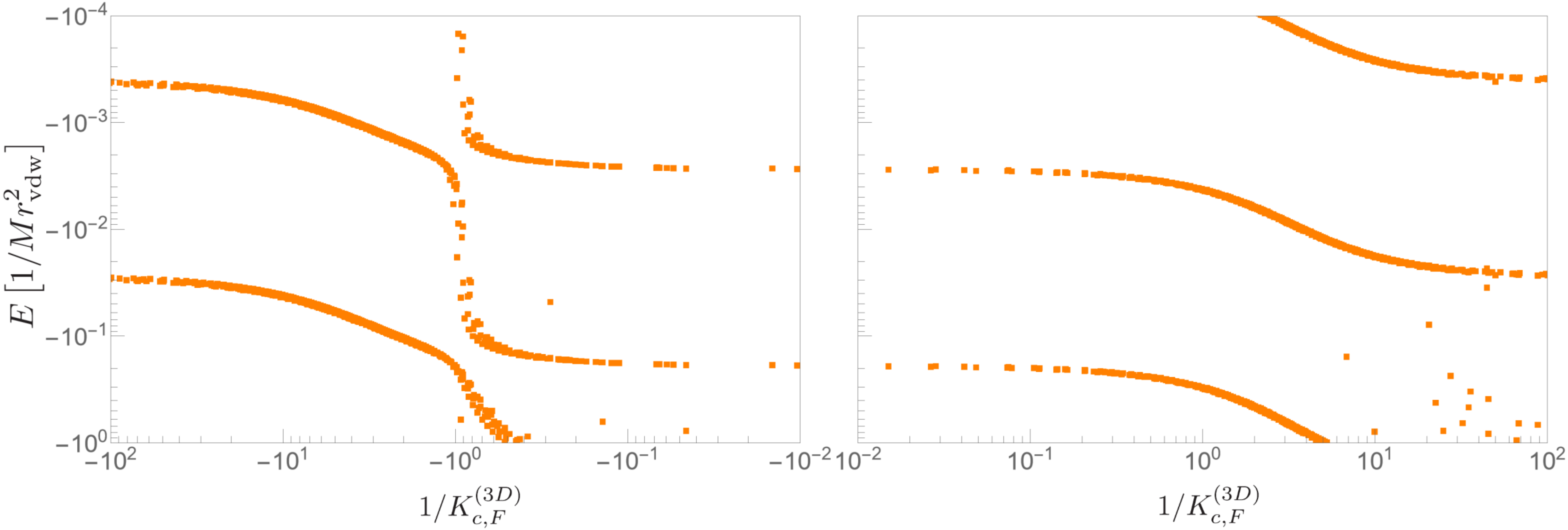
$$r_{\mathrm{vdw}}\equiv\frac{1}{2}(MC_6)^{\frac{1}{4}}$$

$$a_{dd}\equiv\frac{MC_{dd}}{3}$$

Obtain relation between $K_{c,F}^{(3D)}$ and Energy

$K_{c,F}^{(3D)}$ and Energy

$a_{dd}/r_{\mathrm{vdw}} = 0.87054$ (^{167}Er- ^{167}Er) $r_{\mathrm{vdw}} \equiv \frac{1}{2}(MC_6)^{\frac{1}{4}}$ $a_{dd} \equiv \frac{MC_{dd}}{3}$



3-body parameter(binding energy)
determined by $K_{c,F}^{(3D)}$, r_{vdw}, a_{dd}

■ Numerical (coupled channel)

Conclusion

Non dipole system

- 3-body parameter universally determined $r_{\rm vdw}$, $a^{\rm (HH)}$($v_p^{\rm (HH)}$)
- Analytical curve of Efimov binding energy obtained by QDT

With Dipole int.

- Renormalized van der Waals universality

  ➡ QDT analytical curve (vdw only) universally reproduces Efimov binding energy, even with dipole int. Once the dipole effect is renormalized into s-wave(p-wave) scattering parameter

- Comparison with experiment a_-

Strong Dipole int.

- 3-body parameter universally described by 1D-scattering parameter.

Oi, Naidon, Endo, Phys. Rev. A 110, 033305