# A New Quantization of Particle Transfer Potentials

# & a New Definition of the Three-Body Force

*A New Horizon in Few-Body Systems*

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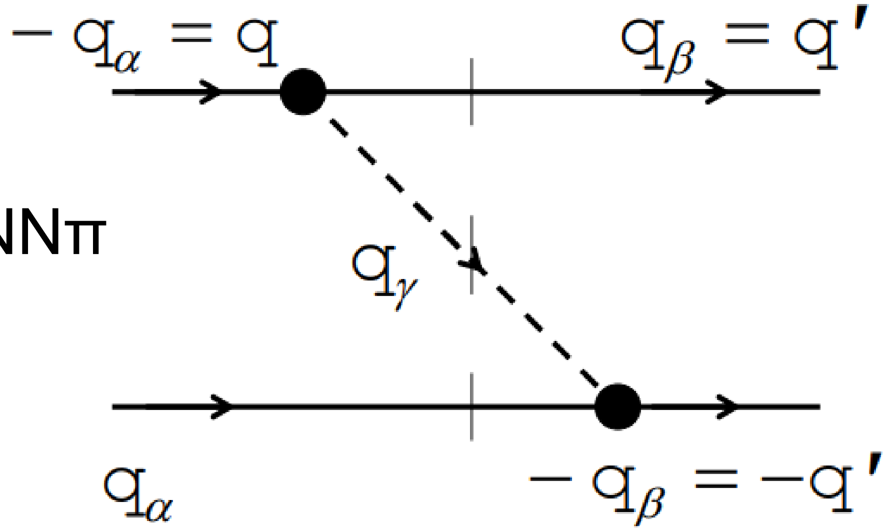
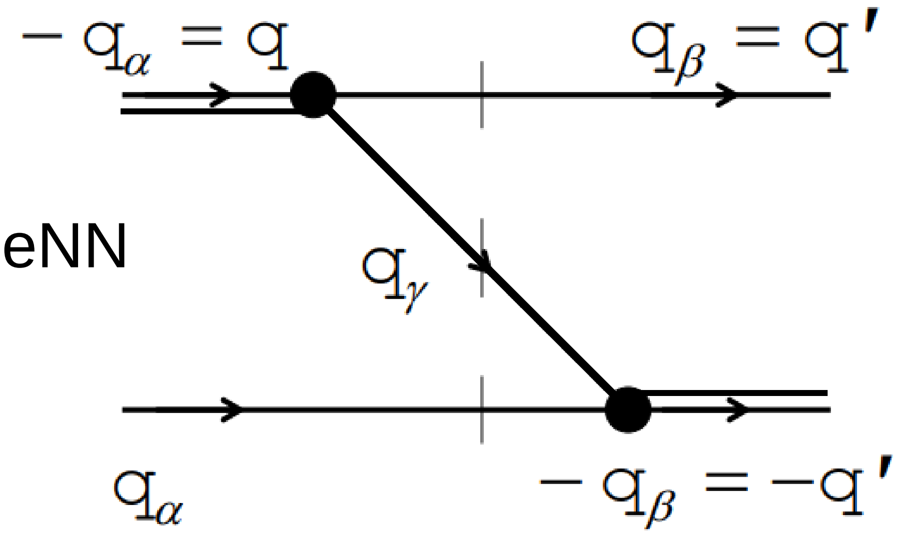
One particle transfer potentials

NNπ




$^4$HeNN



In a three-nucleon system, the AGS Born term: $Z_{\alpha\beta}(q,q';E)$ is given by

$$Z_{\alpha\beta}(q,q';E) = \bar{\delta}_{\alpha\beta} < \psi_\alpha | V_\alpha \frac{1}{E-H_0} V_\beta | \psi_\beta > \tag{1}$$

$$= \bar{\delta}_{\alpha\beta} \frac{g_\alpha^\dagger g_\beta}{E - q^2/2\mu - p^2/2\nu} = -2\mu\bar{\delta}_{\alpha\beta} \frac{g_\alpha^\dagger g_\beta}{q^2+\sigma^2}, \tag{2}$$

$$\text{with} \quad \sigma^2 = 2\mu(p^2/2\nu - E), \quad \text{for} \quad E \leq p^2/2\nu \tag{3}$$

$$\text{and} \quad \sigma^2 = 2\mu(-\epsilon_B - E), \quad \text{for} \quad \epsilon_B \leq -E = |E|, \ \text{with} \ p^2/2\nu \to -\epsilon_B \tag{4}$$

① σ has an energy dependent structure.

② σ=0 is a Coulomb-like potential

③ 0＜σ is satisfied in the following regions

The Fourier transformation of Eq.(2) is given by

$$\mathcal{F}\{Z\} = V_0(\alpha,\beta,r)\frac{e^{-\sigma r}}{r} = V_0(\alpha,\beta,r)\frac{e^{-\sqrt{2\mu(-\epsilon_B+|E|)}r}}{r} \qquad \text{for} \quad \epsilon_B \leq |E|, \quad (5)$$

$$\mathcal{F}\{Z\} = V_0(\alpha,\beta,r)\frac{e^{-\sigma r}}{r} = V_0(\alpha,\beta,r)\frac{e^{-\sqrt{2\mu(p^2/2\nu-E)}r}}{r} \qquad \text{for} \quad E \leq p^2/2\nu. \quad (6)$$

Above discussion concerns the case $0 \leq \sigma^2$, (or $E \leq -\epsilon_B$ and $0 \leq E$) in Eq.(5) and Eq.(6). While, for the case of $\sigma^2 \leq 0$ (or $-\epsilon_B \leq E \leq 0$), if we take $\sigma = \mp i|\sigma|$ and $a = \pm i|a|$, then a relation $a\sigma = |a\sigma| = |a||\sigma|$ is obtained.

Therefore, 0<|σ| is satisfied for all the energy region.

$$\mathcal{F}\{Z\} = V_0(\alpha,\beta,r)\frac{e^{-\sigma r}}{r} = V_0(\alpha,\beta,r)\frac{e^{-|a||\sigma|r/|a|}}{r/|a|}|a|. \tag{7}$$

The Euler integral of the second kind of Eq.(7) is rewritten for the energy region $-\infty \leq E \leq \infty$, or $-\infty \leq \sigma^2 \leq \infty$ by,

$$\mathcal{E}\left[\mathcal{F}\left\{Z(\boldsymbol{q},\boldsymbol{q}';E)\right\}\right] \equiv V(\alpha,\beta;a,n,\boldsymbol{r}) \tag{8}$$

$$= \frac{1}{\mathcal{P}}\int_0^\infty \sigma^{n-2}e^{-a\sigma}\left\{V_0(\alpha,\beta;r)\frac{e^{-\sigma r}}{r}\right\}d\sigma \tag{9}$$

(7) Is satisfied for all the energy region

$$= \frac{1}{|a|}\left[\frac{1}{|a|^{n-1}\mathcal{P}}\int_0^\infty (|a||\sigma|)^{n-2} e^{-|a||\sigma|}\left\{V_0(\alpha,\beta;\boldsymbol{r})\frac{e^{-|a||\sigma|(r/|a|)}}{(r/|a|)}\right\} d(|a||\sigma|)\right] \quad (10)$$

$$\equiv \Omega(\alpha,\beta;n,\boldsymbol{r})\frac{1}{|a|}\left[\frac{1}{\{r/|a|\}(\{r/|a|\}+1)^{n-1}}\right]$$

$$= \Omega(\alpha,\beta;n,\boldsymbol{r})\frac{|a|^{n-1}}{r(r+|a|)^{n-1}} \quad (11)$$

$$\rightarrow V_0(\alpha,\beta;n,\boldsymbol{r})\frac{|a|^{n-1}}{r(r+|a|)^{n-1}}, \quad (12)$$

Since $V_0$ is independent of σ, therefore Ω is analytically obtained

We call this formulae a "quantized general particle transfer" potential from an atom-molecule system to a quark-gluon system.

with

$$\mathcal{P} = \int_0^\infty \sigma^{n-2} e^{-a\sigma} d\sigma = \int_0^\infty (a\sigma)^{n-2} e^{-a\sigma} d(a\sigma)/a^{n-1} = \frac{\Gamma(n-1)}{a^{n-1}}, \qquad (13)$$

where $V_0(\alpha, \beta; n, r)$ is the eigenvalue of operator $\Omega(\alpha, \beta; n, \boldsymbol{r})$. Eq.(11) is the so-called GPT potential.

If the momentum dependence of form factor $g_\alpha, g_\beta$ is very small, then Eq.(11) becomes by omitting $\alpha, \beta$,

Compare indecies

$$V(\alpha, \beta; a, n, r) \rightarrow V_0(n)\frac{e^{-(n-1)r/a}}{r} \qquad (14)$$

$$\equiv V_0(n)\frac{e^{-\mu r}}{r} \quad \text{for} \quad r \ll a \qquad (15)$$

$$\rightarrow V_0'(n)\frac{a^{n-1}}{r^n} \quad \text{for} \quad a \ll r. \qquad (16)$$

Comparing Eq.(14) and Eq.(15), we obtain a quantum number $n$,

$$n = a\mu + 1. \tag{17}$$

Comparison indecies

Therefore, the "quantization is completed" by using a *pion gauge*: $a = 1/m_\pi$ with pion mass $m_\pi$ in the hadron system,

The quantum number is defined by the exchange particle mass

$$n = \mu/m_\pi + 1, \tag{18}$$

where $\mu$ is an exchange particle mass, where $n = 1$ gives a Coulomb-type potential which corresponds to the quasi-two-body threshold (Q2T) with $\sigma = 0$ or $E = -\epsilon_B$ where the right-hand (scattering) cut is started from this branch point, although the traditional three-body calculation is started from $E = 0$. While $n = 2$ indicates one-pion exchange threshold or the branch point of the left-hand (potential) cut, where $E = 0$ is the three-body break up threshold (3BBT).

Let us call the present new quantization of a particle transfer potential (or a primitive potential) "the GPT potential" and also for this treatment "the GPT theory".

Therefore, we can conclude:

Electron transfer: covalent bond,  zero mass transfer: ion bond

**Table 1.** The GPT potential $\Omega(\mathbf{r};\, n)a^{n-1}/[r(r+a)^{n-1}]$ is illustrated. The potential properties for the longer and shorter ranges are shown with respect to the parameter $n$. The potential depths $V_0$ and $V_0{}'$ are given for the short-range and the long-range expansion for the eigenvalue of the operator $\Omega(\mathbf{r};n)$.

| $n$ | $r \ll a$ | GPT- potential | $a \ll r$ |
|---|---|---|---|
| 1 | $V_0(1)/r$ | $\Omega(\mathbf{r};\, 1)/r$ | $V'_0(1)/r$ |
| 2 | $V_0(2)\, e^{-(r/a)}/r$ | $\Omega(\mathbf{r};\, 2)a/[r(r+a)]$ | $V'_0(2)a/r^2$ |
| 3 | $V_0(3)\, e^{-(2r/a)}/r$ | $\Omega(\mathbf{r};\, 3)a^2/[r(r+a)^2]$ | $V'_0(3)a^2/r^3$ |
| 4 | $V_0(4)\, e^{-(3r/a)}/r$ | $\Omega(\mathbf{r};\, 4)a^3/[r(r+a)^3]$ | $V'_0(4)a^3/r^4$ |
| 5 | $V_0(5)\, e^{-(4r/a)}/r$ | $\Omega(\mathbf{r};\, 5)a^4/[r(r+a)^4]$ | $V'_0(5)a^4/r^5$ |
| 6 | $V_0(6)\, e^{-(5r/a)}/r$ | $\Omega(\mathbf{r};\, 6)a^5/[r(r+a)^5]$ | $V'_0(6)a^5/r^6$ |
| 7 | $V_0(7)\, e^{-(6r/a)}/r$ | $\Omega(\mathbf{r};\, 7)a^6/[r(r+a)^6]$ | $V'_0(7)a^6/r^7$ |
| … | … | … | … |
| … | | … | |
| $n$ | $V_0(\mathrm{n})\, e^{-(n-1)r/a}/r$ | $\Omega(\mathbf{r};\, n)\dfrac{a^{n-1}}{[r(r+a)^{n-1}]}$ | $V'_0(n)a^{n-1}/r^n$ |

Pion transfer potentials

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**Table 2.** The GPT potential for the unphysical Riemann energy plane. By putting $n \to -|n|$ in the original GPT potential: $\Omega(\mathbf{r};\ n)a^{n-1}/[r(r+a)^{n-1}] \to \Omega(\mathbf{r};\ |n|)(r+a)^{|n|+1}/[a^{|n|+1}r]$ is illustrated, which corresponds to the dotted path for the second Riemann sheet in figure 2. The potential properties for the long- and short-ranges are shown with respect to the parameter $a$ and $n$. $V_0(|n|)$ and $V_0{'}(|n|)$ are constants.

| $\|n\|$ | $r \ll a$ | GPT potential | $a \ll r$ |
|---|---|---|---|
| 0 | $V_0(0)/r$ | $\Omega(\mathbf{r};\ 0)[(r+a)/ar]$ | $V'_0(0)/a$ |
| 1 | $V_0(1)/r$ | $\Omega(\mathbf{r};\ 1)[(r+a)^2/a^2r]$ | $V'_0(1)r/a^2$ |
| 2 | $V_0(2)/r$ | $\Omega(\mathbf{r};\ 2)[(r+a)^3/a^3r]$ | $V'_0(2)r^2/a^3$ |
| 3 | $V_0(3)/r$ | $\Omega(\mathbf{r};\ 3)[(r+a)^4/a^4r]$ | $V'_0(3)r^3/a^4$ |
| 4 | $V_0(4)/r$ | $\Omega(\mathbf{r};\ 4)[(r+a)^5/a^5r]$ | $V'_0(4)r^4/a^5$ |
| 5 | $V_0(5)/r$ | $\Omega(\mathbf{r};\ 5)[(r+a)^6/a^6r]$ | $V'_0(5)r^5/a^6$ |
| 6 | $V_0(6)/r$ | $\Omega(\mathbf{r};\ 6)[(r+a)^7/a^7r]$ | $V'_0(6)r^6/a^7$ |
| 7 | $V_0(7)/r$ | $\Omega(\mathbf{r};\ 7)[(r+a)^8/a^8r]$ | $V'_0(7)r^7/a^8$ |
| … | … | … | … |
| … | … | … | … |
| $\|n\|$ | $V_0(\|n\|)/r$ | $\Omega(\mathbf{r};\ \|n\|)\left[\frac{(r+a)^{\|n\|+1}}{a^{\|n\|+1}r}\right]$ | $V'_0(\|n\|)r^{\|n\|}/a^{\|n\|+1}$ |

Confinement potential

Here, it should be stressed that the generation of the long-range potential Eq.(16) could compete with the four fundamental interactions. Anyhow, we do have a concern about the concept of traditional potentials starting that "a particle is the outcome of a particle exchange". If and only if, one could be allowed to propose a new concept that "a (primitive) potential exists first", and then a quantization of the potential could generate particles and forces (or a quantized potential) which could be illustrated by the GPT formulation where the generated particles move according to the quantum mechanical laws. The primitive potential is not known but it could contain a lot of fundamental characteristics where the spin and parity of the generated particle could be obtained by the operator $\Omega(\alpha, \beta; n, \boldsymbol{r})$ in Eq.(11).

Since the quantum number gives a branch point of the potential cut, therefore the total potential is obtained by a proper sum over the quantum number $n$, with $\boldsymbol{r}_{\alpha\beta} = \boldsymbol{r}_\alpha - \boldsymbol{r}_\beta$, $\boldsymbol{r}_{\beta\gamma} = \boldsymbol{r}_\beta - \boldsymbol{r}_\gamma$, $\boldsymbol{r}_{\gamma\alpha} = \boldsymbol{r}_\gamma - \boldsymbol{r}_\alpha$, i.e.,

$$\begin{aligned} V_\alpha(\boldsymbol{r}_{\beta\gamma}) &= \sum_{n=2} V(\beta, \gamma; a = 1/m_\pi, n, \boldsymbol{r}_{\beta\gamma}) && (19) \\ &\equiv \sum_{n=2} \Omega(\beta, \gamma; n, \boldsymbol{r}_{\beta\gamma}) \frac{(1/m_\pi)^{n-1}}{r_{\beta\gamma}(r_{\beta\gamma} + (1/m_\pi))^{n-1}} && (20) \\ &\to \sum_{n=2} V_0(\beta, \gamma; n, r_{\beta\gamma}) \frac{(1/m_\pi)^{n-1}}{r_{\beta\gamma}(r_{\beta\gamma} + (1/m_\pi))^{n-1}}. && (21) \end{aligned}$$

However, in this paper, we will take only $n = 2$ for simplicity.

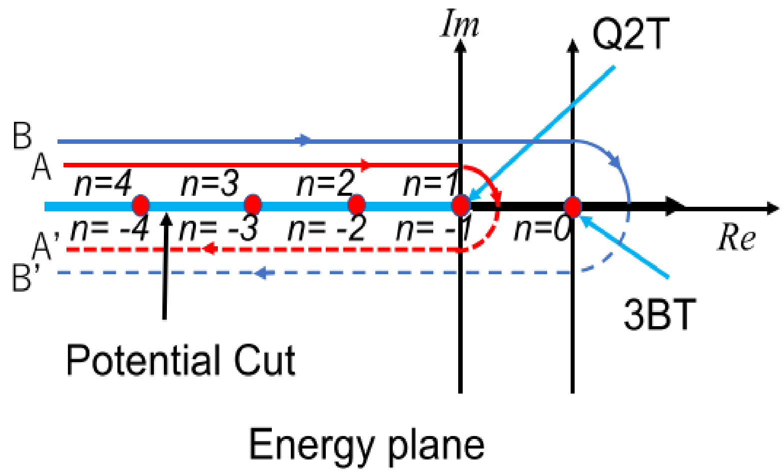
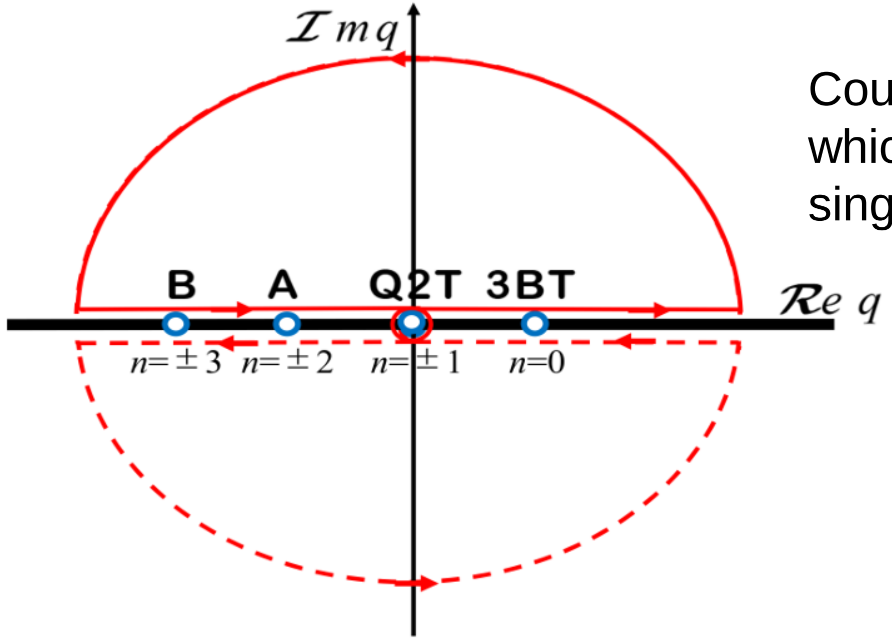


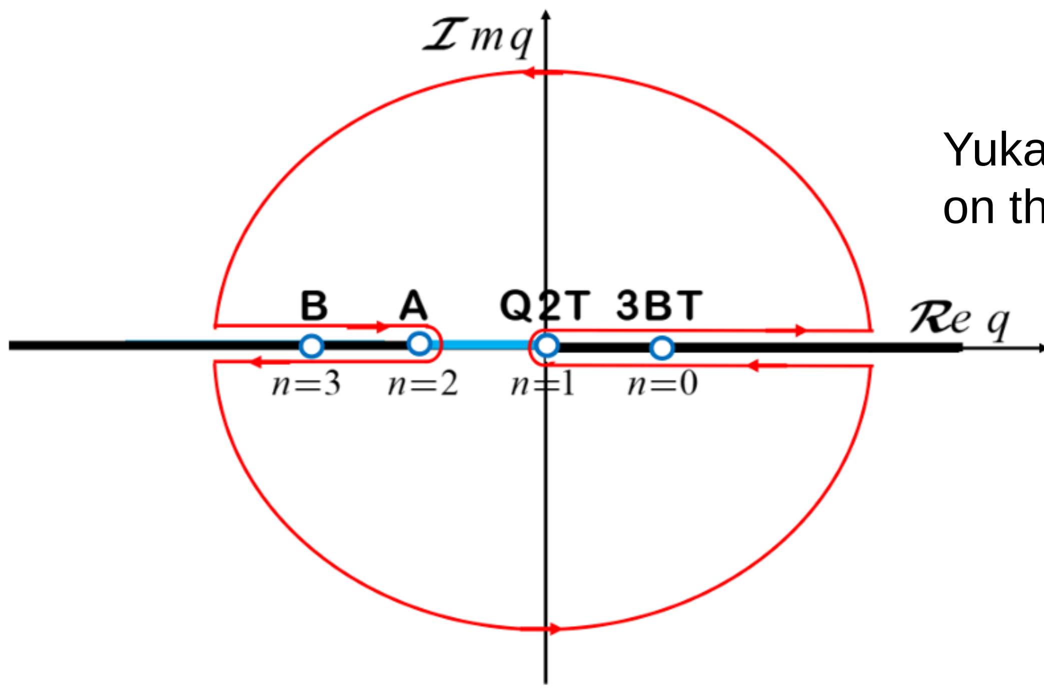

**Figure 2.** The Riemann plane is illustrated for the hadron system, where the index number $n$ of the GPT potential is represented for one-pion transfer (with $n = 2$), two-pion transfer (with $n = 3$), and the three-pion transfer (with $n = 4$) and so on. The number $n$=1 does not only indicate the Q2T, but also a massless particle transfer which is unknown particle, however it could be understood as a particle-antiparticle pair such as $\pi\overline{\pi}$ which is an analogy in the chemical potential for $n$=1 where an ionic-bond is given by the GPT potential with $e^- e^+ \rightarrow \gamma\gamma$. The number $n = 0$ represents the 3BT but gives an unknown potential which is shown in table 2. Therefore, the potential cut (or the left-hand cut) has some possibilities of the starting points with $n = 2$, $n = 1$ and $n = 0$, respectively. Hence, the scattering cut (or the right-hand cut) could depend on the potential cut. In this context, the solid thickish line (A) or the solid thread line (B) are the integral paths on the 1st Riemann sheet (or the 'physical' plane) which turns at the Q2T or the 3BT, and goes down to the 'unphysical' (2nd Riemann) sheet by each dotted line to A′ or B′, which gives the negative index number and generates a new potential as shown in table 2 which seems to be the quark-quark interaction. This fact suggests that the quark-quark scattering (with $-|n|<0$) could not be observed.

For the NN scattering by pion transfer larger than 2=n, however for a higher energy NN scattering nucleon could be considered by a quark cluster where a quark-quark interaction could be realized and mass less gluon or n=1 could occur. The left hand cut is defined bellow the Q2T (n=1), while the right hand cut is defined above the Q2T(n=1). Therefore, the integral contour goes into the second Riemann sheet where the quark and gluon could not be observed because of unphysical sheet.

# Riemann plane

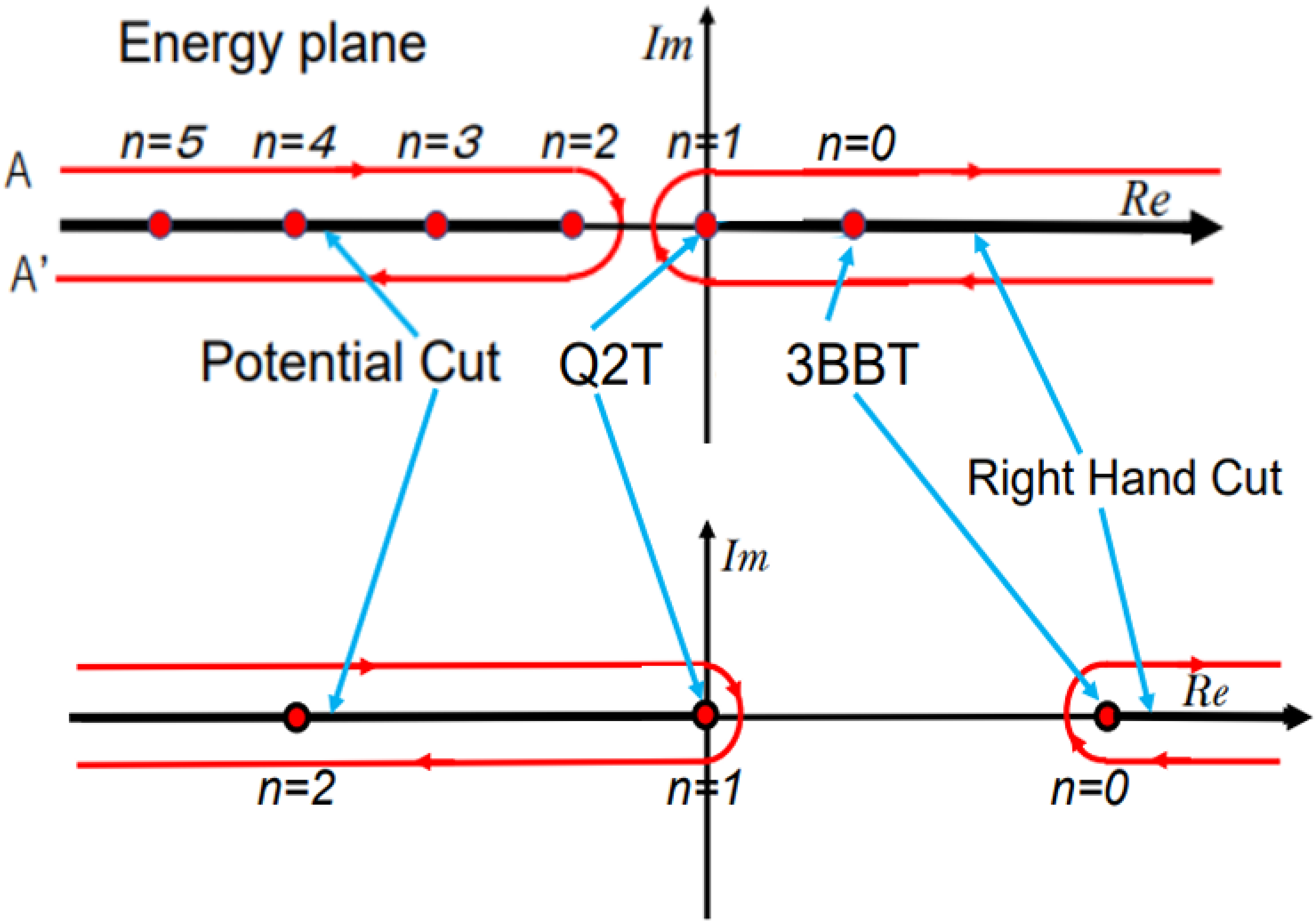



Coulomb-like potential which has a pinching singularity at Q2T




Yukawa-like potential on the Riemann sheet

# Usual hadron-π-hadron systems




Borromean systems

# The Pade approximation & the continued fluxion method;

$$
\begin{aligned}
f(x) &= R(x) + O(x^{m+n+1}) = \left[\frac{m}{n}\right]_f(x) + O(x^{m+n+1}) \\
&= \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m}{1 + b_1 x + b_2 x^2 + \cdots + b_n x^n} + O(x^{m+n+1}),
\end{aligned}
$$

$$
R(b,d,f,h) = a + \cfrac{b}{c + \cfrac{d}{e + \cfrac{f}{g + \cfrac{h}{i}}}} \rightarrow a + \cfrac{x}{c + \cfrac{x}{e + \cfrac{x}{g + \cfrac{x}{i}}}} = \left[\frac{2}{2}\right]_f(x) = R_{[2/2]}(x).
$$

$$
\begin{aligned}
R_{[2/2]}([1/V_\alpha + 1/V_\beta]) &= \cfrac{1}{b_{\alpha\beta} + \cfrac{[1/V_\alpha + 1/V_\beta]}{c_{\alpha\beta} + \cfrac{[1/V_\alpha + 1/V_\beta]}{d_{\alpha\beta} + \cfrac{[[1/V_\alpha + 1/V_\beta]}{e_{\alpha\beta} + \cfrac{[1/V_\alpha + 1/V_\beta]}{f_{\alpha\beta}}}}}} \\
&\equiv \left[\frac{2}{2}\right]_f([1/V_\alpha + 1/V_\beta]).
\end{aligned}
$$

Definition of three-body GPT potential by two-body GPT potentials by the [0/1] Pade:

Hence, a truncated Padé: $R_{[0/1]}(x)$ becomes

$$R_{[0/1]}([1/V_\alpha + 1/V_\beta]) = \frac{1}{b_{\alpha\beta} + [1/V_\alpha + 1/V_\beta]} \quad (46)$$

$$= \frac{V_\alpha V_\beta}{b_{\alpha\beta} V_\alpha V_\beta + V_\alpha + V_\beta} \quad (47)$$

$$\equiv [V_{3\mathrm{GPT}}]_{\alpha\beta} \quad (48)$$

with a constant $b_{\alpha\beta}$ where the time ordering: $[V_\alpha \cdot V_\beta]$ and $[V_\beta \cdot V_\alpha]$ are neglected.

Let us define a generalized three-body GPT (3GPT) potential by

$$V_{3\mathrm{GPT}} = [V_{3\mathrm{GPT}}]_{\alpha\beta} + [V_{3\mathrm{GPT}}]_{\beta\gamma} + [V_{3\mathrm{GPT}}]_{\gamma\alpha}. \quad (49)$$

Make 3-body GPT potential by 2-body GPT potential by a Pade

$$R_{[0/1]}([1/V_\alpha + 1/V_\beta]) \;=\; \frac{1}{b_{\alpha\beta} + [1/V_\alpha + 1/V_\beta]} = \frac{V_\alpha V_\beta}{b_{\alpha\beta} V_\alpha V_\beta + V_\alpha + V_\beta},$$

$$b_{\alpha\beta} \sim \bar{\delta}_{\alpha\beta} V_\gamma(\boldsymbol{r}_{\alpha\beta}) / [V_\alpha(\boldsymbol{r}_{\beta\gamma}) V_\beta(\boldsymbol{r}_{\gamma\alpha})],$$

Let us convert

A linearity of potential condition

makes an AGS—Born

$$\Big[V_{3\mathrm{GPT}}(\boldsymbol{r}_{\alpha\beta}, \boldsymbol{r}_{\beta\gamma}, \boldsymbol{r}_{\gamma\alpha})\Big]_{\alpha\beta}$$

$$\to \bar{\delta}_{\alpha\beta}\Big[\frac{V_\alpha(\boldsymbol{r}_{\beta\gamma}) V_\beta(\boldsymbol{r}_{\gamma\alpha})}{V_\alpha(\boldsymbol{r}_{\beta\gamma}) + V_\beta(\boldsymbol{r}_{\gamma\alpha}) + V_\gamma(\boldsymbol{r}_{\alpha\beta})}\Big], \quad (54)$$

$$= \bar{\delta}_{\alpha\beta}\Big[\frac{V_\alpha(\boldsymbol{r}_{\beta\gamma}) V_\beta(\boldsymbol{r}_{\gamma\alpha})}{E_{\mathrm{had}} - H_0 + i\epsilon}\Big] \equiv [V_{3\mathrm{BFF}}]_{\alpha\beta} \quad (55)$$

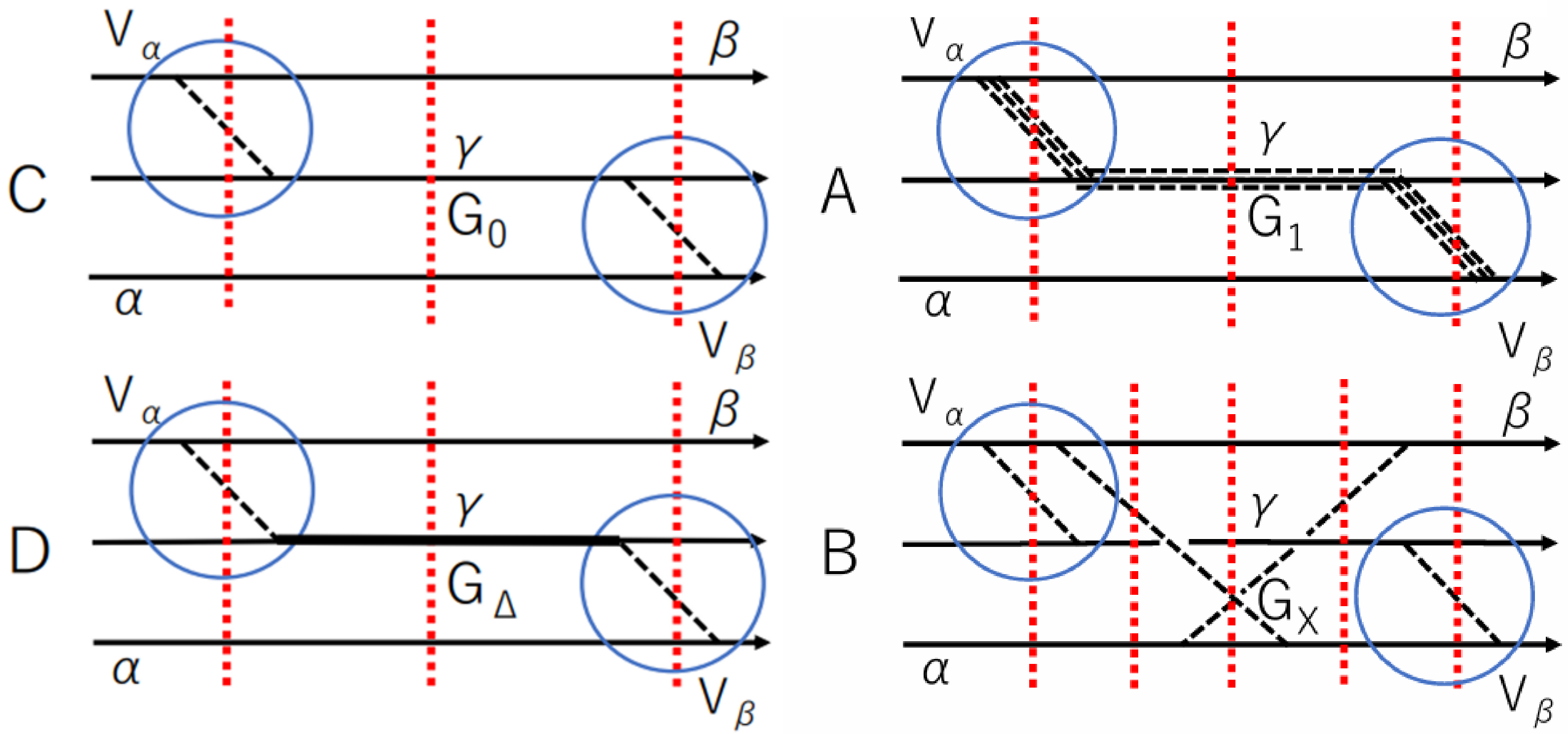
A definition of Operator-Production is given by a complete set of hadron which makes a short range 3-body potential with a small angular dependence.

$$[V_{\rm 3BFF}(\boldsymbol{q}_\alpha, \boldsymbol{q}_\beta; E_{\rm had})]_{\alpha\beta} \to \bar{\delta}_{\alpha\beta} \times \left[ \frac{\sum_i < \psi_\alpha(\boldsymbol{q}_\alpha)|V_\alpha(\boldsymbol{r}_{\beta\gamma})|s_i >< s_i|V_\beta(\boldsymbol{r}_{\gamma\alpha})|\psi_\beta(\boldsymbol{q}_\beta) >}{E_{\rm had} - H_0 + i\epsilon} \right] \quad (56)$$

A 3-body short range potential by a complete hadron set

$Z_{\alpha\beta}(\boldsymbol{q}_\alpha, \boldsymbol{q}_\beta; E_0)$ could be redefined by

$$\mathcal{Z}_{\alpha\beta}(\boldsymbol{q}_\alpha, \boldsymbol{q}_\beta; E_0) \equiv \mathcal{F}\Big[V_{\rm 3GPT}(\boldsymbol{r}_\alpha, \boldsymbol{r}_\beta, \boldsymbol{r}_\gamma)\Big]_{\alpha\beta}. \quad (57)$$




So, the Green‘s function is modified by the three-body force.

Fig. C represents usual three-body Green's function between two-body form factors for the three-nucleon systems.

Fig. D illustrates the well-known Δ-isobar transition Green's function between two-body form factors.

Fig. A is one nucleon transfer which wears several pions.

Fig. B indicates a special Green's function with three nucleons and some pions where a time ordering of pion creation and annihilation crossovers with the two-body interactions $V_\alpha$ and $V_\beta$.

1) Fig. C is the Green's function of original Faddeev equation.
2) Fig. D is the origin of the Fujita-Miyazawa three-body force.
3) Fig. A is one of the Green's function which includes an excited hadron.
4) Figs. C, D, A belong to a linear three-body Green's function.
5) Fig. B belongs to a nonlinear Green's function where the 3BF cannot be separated into the two-body forces where pions occur an entanglement in the three-body systems.

$$\mathcal{E}[\mathcal{F}\{Z\}] = \sum_{n=2}^{\text{nmax}} \Omega(i\alpha_i, j\beta_j, \boldsymbol{r}; n) \frac{a^{n-1}}{r(r+a)^{n-1}} \quad (15)$$

$$\rightarrow \sum_{n=2}^{\text{nmax}} V_0(i\alpha_i, j\beta_j; n) \frac{e^{-\mu r}}{r} \quad \text{for} \quad r \ll a \quad (16)$$

$$\rightarrow \sum_{n=2}^{\text{nmax}} V_0'(i\alpha_i, j\beta_j; n) \frac{a^{n-1}}{r^n} \quad \text{for} \quad a \ll r \quad (17)$$

# 3BLF Appl.

*For n = v*

truncation for short range part by *b*

$$[V_{3\mathrm{BF}}(\rho)]_{ij} = V_{3\mathrm{BF}}^{ij}(\rho)$$

$$\rightarrow \frac{V_0'(i\alpha_i, j\beta_j; \nu) a^{\nu-1}}{r^\nu + b_{ij}^\nu} \bar{\delta}_{ij} \quad (\mathrm{B12})$$

$$= \left[\frac{8A_j}{3}\right]^{\nu/2} \frac{V_0'(i\alpha_i, j\beta_j; \nu) a^{\nu-1}}{\rho^\nu + \rho_3^\nu} \bar{\delta}_{ij}$$

$$\rightarrow \frac{V_3 \rho_3^\nu}{\rho^\nu + \rho_3^\nu} \bar{\delta}_{ij} \delta_{\alpha_i \beta_j}, \quad (\mathrm{B13})$$

Non-diagoinal 3BLF

# OPEP: (with Yukawa's statistic approximation)

$$W_2 \approx \sum_{\boldsymbol{k}} \left\{ \frac{< 0|H'_{N_2\cdot\pi}|m >< m|H'_{N_1\cdot\pi}|0 >}{-\omega_k} + (1 \leftrightarrow 2) \right\},$$

$E - (H_0 + \omega_k) \fallingdotseq - \omega_k$

where two-nucleon's recoil effects are omitted.

$$V^\pi(\boldsymbol{r}) = \Omega^\pi(\boldsymbol{r};2)\frac{e^{-m_\pi r}}{r},$$

This is the simplest form of Eq.(16) in Page 20.

$$\Omega^\pi(\boldsymbol{r};2) = \frac{f^2}{3}\left\{\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2} + \left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2}\right) S_{12}\right\}$$

$$S_{12} = 3\frac{(\boldsymbol{\sigma_1}\cdot\boldsymbol{r})(\boldsymbol{\sigma_2}\cdot\boldsymbol{r})}{r^2} - \sigma_1\cdot\sigma_2.$$

In the three-body $^4$He-n-n system, the 3BLF is a part of the three-body potential: $\mathcal{E}[\mathcal{F}\{\mathcal{Z}\}]$ with the AGS-Born term $Z_{i\alpha_i,j\beta_j}$ where the short range part should be truncated by using a factor $b_{ij}$ for Eq.(17),

Page 20

$$V_0'(i\alpha_i, j\beta_j; \nu)\frac{a^{\nu-1}}{r^\nu} \to \frac{V_0'(i\alpha_i, j\beta_j; \nu)a^{\nu-1}}{r^\nu + b_{ij}^\nu}. \qquad (19)$$

Finally, the Hamiltonian is given by

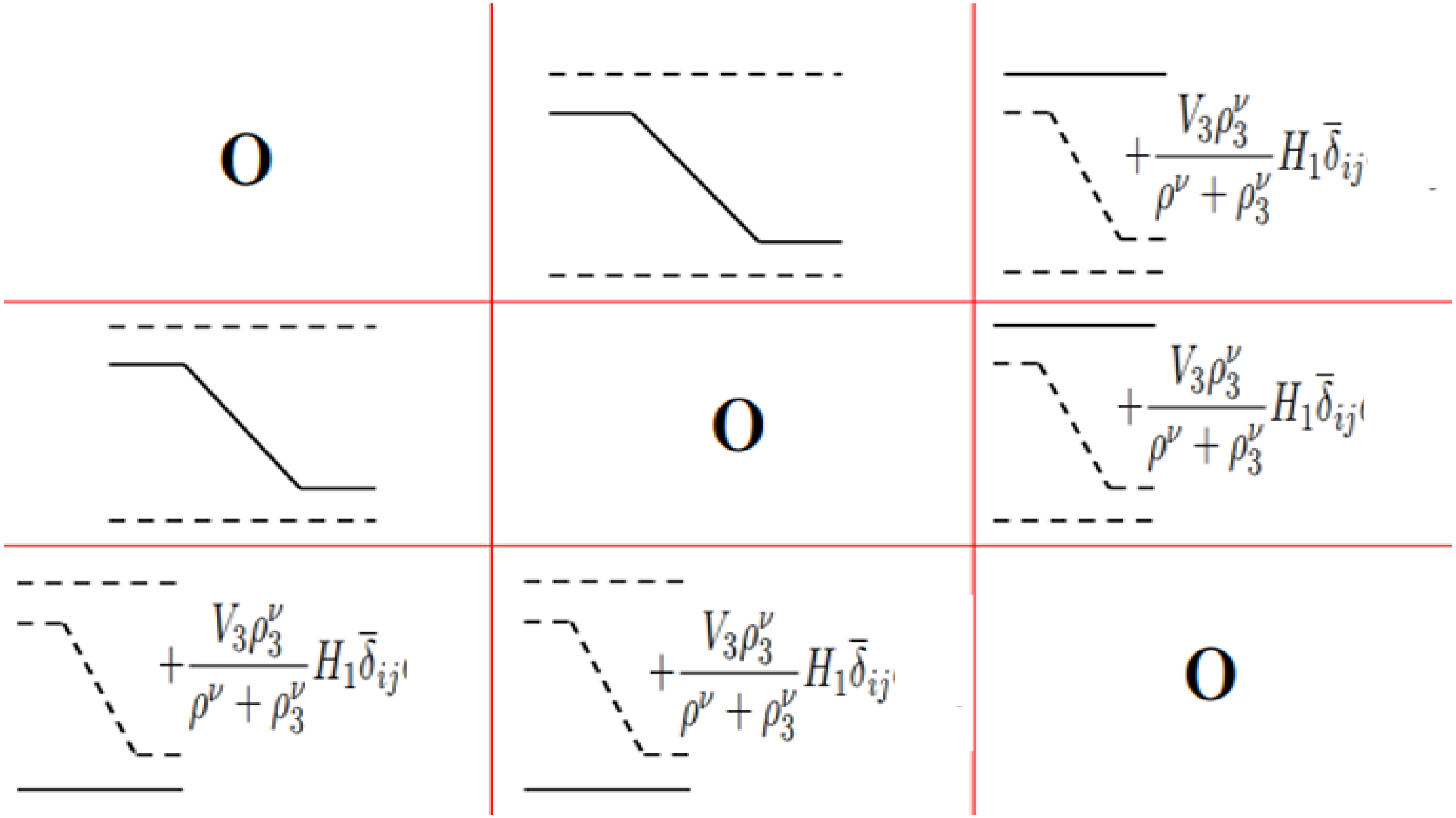
$$
\begin{aligned}
H &= H_0 + V_{3\mathrm{BF}}(\rho) \\
&+ V^{ij}(r_{ij}) + V^{ij}_{\mathrm{GPT}}(r_{ij}) \\
&+ V^{ki}(r_{ki}) + V^{ki}_{\mathrm{GPT}}(r_{ki}) \\
&+ V^{jk}(r_{jk}) + V^{jk}_{\mathrm{GPT}}(r_{jk}),
\end{aligned}
$$

Three-body GPT potential

Usual two-body potentials

Two-body GPT potentials

# 3BLF should be added to the traditional AGS-Born terms




where $H_1$ is the Heaviside step function which is defined in page 37 for the short range truncation.

TABLE I. The binding energy, the matter radius of $^6$He are shown for several cases: the first line by ref.[12], the second by our method, the third by our different parameter choice. The fourth line

[12] Thompson I. J., Danilin B. V., Efros V. D., Vaagen J. S., Bang J. M., Zhukov M. V., Pauli blocking in three-body models of halo nuclei, Phys Rev, **C 61** 024318 (2000).

us,

proper depth parameters $V_3$ and $\rho_3$ are chosen, respectively. The experimental data is $2.30 \pm 0.07$ [fm][31].

$^4$He+n+n →$^6$He

| $\nu$ | $V_3$ [MeV] | $\rho_3$ [fm] | binding energy [MeV] | matter radius [fm] |
|---|---|---|---|---|
| 3 [12] | $-2.4000$ | 5.0 | $-0.98$ | 2.55 |
| 3 | $-2.4000$ | 5.0 | $-0.98$ | 2.56 |
| 7.96* | $-7.2960$ | 2.5 | $-0.98$ | 2.35 |

Thompson et al

Thompson like

2*π*-transfer

Neutron transfer GPT *n*→*ν* =7.96

where $V_3$ and $\rho_3$ should be analytically obtained.

Therefore, the GPT theory is completed, coherent and predictable.

TABLE II Pion effects for the $^6$He binding energy and the matter radius of neutron transfer case in TABLE I with respect of two-body GPT potentials in the Hamiltonian of page 23.
nmax is defined by Eq.(17) of page 20.

| nmax | binding energy [MeV] | matter radius [fm] |
|---|---|---|
| 2 | −0.89 | 2.34 |
| 3 | −0.93 | 2.35 |
| 4 | −0.96 | 2.35 |
| 5 | −0.97 | 2.35 |
| 6 | −0.98 | 2.35 |
| 7 | −0.98 | 2.35 |
| 8 | −0.98 | 2.35 |
| 9 | −0.98 | 2.35 |

The results show that the pion transfer effects in the two-body form factors (or the two-body GPT potentials) are very small.

# Summary

① There are two-types of 3-body forces, one is the 3-body short range force: 3BSF and another is the 3-body long range force: 3BLF.

② The three-body Coulomb force (3BCF) is generated by a kind of "photon entanglement" in the long range region.

Therefore, the 3BCF can not be separated into the two-body Coulomb potentials, but a 3BLF in the three-body systems.

③ By the same way, the 3BSF and 3BLF occur in the three-body very short renge and a very long range region.

They can not be separated into two-body potentials.

④ The 3-body Faddeev equation is only used for a linear three-body space in the middle range region.
Therefore, the three-body forces are additional force in the entire region.
The three-body primitive potential given by the three-body Faddeev kernel is generalized by using a quantized 3-body GPT potential.

⑤ The 3-body Faddeev equation is represented by a multi-channel quantized Lippmann Schwinger equation.

$$\mathcal{X}(\overline{q},\overline{q}';E_{\rm cm}) = \mathcal{Z}(\overline{q},\overline{q}';E_{\rm cm}) + \int_0^\infty \mathcal{Z}(\overline{q},\overline{q}'';E_{\rm cm})\tau(\overline{q}'';E_{\rm cm})\mathcal{X}(\overline{q}'',\overline{q}';E_{\rm cm})d\overline{q}''$$

$$0 \le E_{\rm cm} = E + \epsilon_B < \infty; \qquad 0 \le \overline{q} < \infty.$$

⑥ A quantized potential is made from a primitive potential with Eqs. (1),(2) of page 3 by our GPT method, where the quantized particles and their dynamics are defined.
The new concept may be a counter part to the traditional thought that "a particle exchange makes a potential".

⑦ The quantized potential is defined in an entire space (from a very short range to a very long range) where the Faddeev equation is verified in a linear space of a middle range.
The quantum hadronic meshed area could be represented by a pion-gage, while the quantum electro-magnetic area could be defined by an electron-photon-gauge.

[7] Oryu, S., 2012 Universal structure of the three-body system Phys. Rev. **C86**, 044001.

[8] Oryu, S., 2018 A New Feature of the Efimov-like Structure in the Hadron system -Long Range Force as a Recoil Effect-, Few-Body Syst. **59**:51.

[9] Oryu, S., 2010 Long-range property in time-dependent interaction with three-body structure and new aspect, *Annual Report of Quantum Bio-Informatics Center of Tokyo University of Science*, **5**, 26-34.

[10] Oryu, S., 2013 Long Range Potential Component in the NN Force, Few-Body Syst. **54** 283-286.

[11] Oryu, S., *et al.*, 2017 A New Horizon of Few-Body Problems: Exact Coulomb treatment and the energy-momentum translation of the three-body Faddeev equation, J. Phys. Conference Series **915** 012001.

[12] Oryu, S., 2020 A Possibility of a Long Range Three-Body Force in the Hadron System, Few-Body Syst **61**:33.

[13] Oryu, S., 2021 The General Particle Transfer Potential in a Range of Physical Systems and its Long Range Component, Few-Body Syst **62** 56.

[14] S. Oryu, Generation of long- and short-range potentials from atom-molecules to quark-gluon systems by the GPT potential, J. Phys. Commun. **6** 015009, (2022).

[12]* Thompson I. J., Danilin B. V., Efros V. D., Vaagen J. S., Bang J. M., Zhukov M. V., Pauli blocking in three-body models of halo nuclei, Phys Rev, **C 61** 024318 (2000).

[31] Alkhozov, G. D., et al., Nuclear Matter Distribution2 in $^6$He and $^8$He from Small Angle p-He Scattering in Inverse Kinematics at Intermediate Energy, Phys. ReV. Lett. **78**, 2313 (1997) 33

[3] Oryu S., Watanabe T., Hiratsuka Y., Takeda M., A New Quantization of Hadron System and Ultra-Low Energy Nuclear Fusion, Few-Body Syst (2024) 65:42 https://doi.org/10.1007/s00601-024-01912-6.

END

# Appendix

# Hyper radius formular

$$A_{jk} = \frac{A_j A_k}{A_j + A_k},$$

$$A_{(jk)i} = \frac{(A_j + A_k)A_i}{A_i + A_j + A_k}$$

$$\boldsymbol{x}_i = \sqrt{A_{jk}}\boldsymbol{r}_{jk}$$

$$\boldsymbol{y}_i = \sqrt{A_{(jk)i}}\boldsymbol{r}_{(jk)i}$$

$$\rho^2 = x_i^2 + y_i^2,$$

$$\theta_i = \arctan\left(\frac{x_i}{y_i}\right),$$

$$\rho^2 = x_1^2 + y_1^2 = x_2^2 + y_2^2 = x_3^2 + y_3^2$$

$$\rightarrow \frac{(A_j + A_k)A_i}{A_i + A_j + A_k}\boldsymbol{r}^2 = \frac{8A_j}{3}\boldsymbol{r}^2$$

$$r = \sqrt{3/[8A_j]}\rho.$$

# The definition of three-body potential:

Therefore, a channel component of the 3GPT potential for $\alpha \to \beta$ becomes

$$[V_{\rm 3GPT}(\boldsymbol{r}_\alpha, \boldsymbol{r}_\beta, \boldsymbol{r}_\gamma)]_{\alpha\beta} = \Big[V_{\rm 3GPT}(\boldsymbol{r}_{\alpha\beta}, \boldsymbol{r}_{\beta\gamma}, \boldsymbol{r}_{\gamma\alpha})\Big]_{\alpha\beta}$$

$$= \Big[b_{\alpha\beta} + \big\{V_\alpha(\boldsymbol{r}_{\beta\gamma})\big\}^{-1} + \big\{V_\beta(\boldsymbol{r}_{\gamma\alpha})\big\}^{-1}\Big]^{-1} \quad (50)$$

$$= \frac{V_\alpha(\boldsymbol{r}_{\beta\gamma})V_\beta(\boldsymbol{r}_{\gamma\alpha})}{b_{\alpha\beta}V_\alpha(\boldsymbol{r}_{\beta\gamma})V_\beta(\boldsymbol{r}_{\gamma\alpha}) + [V_\alpha(\boldsymbol{r}_{\beta\gamma}) + V_\beta(\boldsymbol{r}_{\gamma\alpha})]} \quad (51)$$

$$= \Big[b_{\alpha\beta} + \sum_{n=n_{0\alpha}} \Big\{\frac{\Omega_{\beta\gamma}(\boldsymbol{r}_{\beta\gamma}; n)^{n-1}}{r_{\beta\gamma}(r_{\beta\gamma}+a)^{n-1}}\Big\}^{-1}$$

$$+ \sum_{n=n_{0\beta}} \Big\{\frac{\Omega_{\gamma\alpha}(\boldsymbol{r}_{\gamma\alpha}; n)a^{n-1}}{r_{\gamma\alpha}(r_{\gamma\alpha}+a)^{n-1}}\Big\}^{-1}\Big]^{-1}, \quad (52)$$

where $n_{0\alpha}$ and $n_{0\beta}$ depend on the transfered particle masses.

In Eq.(51), if we choose a constant by

$$b_{\alpha\beta} \sim \bar{\delta}_{\alpha\beta}V_\gamma(\boldsymbol{r}_{\alpha\beta})/[V_\alpha(\boldsymbol{r}_{\beta\gamma})V_\beta(\boldsymbol{r}_{\gamma\alpha})], \quad (53)$$

where the 3GPT potential is a linear potential as the Faddeev's kernel.

In the former paper [1], we defined the three-body GPT (3GPT) force from the $\alpha$-channel to the $\beta$-channel by

$$\Big[V_{\mathrm{3GPT}}(\boldsymbol{r}_\alpha,\boldsymbol{r}_\beta,\boldsymbol{r}_\gamma)\Big]_{\alpha\beta} \equiv \bar{\delta}_{\alpha\beta}\Big[b_{\alpha\beta}+\{V_\alpha(\boldsymbol{r}_{\beta\gamma})\}^{-1}+\{V_\beta(\boldsymbol{r}_{\gamma\alpha})\}^{-1}\Big]^{-1}.$$

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For the long-range limit, Eq.(17) becomes a three-body long-range force (3BLF) for $n$=2

$$\Big[V_{\mathrm{3GPT}}(\boldsymbol{r}_\alpha,\boldsymbol{r}_\beta,\boldsymbol{r}_\gamma)\Big] \equiv \sum_{\alpha\neq\beta\neq\gamma\neq\alpha=1}\Big[V_{\mathrm{3GPT}}(\boldsymbol{r}_\alpha,\boldsymbol{r}_\beta,\boldsymbol{r}_\gamma)\Big]_{\alpha\beta}$$

$$\to \Big[V_{\mathrm{3BLF}}(\boldsymbol{r}_\alpha,\boldsymbol{r}_\beta,\boldsymbol{r}_\gamma)\Big] = V_0\Big[d+a(\boldsymbol{r}_{\beta\gamma})^2+b(\boldsymbol{r}_{\gamma\alpha})^2+c(\boldsymbol{r}_{\alpha\beta})^2\Big]^{-1},$$

where if we take $a=b=c=1$, then $V_{\mathrm{3BLF}}$ is equivalent to that of ref.[4]. It was pointed out that this kind of potential is similar to the Efimov potential in the long-range limit [1],[17][18] where $d$ indicates a truncation of the short-range part.

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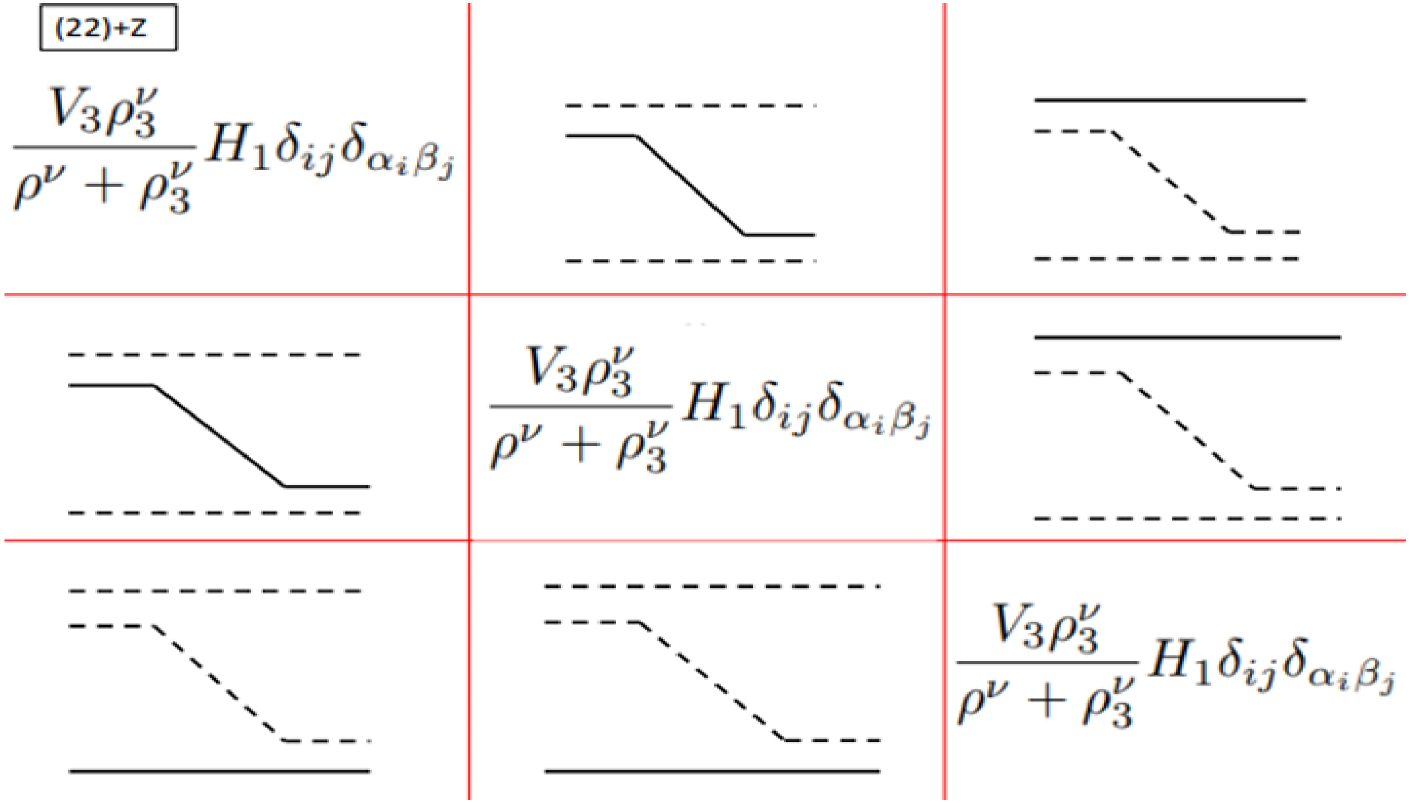
For the next, three kinds of 3BLF were introduced by using Eq.(19) and the Heaviside step function $H_1 \equiv H_1(7.96 - \nu)$ (Appendix B),

X
$$V_{3BF}(\rho) = \frac{V_3 \rho_3^\nu}{\rho^\nu + \rho_3^\nu}, \quad (21)$$

X
$$V_{3BF}(\rho) = \frac{V_3 \rho_3^\nu}{\rho^\nu + \rho_3^\nu} H_1 \delta_{ij} \delta_{\alpha_i \beta_j}, \quad (22)$$

$$V_{3BF}(\rho) = \frac{V_3 \rho_3^\nu}{\rho^\nu + \rho_3^\nu} H_1 \overline{\delta}_{ij} \delta_{\alpha_i \beta_j}. \quad (23)$$

The 3BLF from the α-channel to β-channel should not be a symmetric form but an asymmetric form like (23).
However, the traditional 3BSF in the three-nucleon system adopts almost symmetric form which started from the Fujita-Miyazawa (Δ-isobar bases) type and a point-like interaction. Therefore, even if we adopt a chiral perturbation 3BSF, they are almost symmetric 3BSF as seen in Eqs (21) and (22).




① a neutron transfer potential makes a long range potential with v=7. 96.
an alpha particle transfer GPT potential has almost no effect for the long range.

② The 2GPT based 3GPT potential generates an asymmetric 3BF potential.