Self-bound bosonic droplets

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Universality of Quantum Systems: From Cold Atoms, Nuclei, to Hadron Physics

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He-4 droplets

- He-4 atoms form droplets with any number of atoms
- Binding energy: $B_2 \sim 1$ mK
  $B_3 \sim 100$ mK
  …..
  $B_N \sim N\,(7.1$ K$)$

Universal regime in droplets?

- 2D bosons with weak short-range attraction
- $B_2 \sim \Lambda^2 e^{-4\pi/g}$
- $B_3 = 16.52 B_2, B_4 = 197.2 B_2$
- $B_N \sim 8.576^N$

Unitarity bosons in 3D?

- 3 bosons: infinite number of bound states (Efimov)
- 4 bosons: 2 4-body states for each 3-body state
  - the ground state energy not expected to be universal
- Situation with a large number of bosons not completely clear

Narrow resonance

- One way to regularize the short-distance physics: introduce an effective range r_0
  - ignoring all higher shape parameters
- $r_0 > 0$: unphysical pole in the scattering amplitude (Wigner)
- Assume $r_0 < 0$ and large: narrow resonance

Field theory

- $\mathscr{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2} \right) \psi + \phi^\dagger \left(i\partial_t + \frac{\nabla^2}{4} \right) \phi + \alpha(\psi^\dagger \psi^\dagger \phi + \phi^\dagger \psi \psi) - \nu \phi^\dagger \phi$
  - theory well-defined in the UV
- Matching with scattering amplitude:

$$\alpha = \sqrt{\frac{4\pi}{-r_0}} \qquad \nu = \frac{2}{r_0 a}$$

3-body problem

- The three-body problem with bosons with narrow resonance was recently solved by van Kolck and Griesshammer

- At $a = \infty$, 3-body ground state $B \approx 0.055 \frac{\hbar^2}{mr_0^2}$, infinite tower of Efimov states

- 3-body bound state exists in a finite range of 1/a:

$$-\frac{c_1}{|r_0|} < \frac{1}{a} < \frac{c_2}{|r_0|}$$

Droplet: energy minimization

- $N \gg 1$: many bosons in one quantum state → classical field configuration

- Need to minimize the energy

$$H = \int \mathrm{d}\mathbf{x} \left[\frac{|\nabla \psi|^2}{2} + \frac{|\nabla \phi|^2}{4} - \alpha(\psi^\dagger \psi^\dagger \phi + \phi^\dagger \psi \psi) \right]$$

under the constraint $\int d\mathbf{x} \, (\psi^\dagger \psi + 2\phi^\dagger \phi) = N$

Variational ansatz

- Let $f(x)$ be a shape function example: $f(x) = \dfrac{1}{\cosh x}$

$$\psi(r) = \frac{1}{(4\pi I_2)^{1/2}} \sqrt{\frac{cN}{R^3}} f\left(\frac{r}{R}\right)$$

$$I_2 = \int_0^\infty dx\, x^2 f^2(x)$$

$$\phi(r) = \frac{1}{(8\pi I_2)^{1/2}} \sqrt{\frac{(1-c)N}{R^3}} f\left(\frac{r}{R}\right)$$

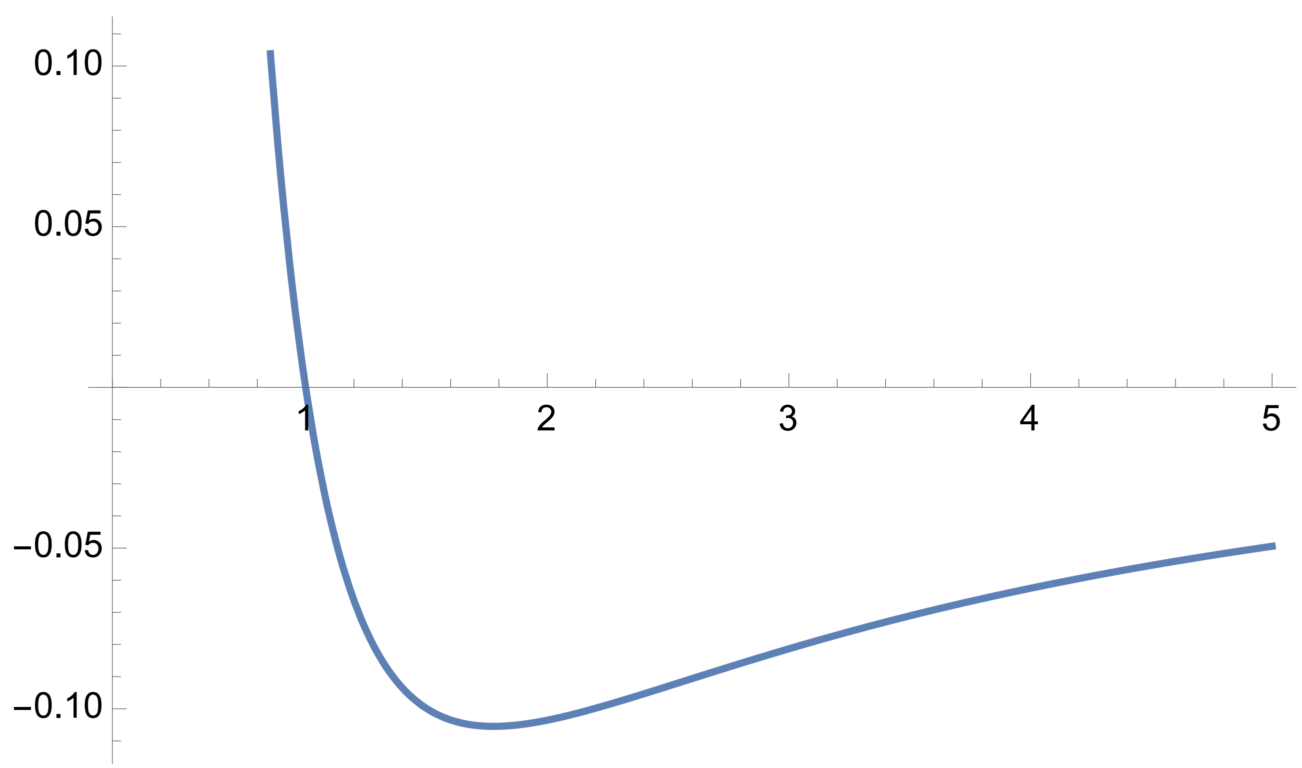
- Two overlapping condensates of same shape and size
cN atoms, $\frac{1}{2}(1-c)N$ molecules

Variational energy

- $$E(R) = a(c)\frac{N}{R^2} - b(c)\frac{N^{3/2}}{R^{3/2}}$$

$$a(c) \sim 3c + 1 \qquad\qquad b(c) \sim c\sqrt{1-c}$$

- Competition between kinetic and Rabi coupling

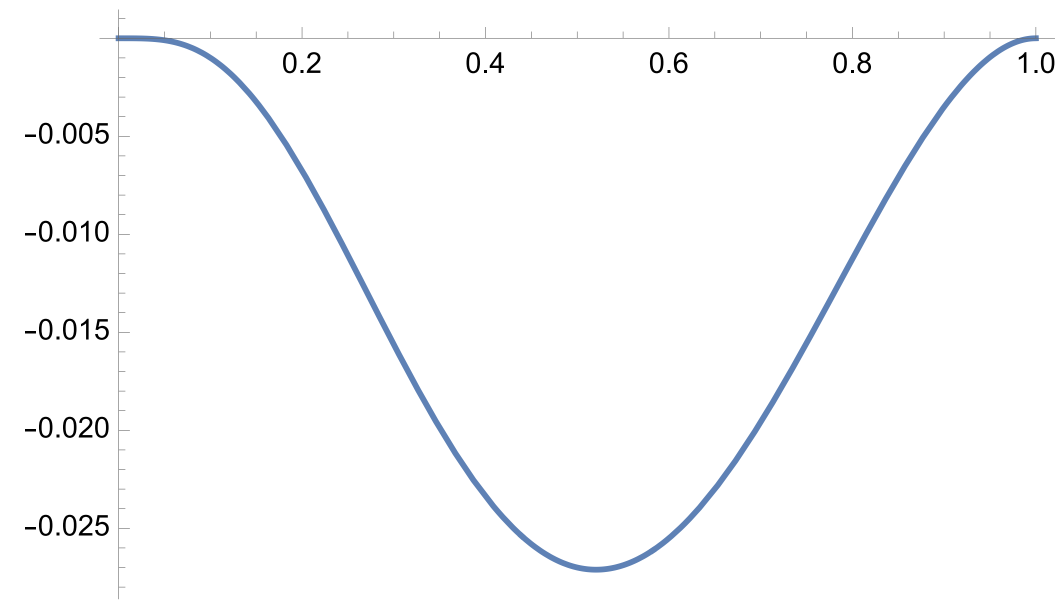


$$R = \frac{16a^2}{9b^2}\frac{1}{N}$$

Variational energy (II)

- After minimizing over R

$$E(c) = -AN^3 \frac{c^4(1-c)^2}{(c+\frac{1}{3})^3}$$



- Minimizing over $c : c \approx 0.521 \rightarrow E = -0.0316N^3$

- Better variational Ansatz:

$$E = -0.0347N^3 \frac{\hbar^2}{mr_0^2}$$

Away from unitarity

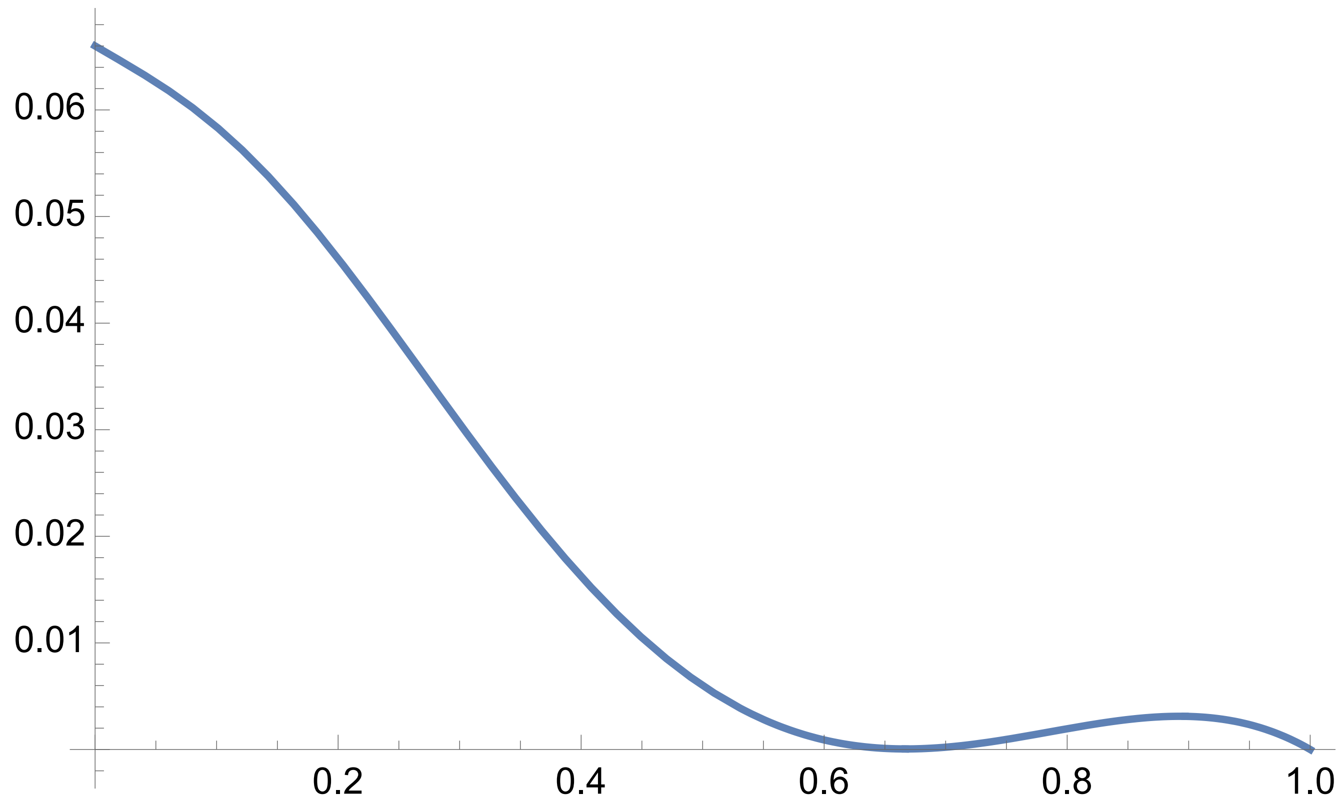
- $H = H_{a=\infty} + \nu \int \mathrm{d}\mathbf{x}\, \phi^\dagger \phi \qquad \nu = -\frac{2\hbar^2}{m|r_0|a}$

- Using single-profile Ansatz:

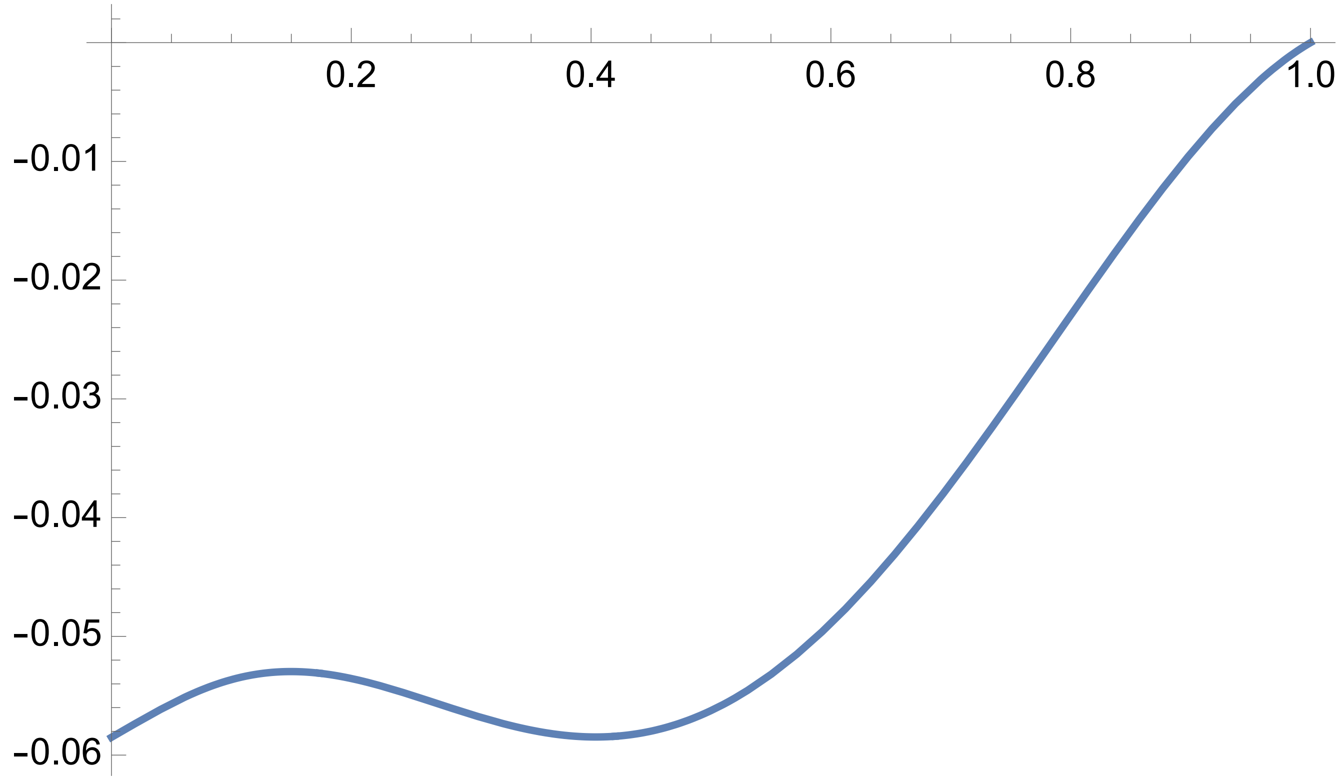
$$E(R) = a(c)\frac{N}{R^2} - b(c)\frac{N^{3/2}}{R^{3/2}} + \frac{\nu}{2}N(1-c)$$

Minimizing over R:

$$E(c) = -N^3 A \frac{c^4(1-c)^2}{(c+\frac{1}{3})^3} + \frac{\nu}{2}N(1-c)$$



$$\frac{2|r_0|}{a} = -0.123A$$



$$\frac{2|r_0|}{a} = 0.117A$$

Summary so far

- Bosons at unitarity, large negative effective range (narrow resonance) seem to form bound state with any N

- At large N the size of the droplet decreases as $1/N$, binding energy increases as N^3

- Tuning away from unitarity: the N-body bound state exists in a finite range of inverse scattering length

$$-0.191N^2 < \frac{2|r_0|}{a} < 0.140N^2$$

Very high N

- At very large N, the density in droplets becomes very large: terms formerly ignored needs to be taken into account

$$H_{\text{bg}} = \frac{1}{2}\int \mathrm{d}\mathbf{x}\,(g_{11}\,|\psi|^4 + 2g_{12}\,|\psi|^2\,|\phi|^2 + g_{22}\,|\phi|^4)$$

- Now a very large droplet has a constant center density

$$n \sim \frac{1}{a_{ij}^2\,|r_0|} \ll \frac{1}{a_{ij}^3}\text{: still dilute}$$

- Droplet binding energy saturates to $E \sim N$

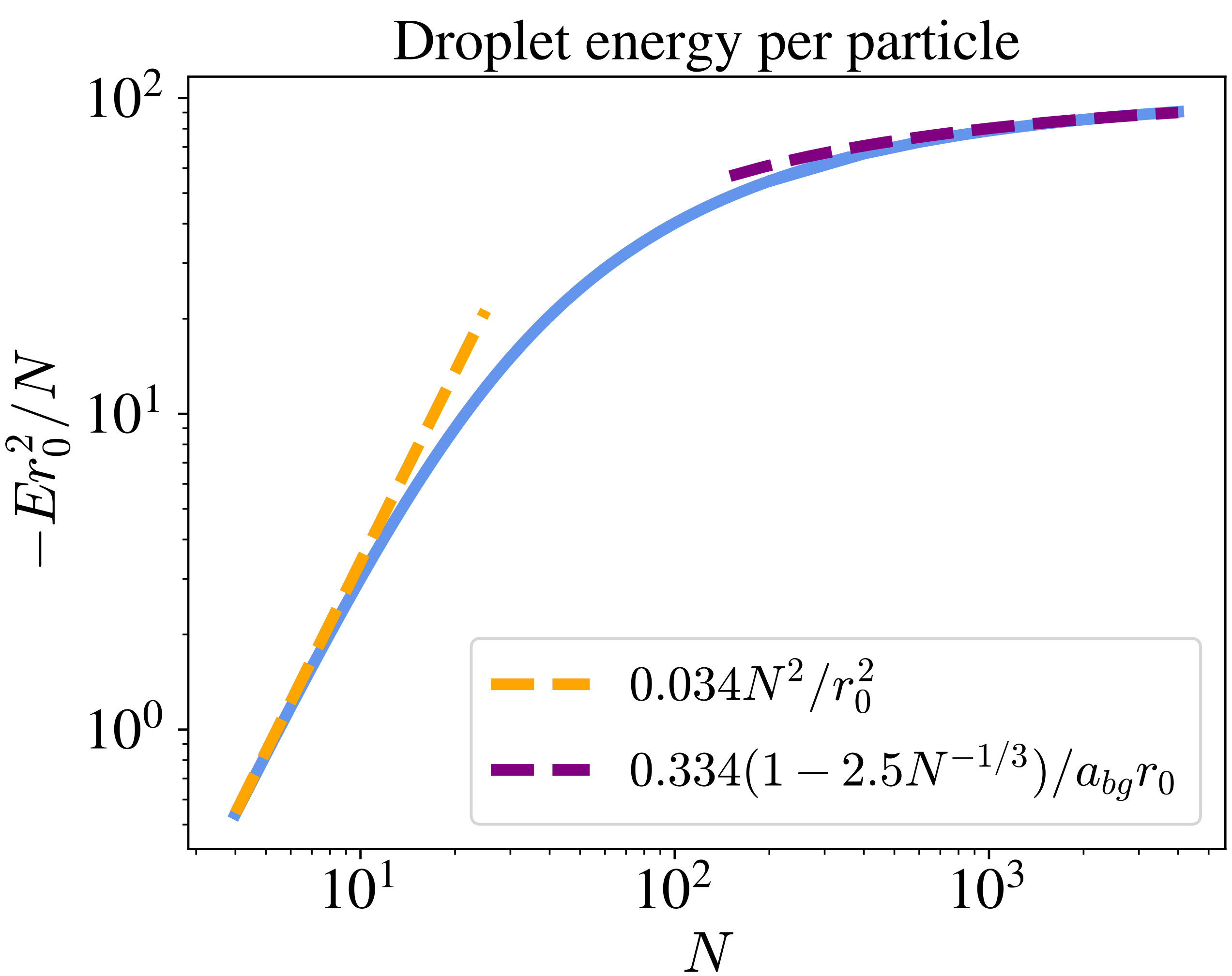


FIG. 1. Droplet binding energy per particle for $g_{11} = 4\pi a_{\rm bg}$, $g_{22} = 2g_{11}/3$, $g_{12} = g_{22}/2$, and $a_{\rm bg} = |r_0|/320$. The fit at large N is motivated by the picture of a droplet with finite surface tension.

Away from unitarity

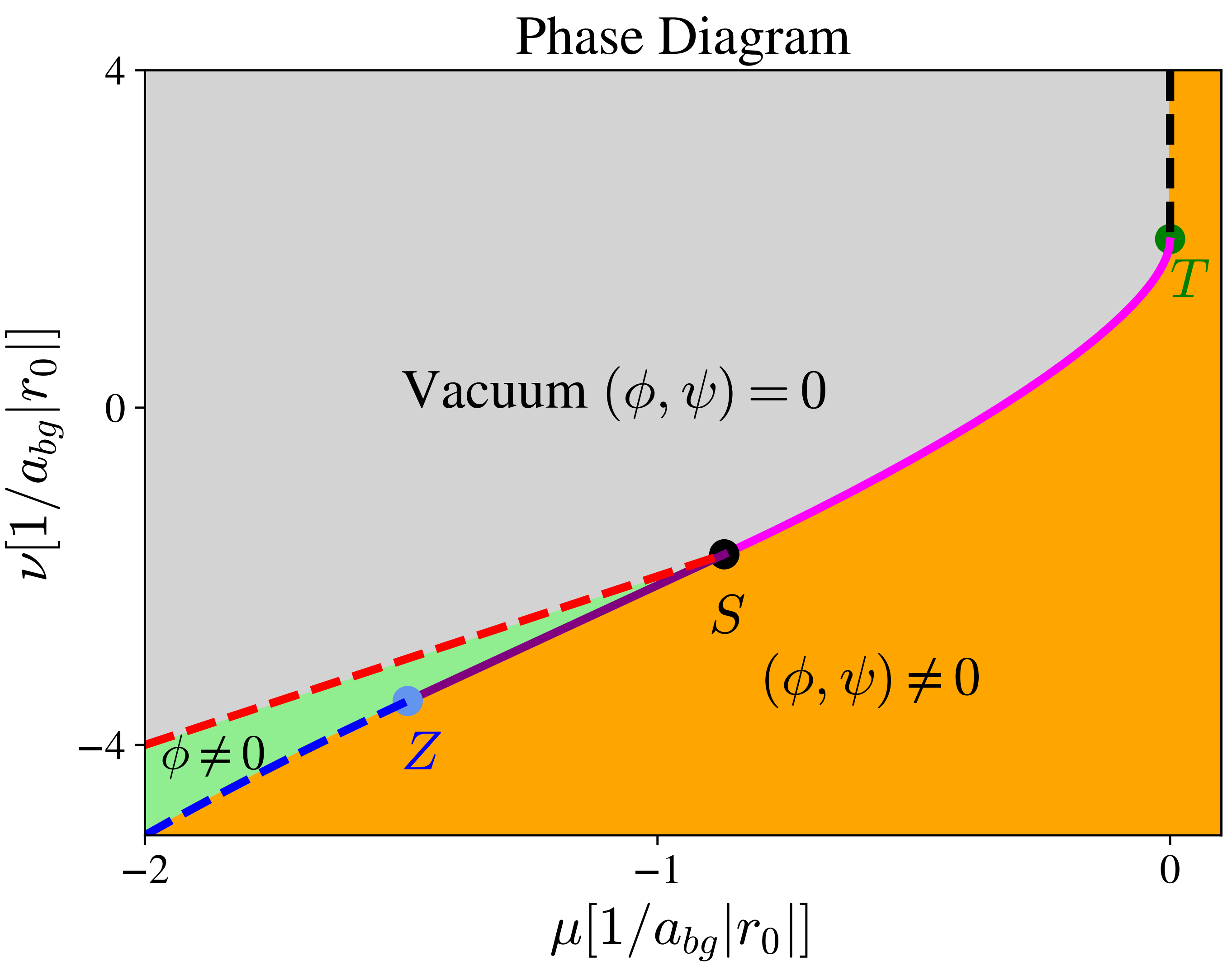


FIG. 2. Phase diagram in the plane of chemical potential μ and detuning ν in units of $1/(a_{\rm bg}|r_0|)$ for $g_{11} = 4\pi a_{\rm bg}$, $g_{22} = 2g_{11}/3$, and $g_{12} = g_{22}/2$. Solid and dashed lines represent first- and second-order transitions, respectively. The

Conclusions

- Bosons at narrow s-wave resonance can form self-bound droplets
- The binding energy of N bosons scales as N^3
- Open questions:
  - quantum (beyond mean field) corrections
  - small-N (N=4, 5 etc) clusters
  - experimental realization?

A remark about self-bound fermonic droplets

- ^{3}He atoms form self-bound clusters at $N > N_{\text{crit}} \sim 20 - 40$
- But ^{3}He is rather far from unitarity: can increase the strength of interaction to reach unitarity
- Presumably N_{crit} will be smaller
- Example of fermions at unitarity, but short-distance physics allow self-bound clusters. Lessons for neutrons?