UTokyo

5th Sep, 2024, UQS2024

Tripling fluctuations and peaked speed of sound in three-color fermions

Hiroyuki Tajima

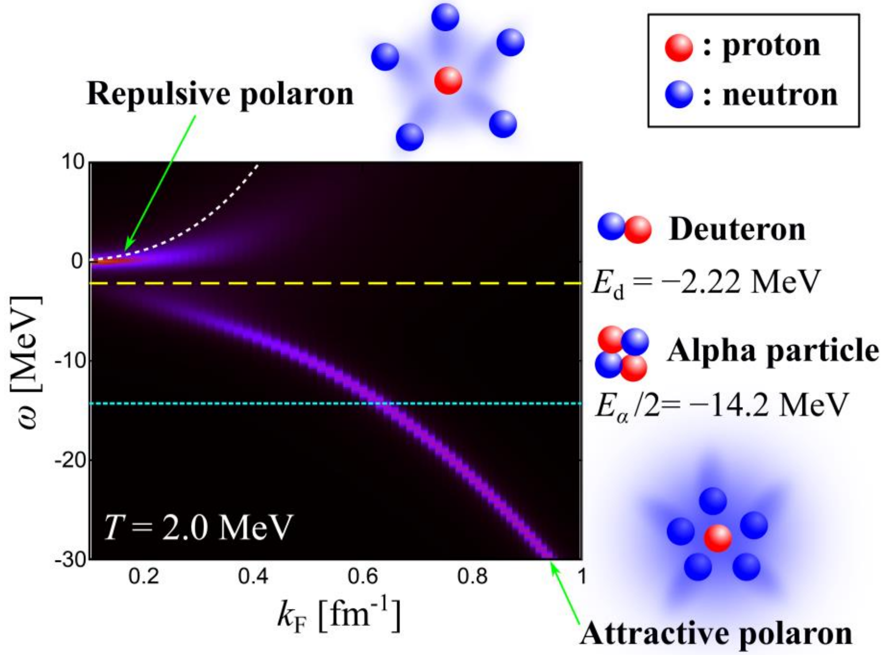
The University of Tokyo, Japan

Collaborators: Kei Iida (Kochi), Toru Kojo (Tohoku), Haozhao Liang (Tokyo)

References:
HT, K. Iida, T. Kojo, and H. Liang, in preparation
HT, K. Iida, and H. Liang, Phys. Rev. C **109**, 055203 (2024).
HT, S. Tsutsui, T. M. Doi, and K. Iida, Phys. Rev. Res. **4**, L012021 (2022).
HT, S. Tsutsui, T. M. Doi, and K. Iida, Phys. Rev. A **104**, 053328 (2021).

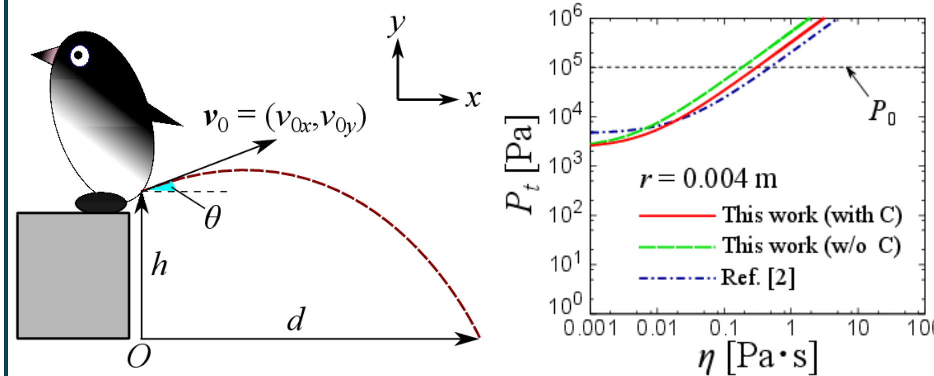
Proton impurities in neutron-rich matter

HT, H. Moriya, W. Horiuchi, E. Nakano, and K. Iida, Phys. Lett. B **851**, 138567 (2024).

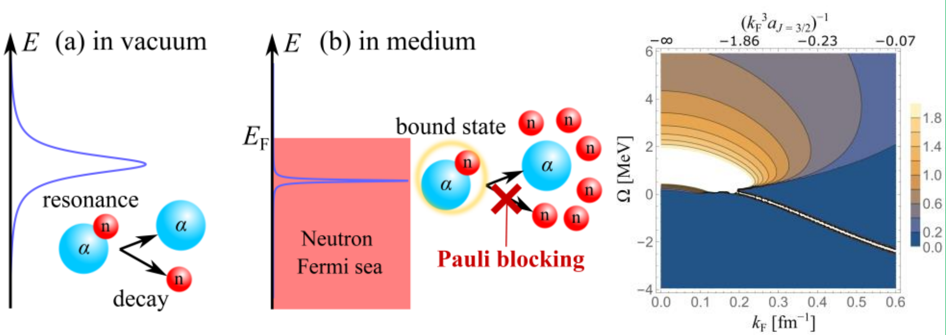


Projectile trajectory of penguin's faeces

HT, and F. Fujisawa, arXiv:2007.00926



Bound ^{5}He in dilute neutron matter and its analogy with *p*-wave Feshbach molecule

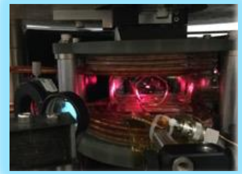


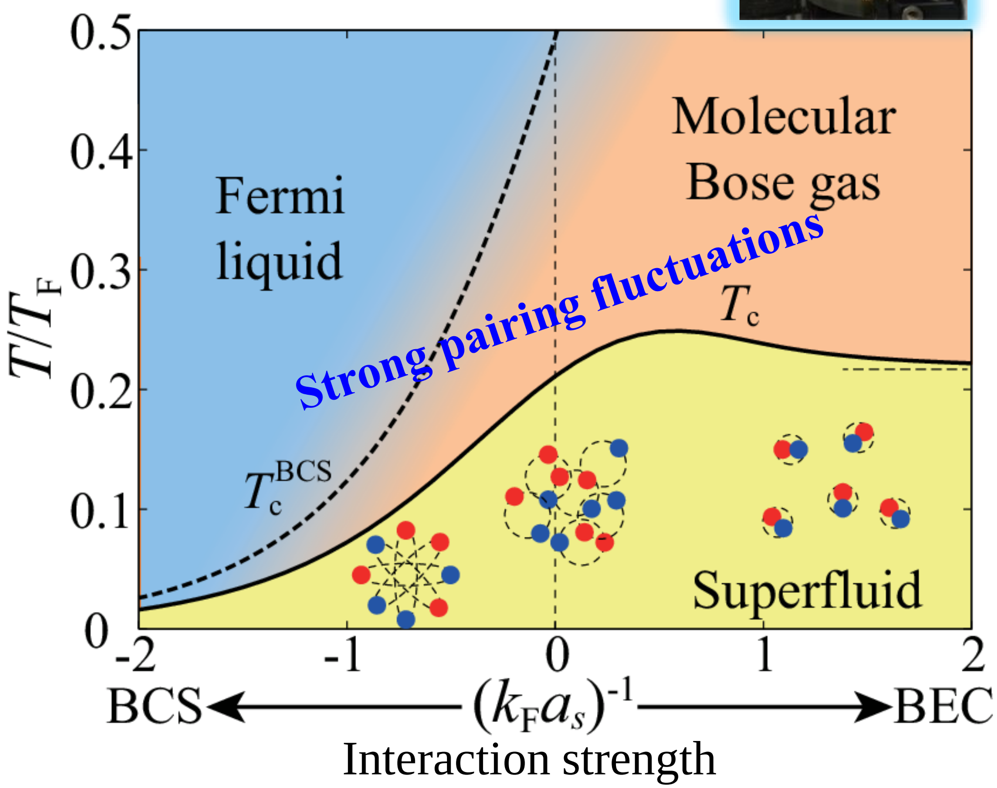
HT, H. Moriya, W. Horiuchi, K. Iida, and E. Nakano, Phys. Rev. C **106**, 045807 (2022).

BEC-BCS crossover phase diagram

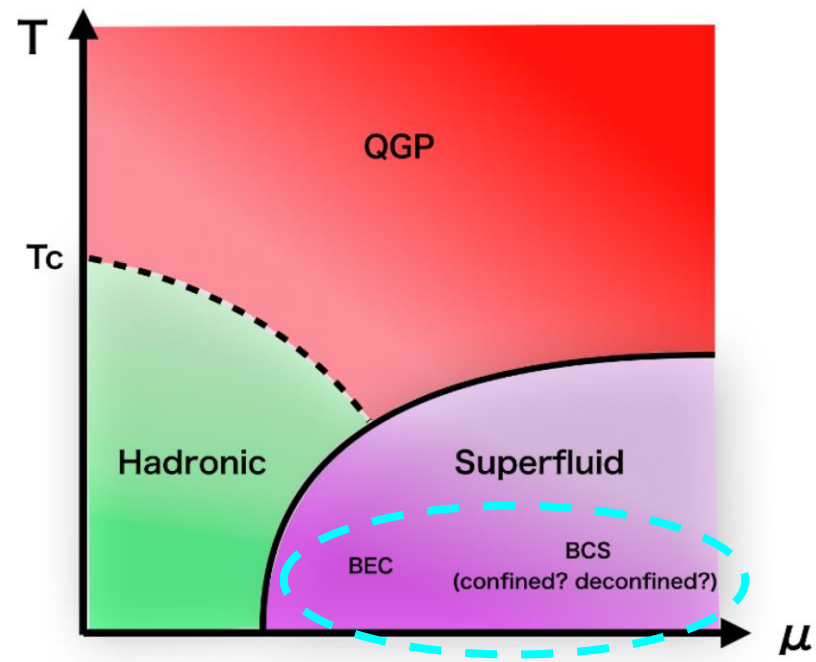
Review: Y. Ohashi, HT, and P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).

BEC-BCS crossover realized in ultracold Fermi gases





BEC-BCS crossover
≃ HQ crossover in "2-color" QCD



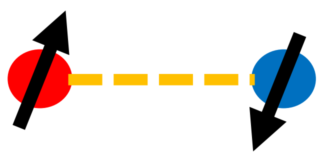
K. Iida, et al., JHEP01(2020)181

Photo: http://www.sci.osaka-cu.ac.jp/phys/laser/research_Li.html

Why pairs or dimers?

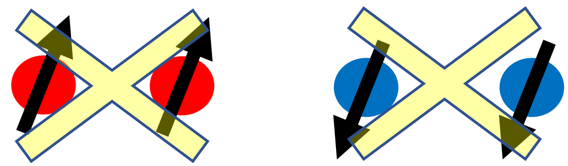
- The quantum cluster formation is related to the internal degrees of freedom and Pauli's exclusion principle

e.g. Spin-1/2 fermions with s-wave interaction

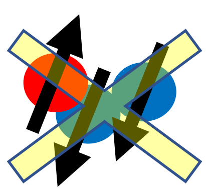


"attraction"

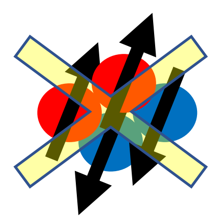
➡Cooper pair or dimer



"Pauli's exclusion principle"



"Trimer"



"Tetramer"

Why pairs or dimers?

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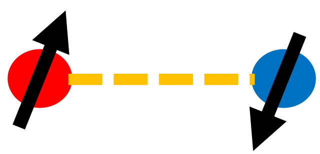

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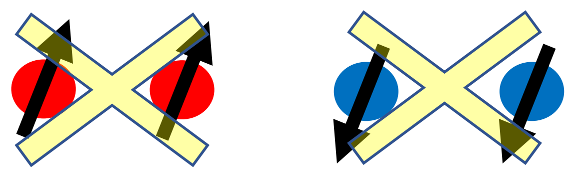

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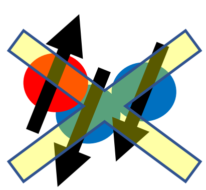
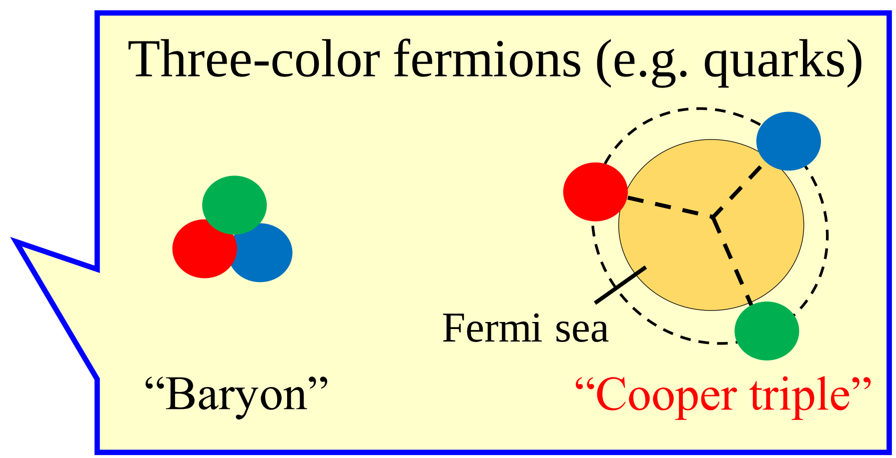


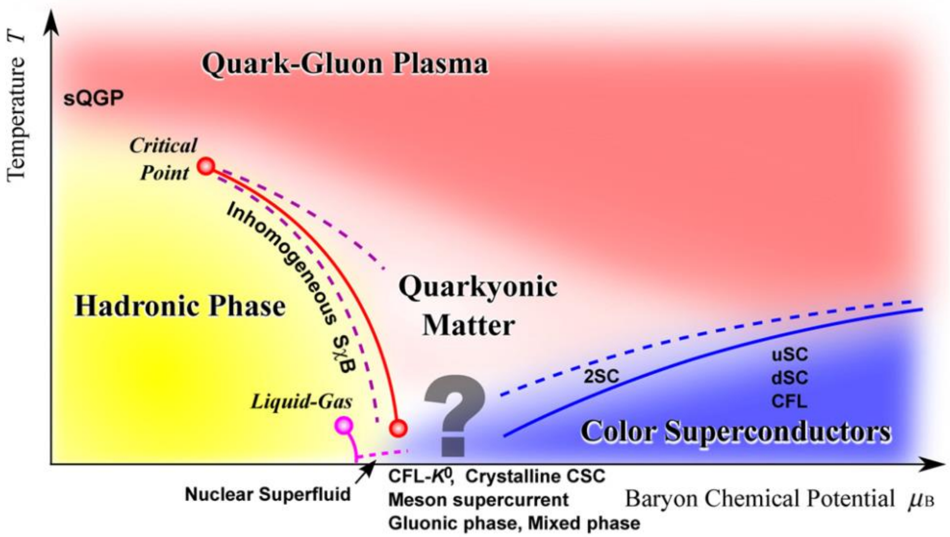
"Trimer"



P. Niemann and H.-W. Hammer
PRA **86**, 013628 (2012).

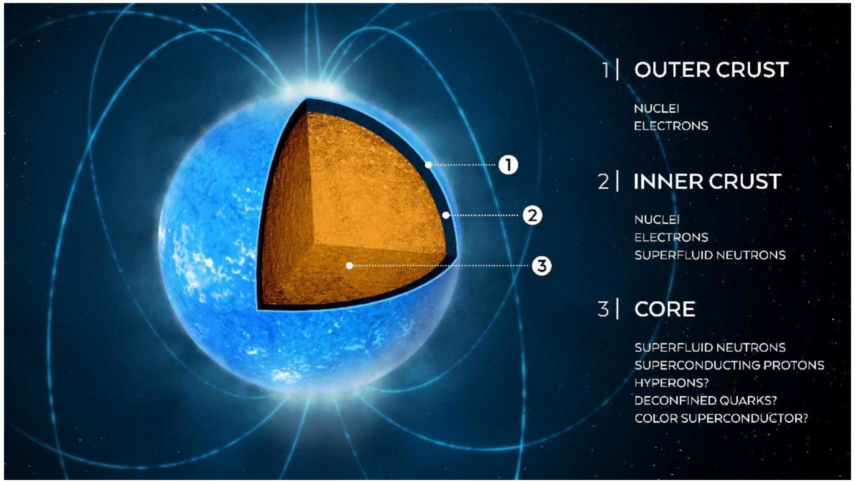
Extremely dense matter

<u>Dense QCD phase diagram</u>



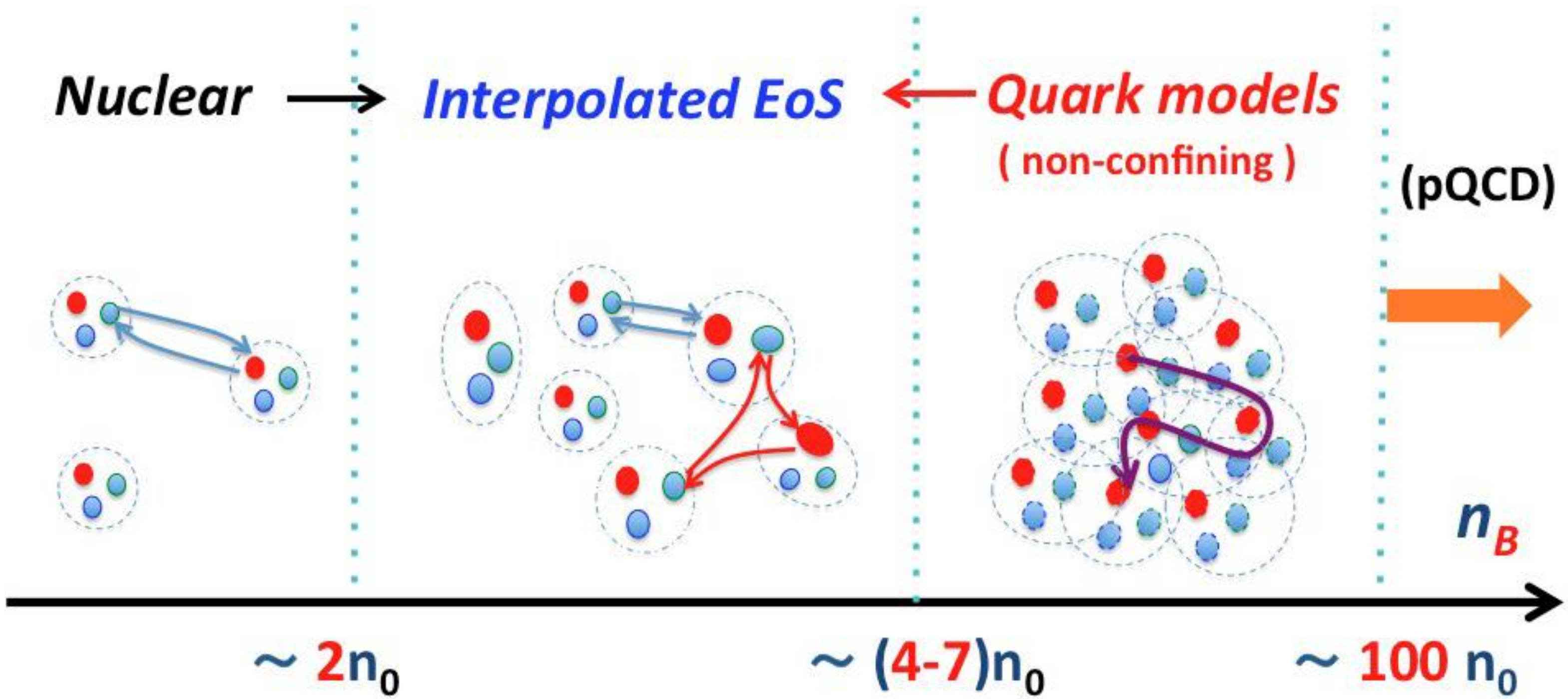
K. Fukushima, *et al.*, Rep. Prog. Phys. **74**, 014001 (2011).

<u>Neutron star as a testing ground of dense matter</u>



A. L. Watts, *et al.*, RMP **88**, 021001 (2016).

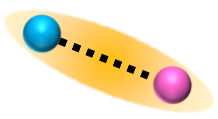
Hadron-quark (HQ) crossover



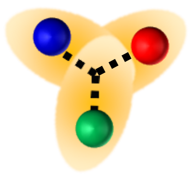
G. Baym, *et al*., Rep. Prog. Phys. **81**, 056902 (2018).

BEC-BCS crossover
≃ HQ crossover in "3-color" QCD?

Dimer
"boson"



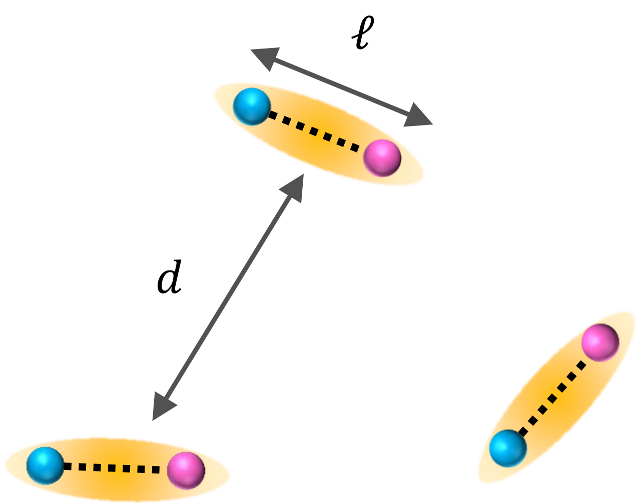
Trimer
"fermion"



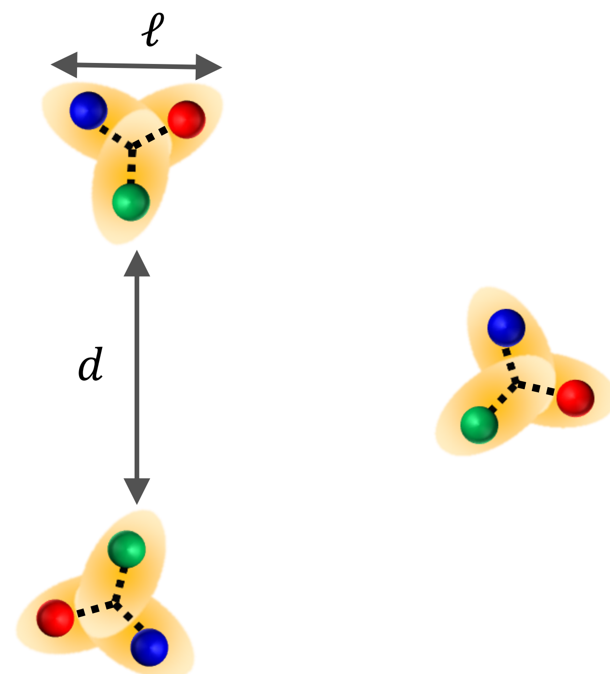
BEC-BCS crossover
$\simeq$ HQ crossover in “3-color” QCD?

Let us consider density evolution

Dimer BEC
($\ell \ll d$)



Trimer Fermi gas
($\ell \ll d$)



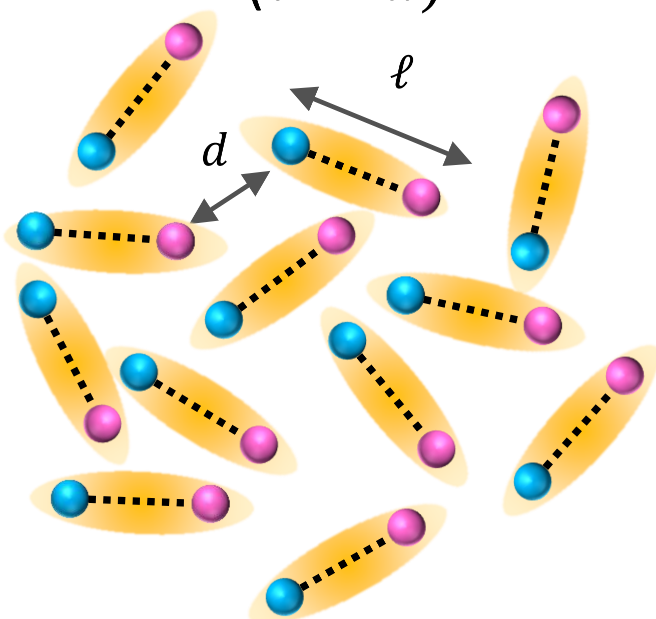
d: interparticle distance

ℓ: molecular size

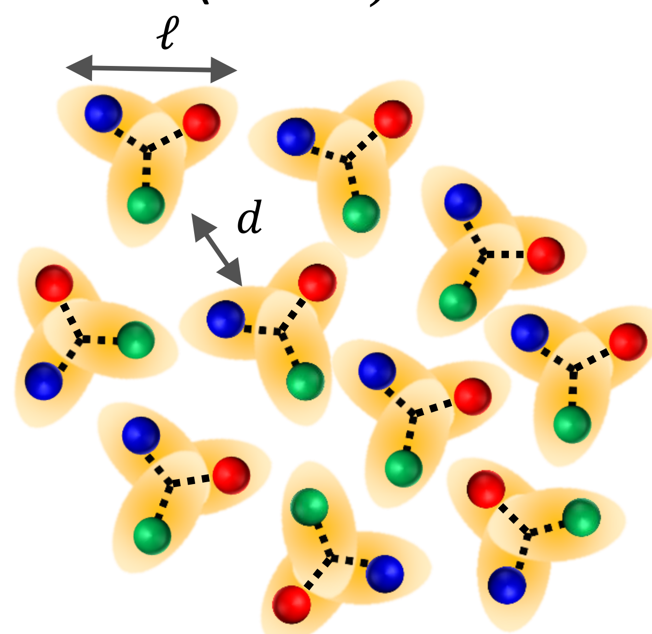
BEC-BCS crossover
$\simeq$ HQ crossover in "3-color" QCD?

Let us consider density evolution

Dense dimer gas
$(\ell \simeq d)$



Dense trimer gas
$(\ell \simeq d)$



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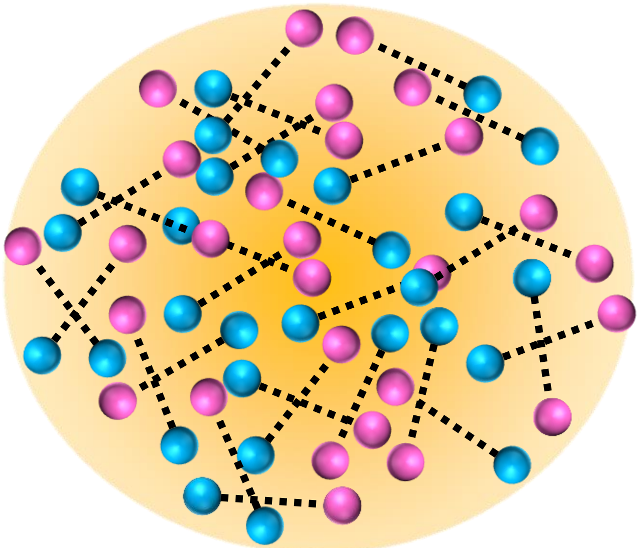
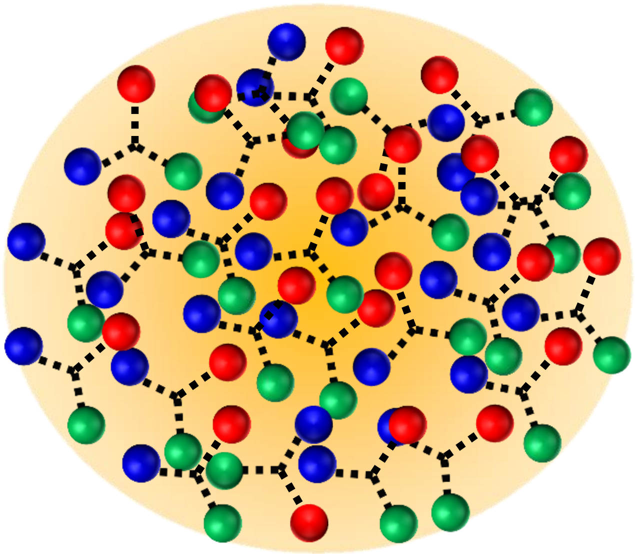
BEC-BCS crossover
$\simeq$ HQ crossover in "3-color" QCD?

Let us consider density evolution

Cooper pairs
$(d \lesssim \ell)$



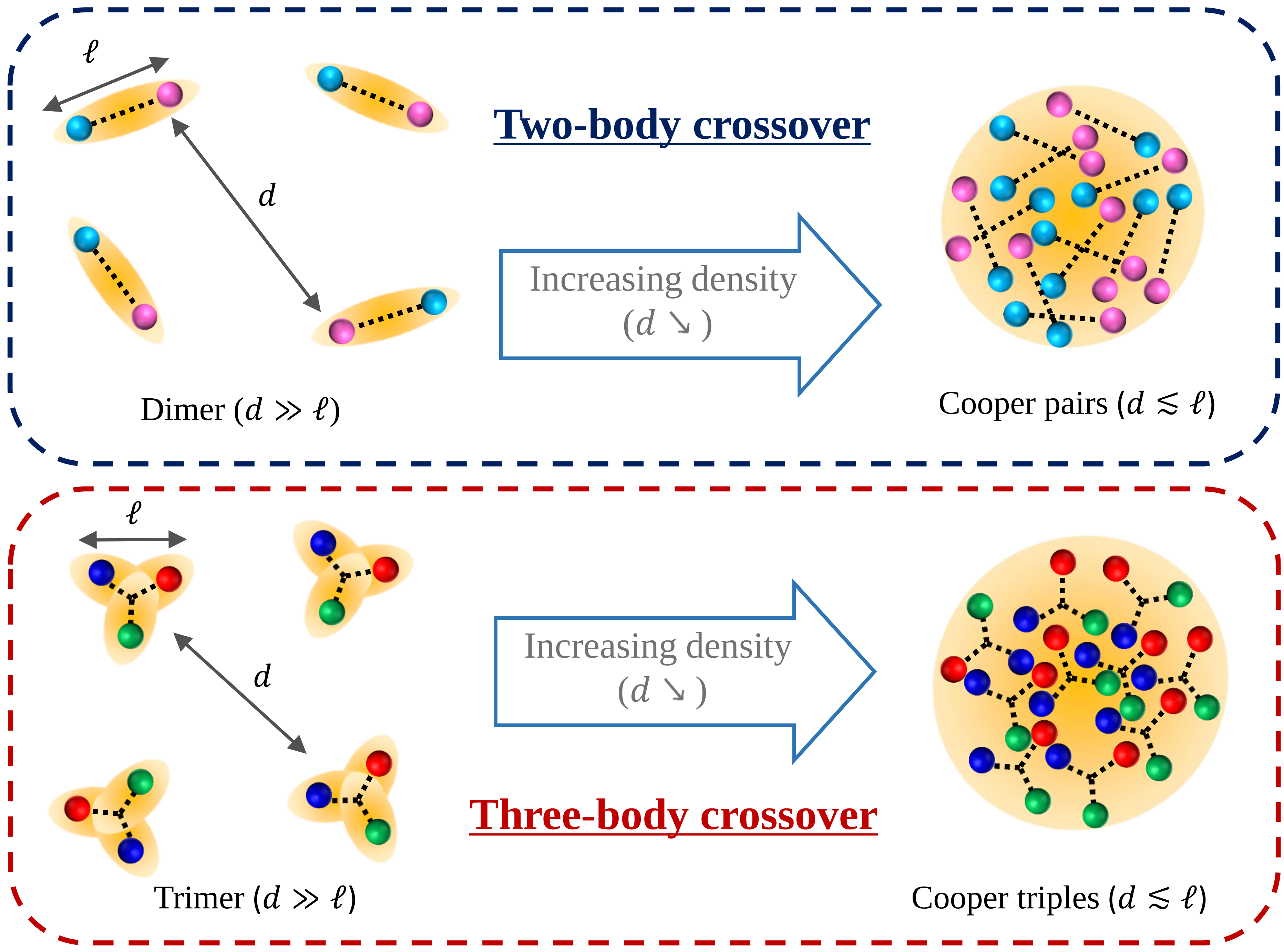
Cooper triples*
$(d \lesssim \ell)$



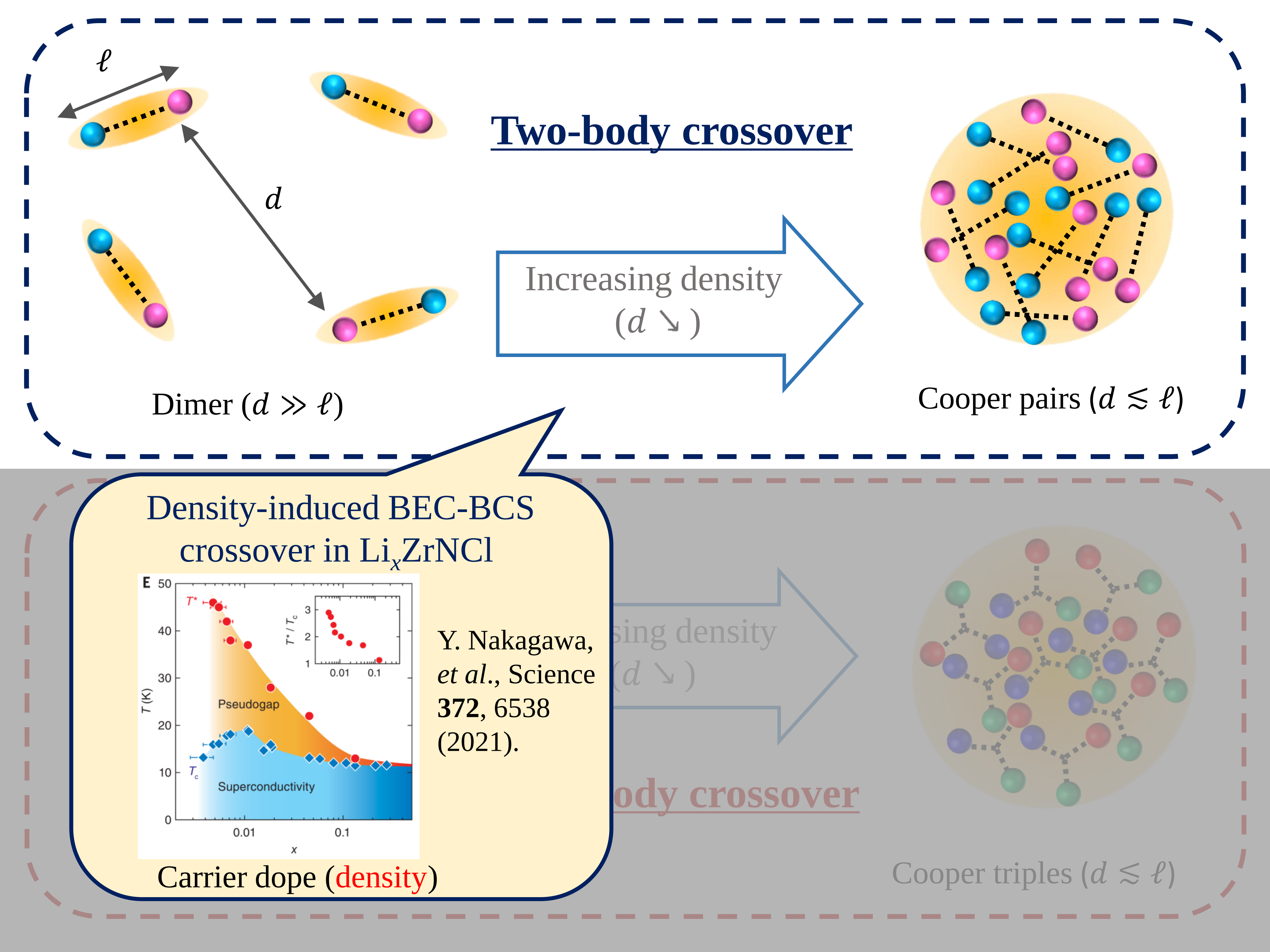
d: interparticle distance

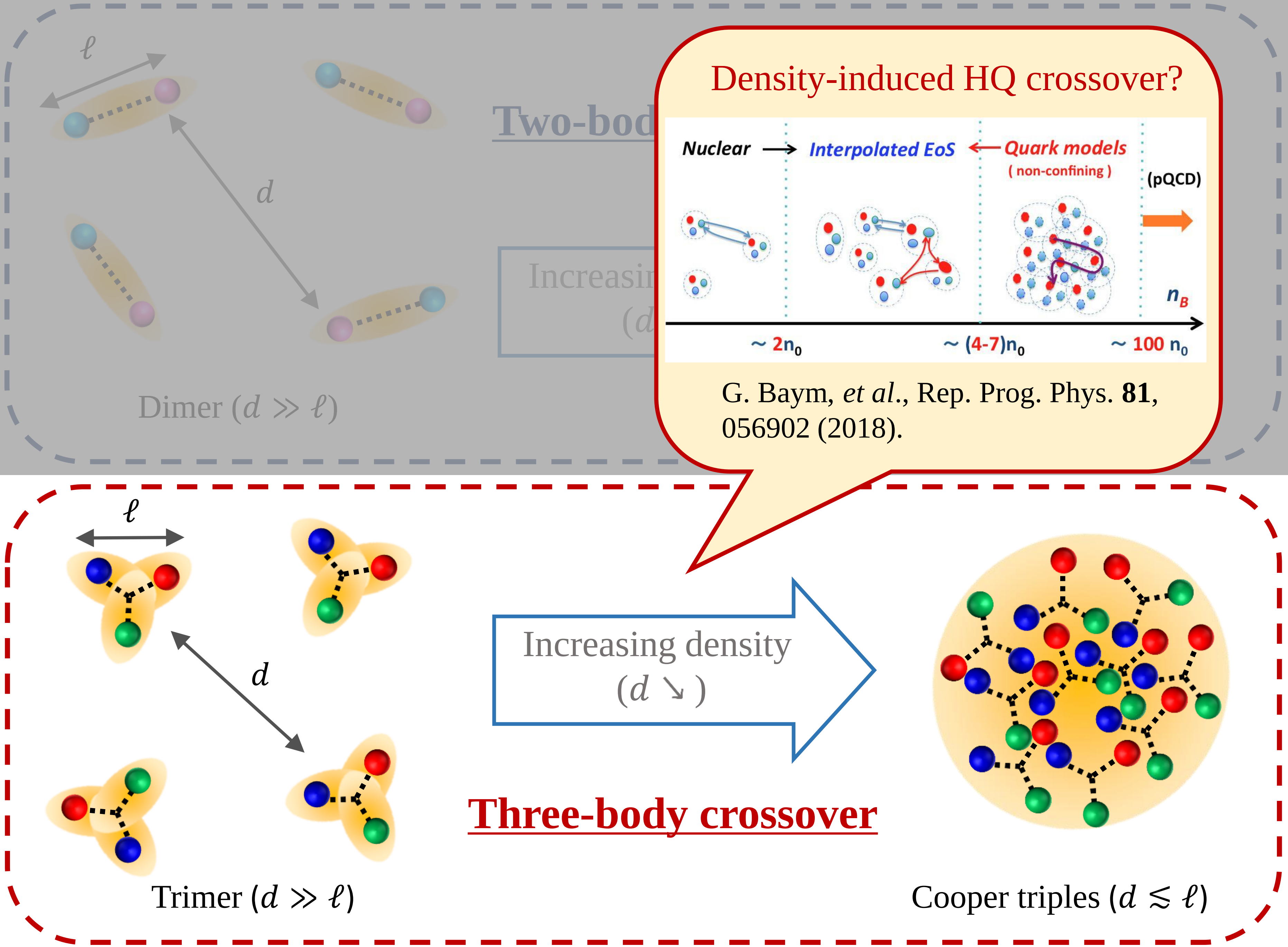
ℓ: molecular size

*P. Niemann and H.-W. Hammer
Phys. Rev. A **86**, 013628 (2012).



Two-body crossover
ℓ
d
Increasing density
($d \searrow$)
Dimer ($d \gg \ell$)
Cooper pairs ($d \lesssim \ell$)
Three-body crossover
ℓ
d
Increasing density
($d \searrow$)
Trimer ($d \gg \ell$)
Cooper triples ($d \lesssim \ell$)

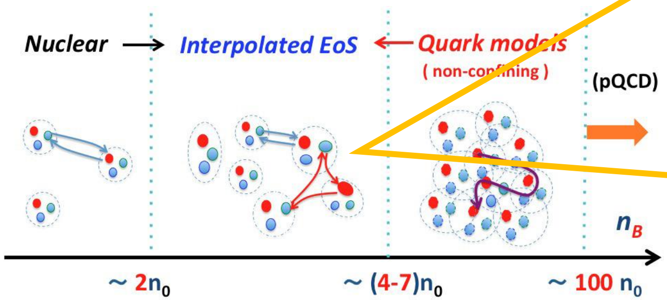

ℓ
d
Two-body crossover
Increasing density
(d ↘)
Dimer (d ≫ ℓ)
Cooper pairs (d ≲ ℓ)
Density-induced BEC-BCS crossover in Li_x ZrNCl
E
T*
T_c
T (K)
Pseudogap
Superconductivity
x
T* / T_c
Y. Nakagawa, et al., Science 372, 6538 (2021).
Carrier dope (density)
sing density
(d ↘)
ody crossover
Cooper triples (d ≲ ℓ)



Dimer ($d \gg \ell$)
Density-induced HQ crossover?
Nuclear
Interpolated EoS
Quark models
(non-confining)
(pQCD)
n_B
~ $2n_0$
~ $(4\text{-}7)n_0$
~ $100\ n_0$
G. Baym, *et al.*, Rep. Prog. Phys. **81**, 056902 (2018).
ℓ
d
Increasing density
($d \searrow$)
Three-body crossover
Trimer ($d \gg \ell$)
Cooper triples ($d \lesssim \ell$)

Can we simulate density-induced three-body crossover?

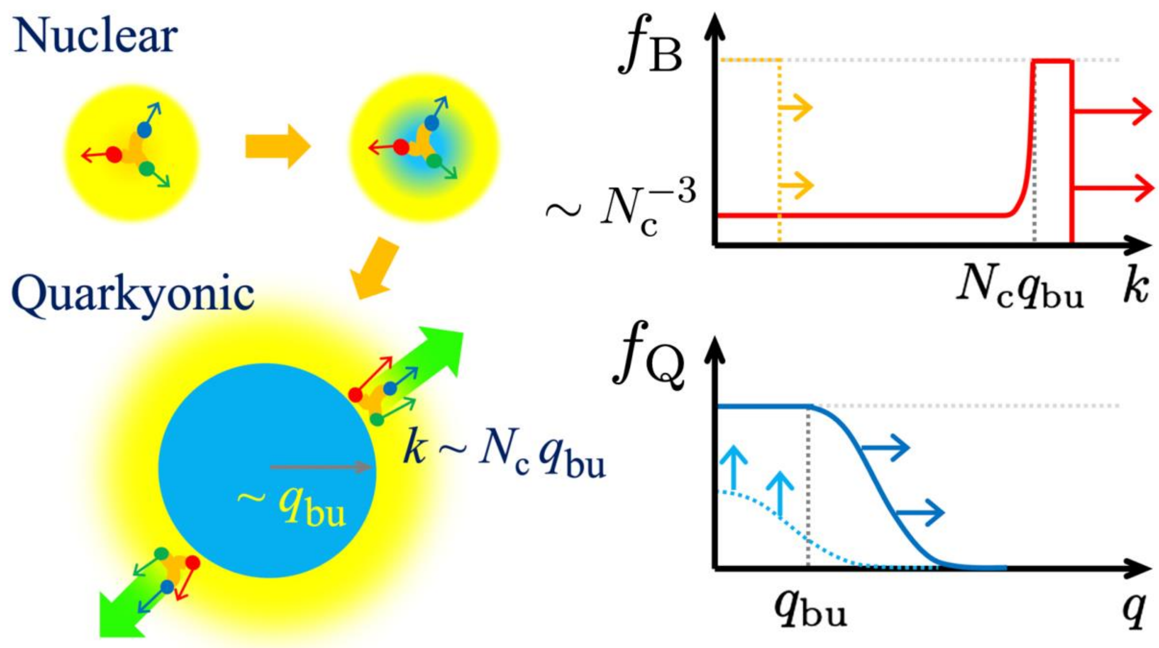
Y. Fujimoto, T. Kojo, and L. D. McLerran, Phys. Rev. Lett. **132**, 112701 (2024).



G. Baym, *et al.*, Rep. Prog. Phys. **81**, 056902 (2018).

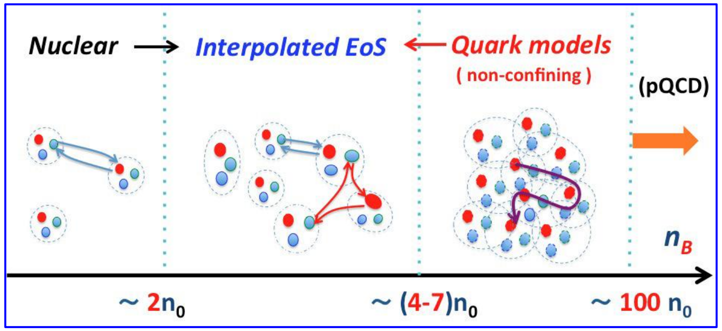
Duality picture and momentum distributions

Ideal gas model except for the baryon confinement



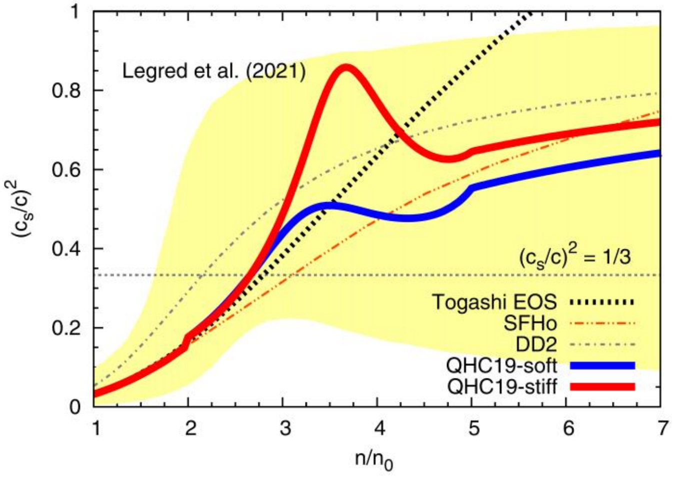
In this talk…

- In analogy with the BEC-BCS crossover, we discuss the possible crossover phenomena from three-body bound states to Cooper triples in nonrelativistic three-color fermions.

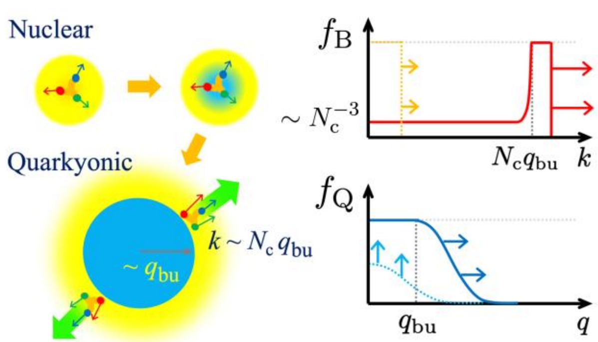


G. Baym, *et al*., Rep. Prog. Phys. **81**, 056902 (2018).

Peaked speed of sound



Momentum shell structure



Y. Fujimoto, *et al*., Phys. Rev. Lett. **132**, 112701 (2024).

Y.-J. Huang, *et al*., Phys. Rev. Lett. **129**, 181101 (2022)

Non-relativistic three-color Fermi gas

Hamiltonian: $H = H_0 + V_2 + V_3$

One-body term

$$H_0 = \sum_{\boldsymbol{k},i} \xi_{\boldsymbol{k},i} c^{\dagger}_{\boldsymbol{k},i} c_{\boldsymbol{k},i}$$

$\xi_{\boldsymbol{k},i} = \frac{k^2}{2m_i} - \mu_i$: non-rela. kinetic energy

$i = \mathrm{r, g, b}$: pseudo-color (hyperfine states)

$c_{\boldsymbol{k},i}$, $c^{\dagger}_{\boldsymbol{k},i}$: fermionic annihilation/creation operator

Two-body interaction

$$V_2 = \sum_{i \neq j} \sum_{\boldsymbol{k},\boldsymbol{q},\boldsymbol{P}} g c^{\dagger}_{\boldsymbol{k}+\frac{\boldsymbol{P}}{2},i} c^{\dagger}_{-\boldsymbol{k}+\frac{\boldsymbol{P}}{2},j} c_{-\boldsymbol{q}+\frac{\boldsymbol{P}}{2},j} c_{\boldsymbol{q}+\frac{\boldsymbol{P}}{2},i}$$

Three-body interaction

$$V_3 = \sum_{\boldsymbol{k},\boldsymbol{q},\boldsymbol{k}',\boldsymbol{q}',\boldsymbol{P}} g_3 c^{\dagger}_{\frac{\boldsymbol{P}}{3}+\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\mathrm{r}} c^{\dagger}_{\frac{\boldsymbol{P}}{\boldsymbol{3}}+\boldsymbol{q},\mathrm{g}} c^{\dagger}_{\frac{\boldsymbol{P}}{3}-\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\mathrm{b}} c_{\frac{\boldsymbol{P}}{3}-\boldsymbol{k}'-\frac{\boldsymbol{q}'}{2},\mathrm{b}} c_{\frac{\boldsymbol{P}}{\boldsymbol{3}}+\boldsymbol{q}',\mathrm{g}} c_{\frac{\boldsymbol{P}}{3}+\boldsymbol{k}'-\frac{\boldsymbol{q}'}{2},\mathrm{r}}$$

HT, S. Tsutsui, T. M. Doi, and K. Iida, Phys. Rev. Research **4**, L012021 (2022).
S. Akagami, HT, and K. Iida, Phys. Rev. A **104**, L041302 (2021).

Cooper problems for three-body states

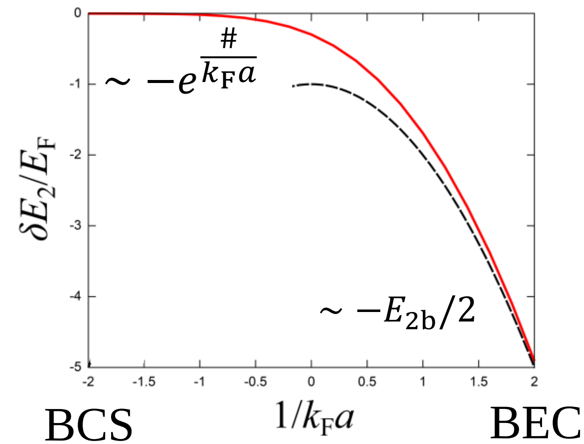
Variational equation: $\delta\langle\psi|E-H|\psi\rangle=0$

Cooper pair on the top of Fermi sea

$$|\Psi_{\mathrm{CP}}\rangle=\sum_{|\boldsymbol{p}|\geqslant k_{\mathrm{F}}}\Phi_{\boldsymbol{p}}\hat{B}^{\dagger}_{\boldsymbol{p}\gamma,-\boldsymbol{p}\gamma'}|\mathrm{FS}\rangle$$

$$\hat{B}^{\dagger}_{\boldsymbol{k}_1\gamma,\boldsymbol{k}_2\gamma'}=\hat{c}^{\dagger}_{\boldsymbol{k}_1,\gamma}\hat{c}^{\dagger}_{\boldsymbol{k}_2,\gamma'}$$

Cooper pair energy per atom measured from E_{F}



Cooper triple on the top of Fermi sea

P. Niemann and H.-W. Hammer, PRA **86**, 013628 (2012).

$$|\Psi_{\mathrm{CT}}\rangle=\sum_{|\boldsymbol{k}_1|\geqslant k_{\mathrm{F}}}\sum_{|\boldsymbol{k}_2|\geqslant k_{\mathrm{F}}}\sum_{|\boldsymbol{k}_3|\geqslant k_{\mathrm{F}}}\mathcal{O}_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}\hat{C}^{\dagger}_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}\delta_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3,\boldsymbol{0}}|\mathrm{FS}\rangle$$

$$\hat{C}^{\dagger}_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}=\frac{1}{6}\sum_{\gamma_1,\gamma_2,\gamma_3}\varepsilon_{\gamma_1\gamma_2\gamma_3}\hat{c}^{\dagger}_{\boldsymbol{k}_1,\gamma_1}\hat{c}^{\dagger}_{\boldsymbol{k}_2,\gamma_2}\hat{c}^{\dagger}_{\boldsymbol{k}_3,\gamma_3}$$

Reproducing usual three-body equation if we replace $|\mathrm{FS}\rangle$ with $|0\rangle$, or in the strong-coupling case where E_{F} is negligible.

Cooper problems for three-body states

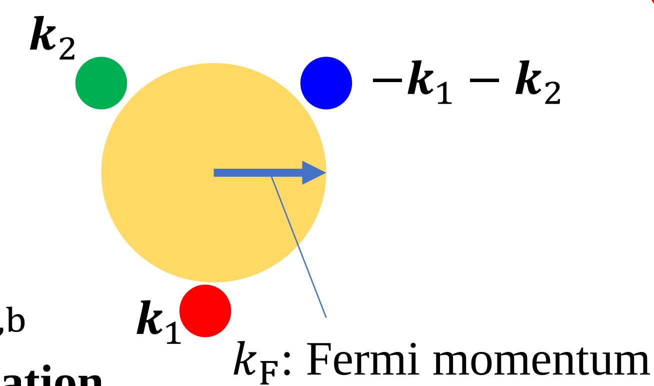
Variational equation: $\delta\langle\psi|E-H|\psi\rangle=0$

<u>Cooper triple on the top of Fermi sea</u>

Antisymmetrized variational wave function

$$|\Psi_{\mathrm{CT}}\rangle=\sum_{|\boldsymbol{k}_1|\geqslant k_{\mathrm{F}}}\sum_{|\boldsymbol{k}_2|\geqslant k_{\mathrm{F}}}\Omega_{\boldsymbol{k}_1,\boldsymbol{k}_2}\hat{F}^{\dagger}_{\boldsymbol{k}_1,\boldsymbol{k}_2}|\mathrm{FS}\rangle$$

Three-body operator: $F^{\dagger}_{\boldsymbol{k}_1,\boldsymbol{k}_2}=c^{\dagger}_{\boldsymbol{k}_1,\mathrm{r}}c^{\dagger}_{\boldsymbol{k}_2,\mathrm{g}}c^{\dagger}_{-\boldsymbol{k}_1-\boldsymbol{k}_2,\mathrm{b}}$



In-medium Skorniakov-Ter-Martirosyan (STM) equation

$$\mathcal{A}_i(\boldsymbol{p})\left[\frac{1}{g}+\sum_{|\boldsymbol{K}|\geqslant k_{\mathrm{F}}}\frac{\theta(\xi_{\boldsymbol{p}+\boldsymbol{K}})}{\xi_{-\boldsymbol{p}-\boldsymbol{K}}+\xi_{\boldsymbol{K}}+\xi_{\boldsymbol{p}}-E_3}\right]$$

$$=-\sum_{|\boldsymbol{K}|\geqslant k_{\mathrm{F}}}\frac{\theta(\xi_{\boldsymbol{p}+\boldsymbol{K}})[\mathcal{A}_k(-\boldsymbol{p}-\boldsymbol{K})+\mathcal{A}_j(\boldsymbol{K})]}{\xi_{-\boldsymbol{p}-\boldsymbol{K}}+\xi_{\boldsymbol{K}}+\xi_{\boldsymbol{p}}-E_3}$$

Reproducing three-body equation in vacuum

$$|\mathrm{FS}\rangle\to|0\rangle$$

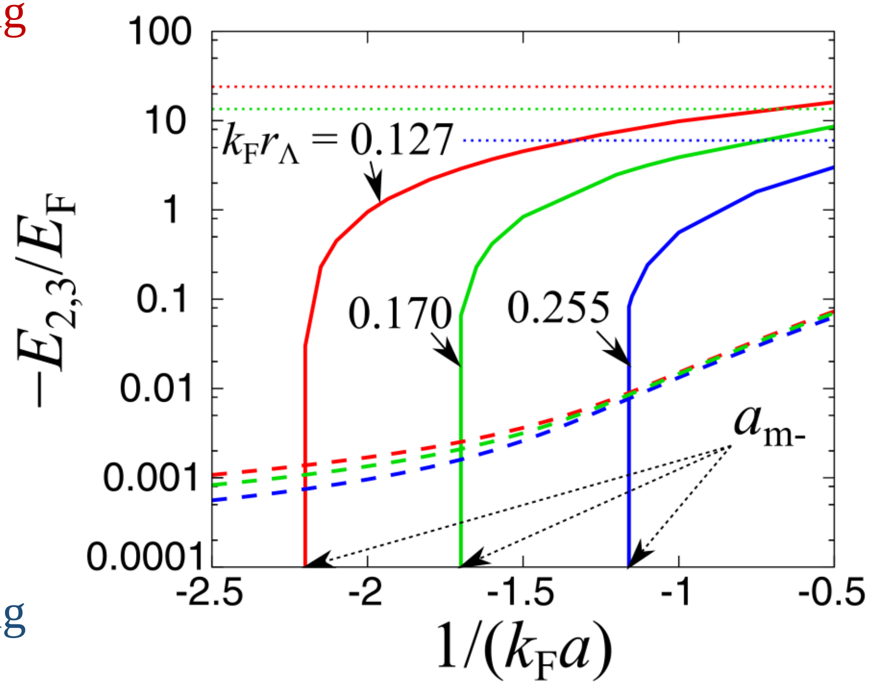
$$\theta\left(\xi_{\boldsymbol{p}+\boldsymbol{K}}\right)\to 1$$

$$\Omega_{\boldsymbol{k}_1,\boldsymbol{k}_2}=-g\frac{\mathcal{A}_1(\boldsymbol{k}_1)+\mathcal{A}_2(\boldsymbol{k}_2)+\mathcal{A}_3(-\boldsymbol{k}_1-\boldsymbol{k}_2)}{\xi_{\boldsymbol{k}_1}+\xi_{\boldsymbol{k}_2}+\xi_{-\boldsymbol{k}_1-\boldsymbol{k}_2}-E_3}$$

Cooper pair and triple binding energies

Competition between pair (dashed line) and triple (solid line) formations

Large binding



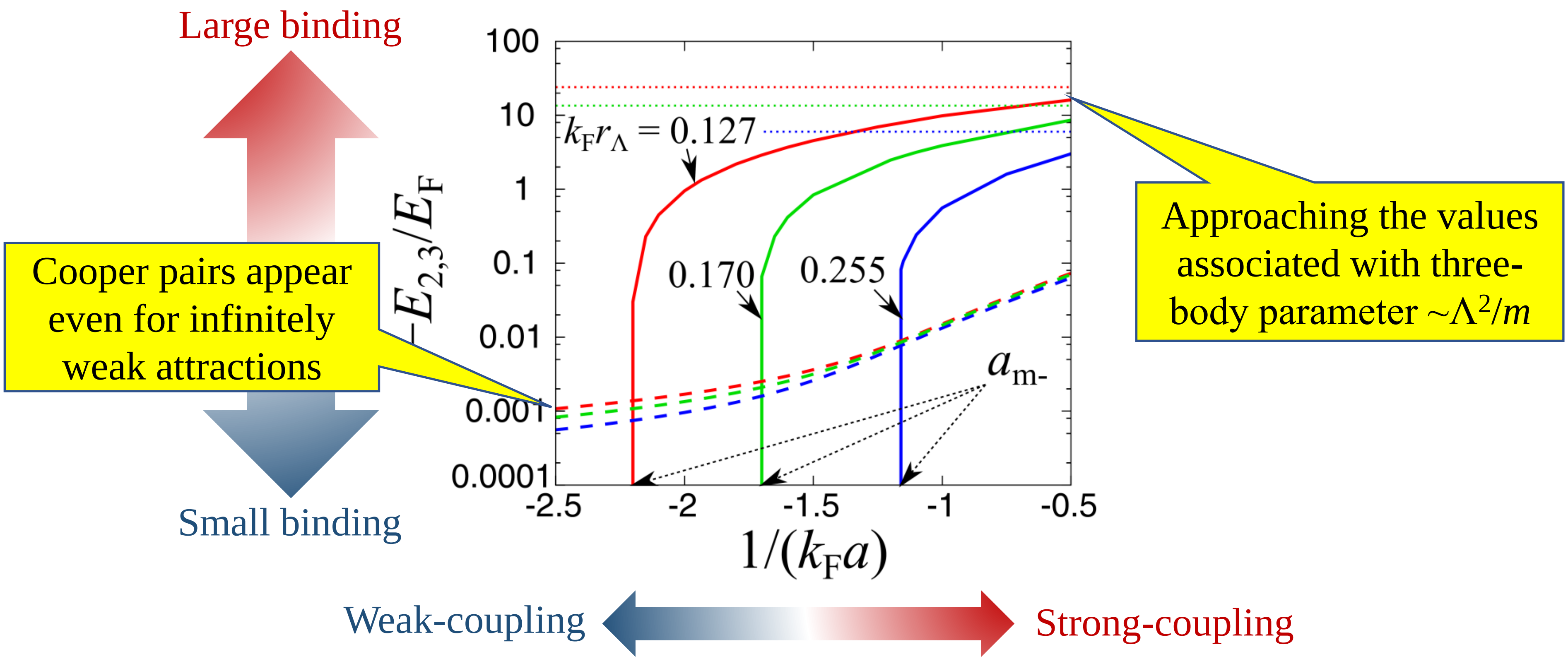
Small binding

Weak-coupling Strong-coupling

$* \; g_{12} = g_{23} = g_{31}, U_{123} = 0$ $r_\Lambda = \frac{4}{\pi\Lambda}$: range parameter Λ: momentum cutoff

Cooper pair and triple binding energies

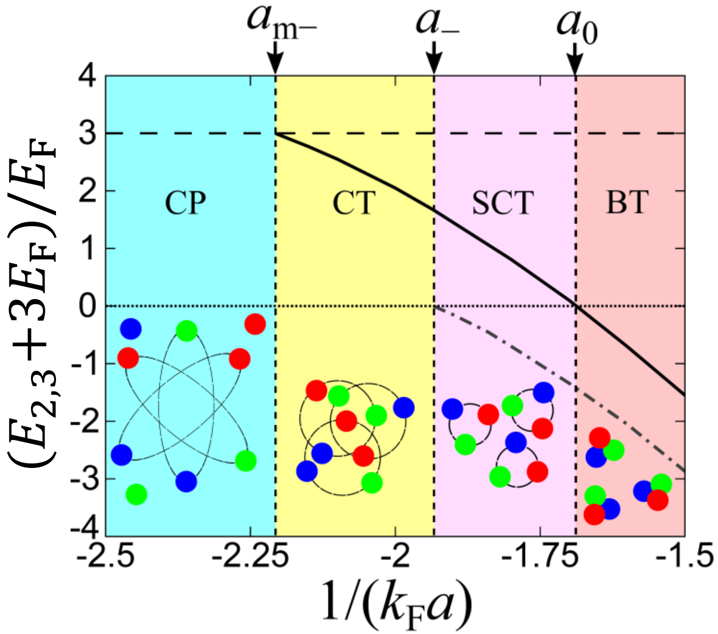
Competition between pair (dashed line) and triple (solid line) formations



$* \; g_{12} = g_{23} = g_{31}, U_{123} = 0$ $\quad$ $r_\Lambda = \frac{4}{\pi\Lambda}$: range parameter $\quad$ Λ: momentum cutoff

Crossover between bound trimer to Cooper triple

HT, S. Tsutsui, T. M. Doi, and K. Iida, Phys. Rev. A **104**, 053328 (2021).



a_-: triatomic resonance (where trimer is bound in vacuum)

Cutoff range: $k_F r_\Lambda = 0.127$

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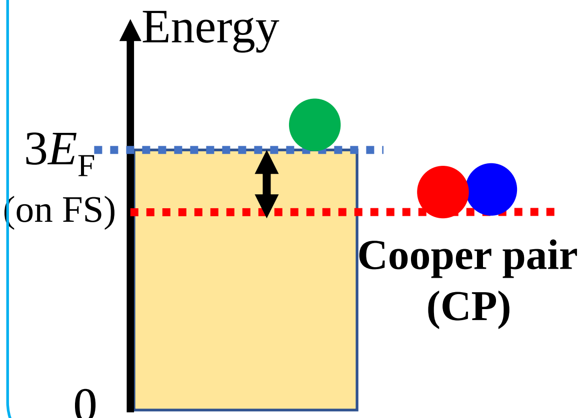
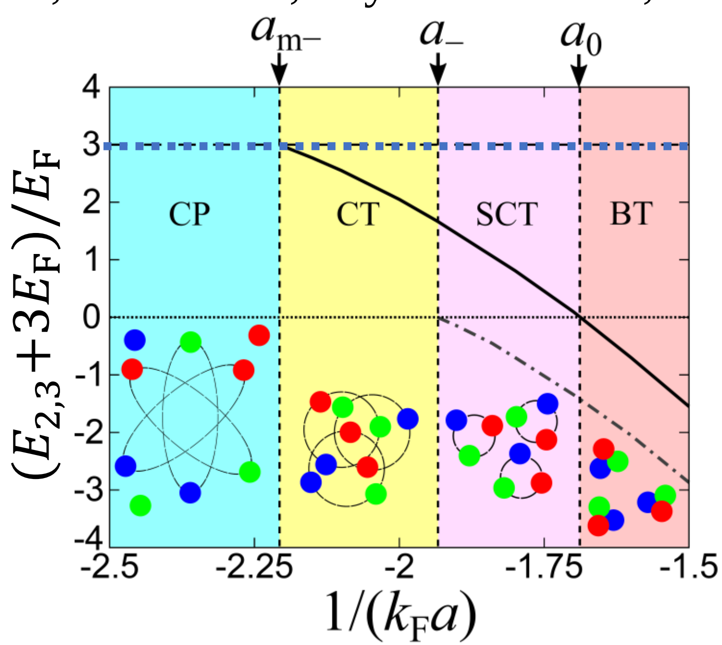
Weak-coupling BCS

$$E_2 = -8E_F e^{\frac{\pi}{k_F a}}$$





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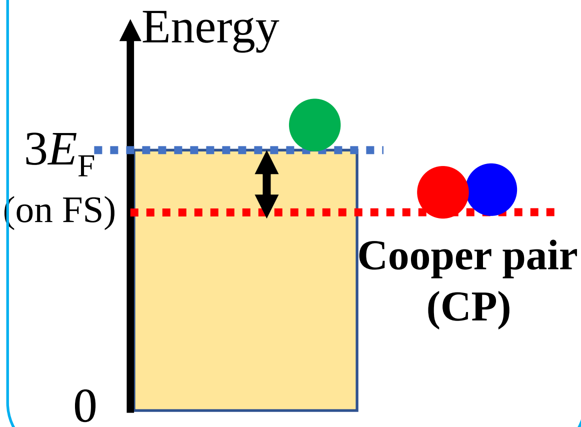
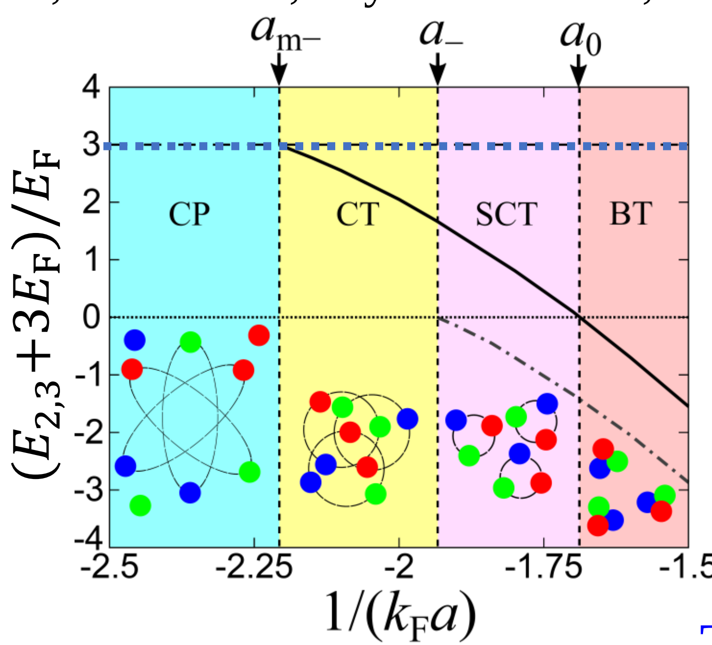
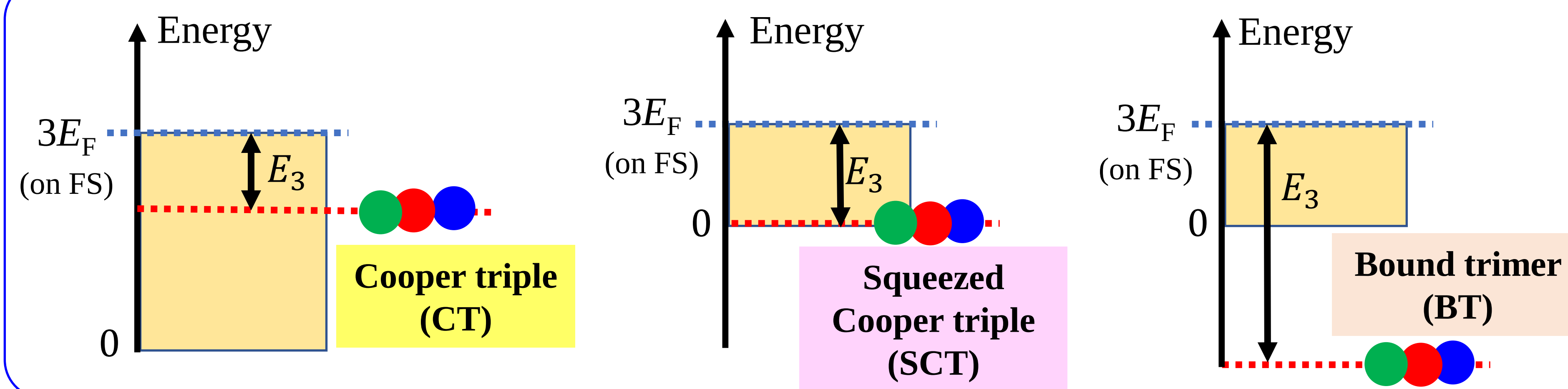




a_-: triatomic resonance (where trimer is bound in vacuum)

Cutoff range: $k_F r_\Lambda = 0.127$

Three-body crossover



Three-body decay rate

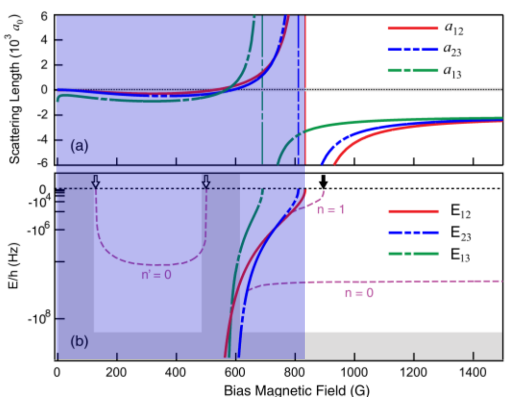
HT, S. Tsutsui, T. M. Doi, and K. Iida, Phys. Rev. A **104**, 053328 (2021).

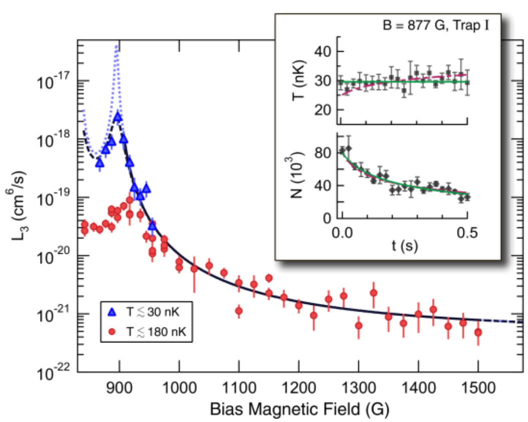
Imaginary three-body term for three-body loss

$$W = -i\gamma \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q}} c^{\dagger}_{\boldsymbol{k},1} c^{\dagger}_{\boldsymbol{p},2} c^{\dagger}_{\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{p},3} c_{\boldsymbol{q}-\boldsymbol{k}'-\boldsymbol{p}',3} c_{\boldsymbol{p}',2} c_{\boldsymbol{k}',1}$$

T. Kirk and M. Parish, Phys. Rev. A **96**, 053614 (2017).

Trimer resonance in ^{6}Li 3-com. Fermi gas (exp.)



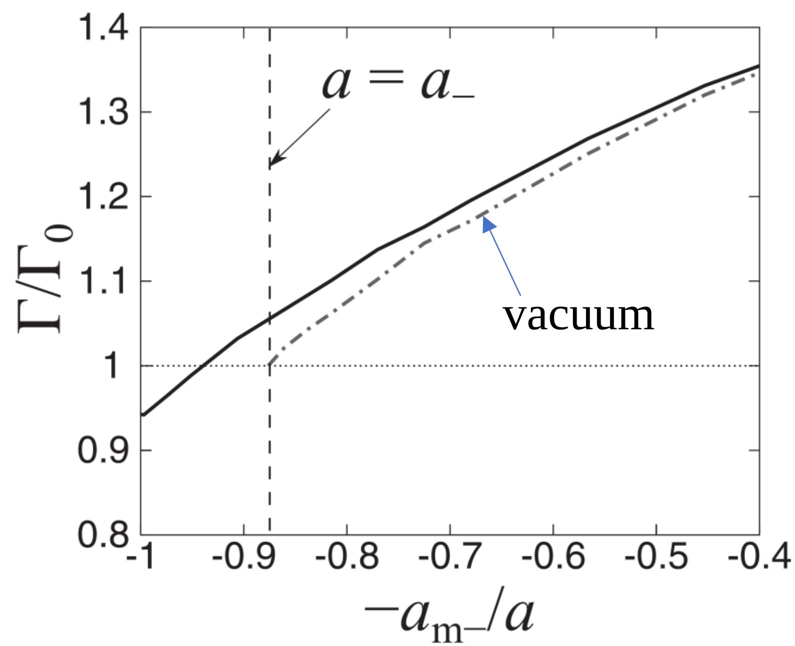


J. R. Williams, *et al*., Phys. Rev. Lett. **103**, 130404 (2009).

Three-body decay rate

$$\Gamma = 2i\langle\Psi_{\rm CT}|W|\Psi_{\rm CT}\rangle$$

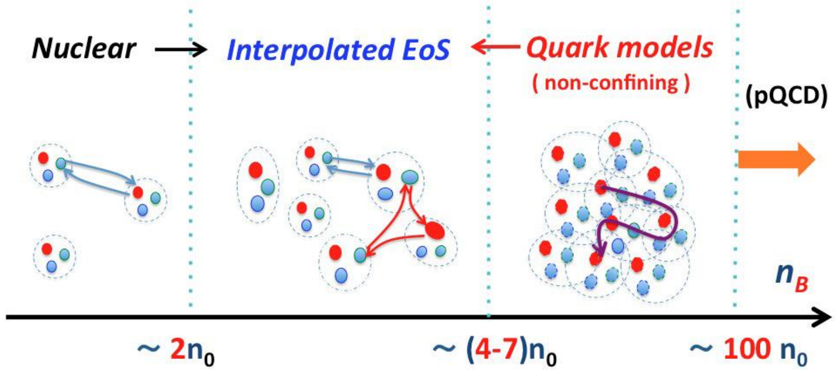
$$\rightarrow \langle\Psi(t)|\Psi(t)\rangle \propto e^{-\Gamma t}$$



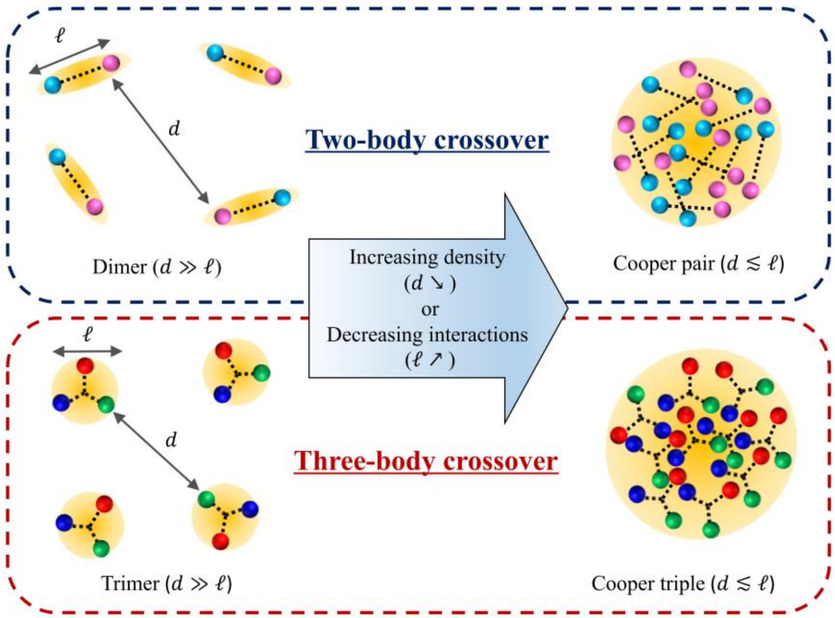
Summary

HT, K. Iida, T. Kojo, and H. Liang, in preparation

- In analogy with the BEC-BCS crossover in two-component Fermi gases, we have discussed the three-body counterpart in three-color fermions, where bound trimer gases change into degenerate Fermi state.



G. Baym, *et al*., Rep. Prog. Phys. **81**, 056902 (2018).



Appendix

Realization of tunable three-body interaction in cold atoms

A. Hammond, *et al.*, Phys. Rev. Lett. **128**, 083401 (2022)

EOS in Rabi-coupled 2-com. 1D BEC

$$\frac{E_{\rm MF}}{V} = -\frac{\hbar\Omega}{2}(\phi_\uparrow^*\phi_\downarrow + \phi_\downarrow^*\phi_\uparrow) + \frac{\hbar\delta}{2}(|\phi_\uparrow|^2 - |\phi_\downarrow|^2) + \sum_{\sigma\sigma'}\frac{g_{\sigma\sigma'}}{2}|\phi_\sigma|^2|\phi_{\sigma'}|^2.$$

Low-energy EFT

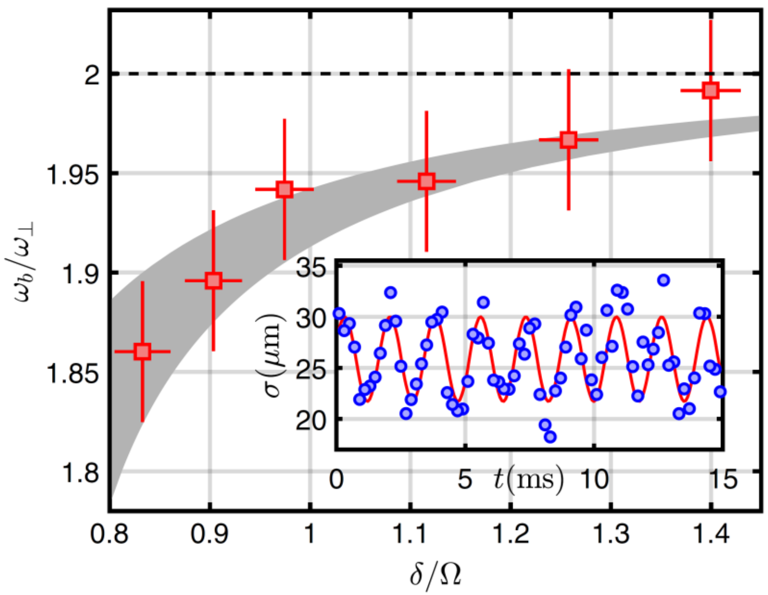
$$\frac{E_{\rm MF}}{N} \approx \epsilon_- + g_2\frac{n}{2} + g_3\frac{n^2}{3}$$

with $g_2 = g - \dfrac{\bar{g}}{1+\delta^2/\Omega^2}$

and $g_3 = -\dfrac{3\bar{g}^2}{\hbar\Omega}\dfrac{\delta^2/\Omega^2}{(1+\delta^2/\Omega^2)^{5/2}}$

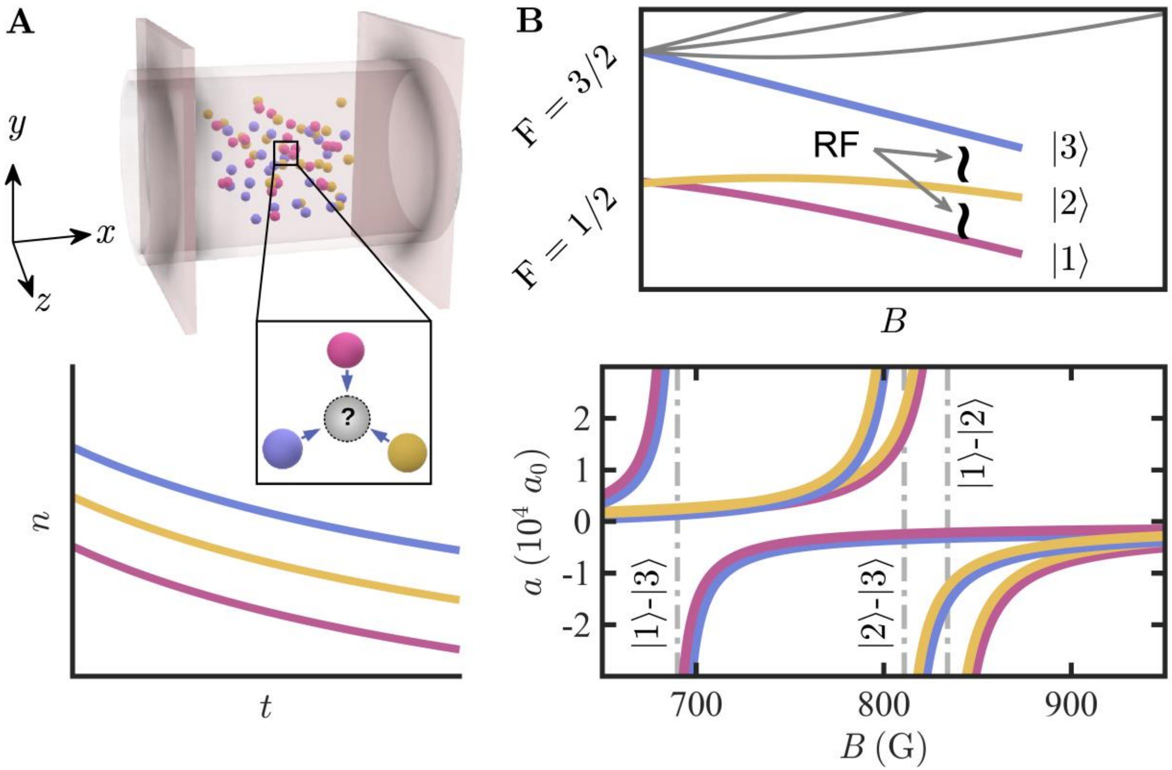
Breezing mode frequency

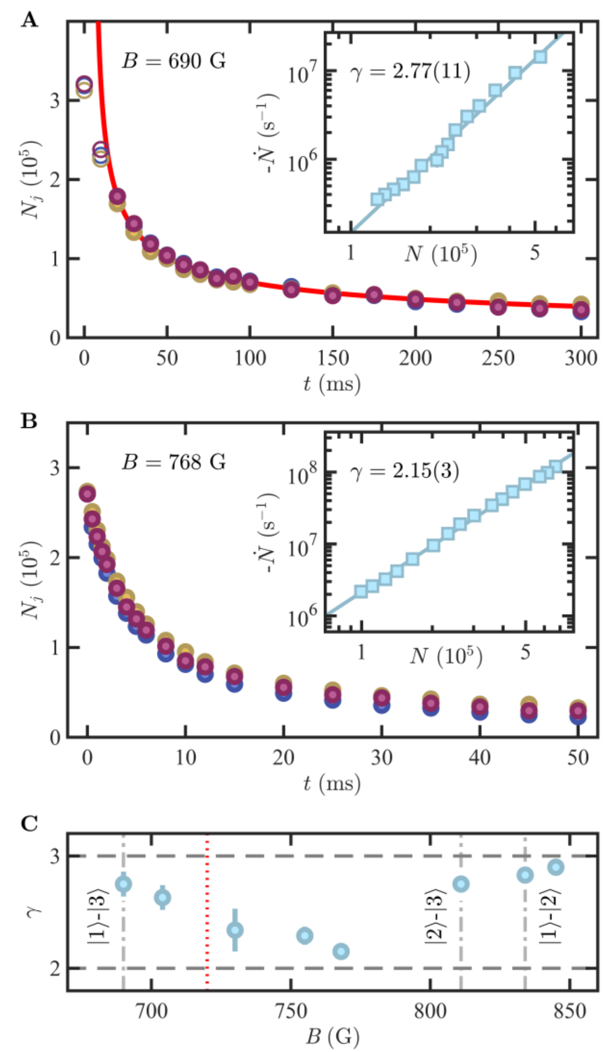
$$\omega_b = 2\omega_\perp\sqrt{1 + E_3/E_{\rm pot}}$$



Recent experiments of three-component Fermi gases

G. L. Schumacher, et al., arXiv:2301.92237





1D nonrelativistic three-color fermions with three-body interaction

- Hamiltonian density: $\hat{H} = \hat{H}_0 + \hat{V}_3$ (No two-body interaction)

One-body kinetic term

$$\hat{H}_0 = \sum_{a=\mathrm{r,g,b}} \psi_a^\dagger \left(-\frac{\partial_x^2}{2m} - \mu \right) \psi_a$$

μ: chemical potential

$a = \mathrm{r, g, b}$: pseudo-color (hyperfine states)

$\psi_a^\dagger, \psi_a$: fermionic field operator

Three-body interaction (classically scale invariant: $x \to \lambda^{-1} x$)

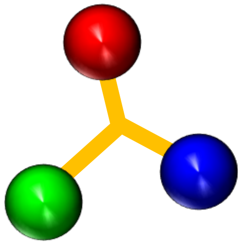
J. Drut, et al., PRL **120**, 243002 (2018).

$$\hat{V}_3 = g_3 (\psi_\mathrm{r}^\dagger \psi_\mathrm{r})(\psi_\mathrm{g}^\dagger \psi_\mathrm{g})(\psi_\mathrm{b}^\dagger \psi_\mathrm{b})$$

$g_3 < 0$: three-body attraction

Trimer binding energy (broken scale invariance)

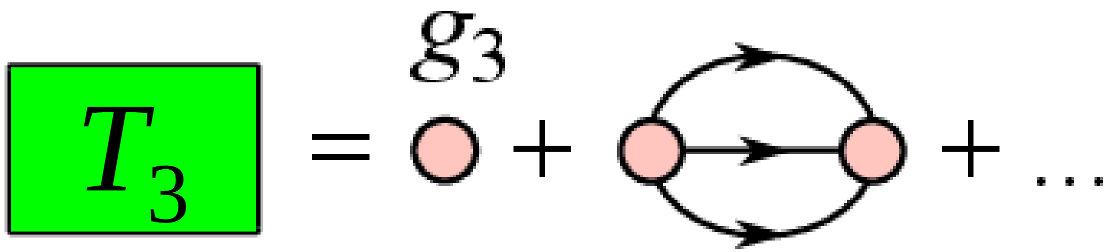
$$E_\mathrm{b} = \frac{\Lambda^2}{m} \exp\left(\frac{2\sqrt{3}\pi}{m g_3} \right)$$



Λ: UV cutoff scale

Three-body T-matrix for three-body interaction

Three-body coupling constant g_3 can be represented by the three-body binding energy ε_B



$$T_3(P,\Omega_+) = \left[\frac{1}{g_3} - \Xi_0(P,\Omega_+)\right]^{-1}$$

Ξ_0: Three-body propagator in vacuum

$$\Xi_0(P,\Omega_+) = \sum_{k,q} \frac{1}{\Omega_+ - \varepsilon_{\frac{P}{3}+k-\frac{q}{2}} - \varepsilon_{\frac{P}{3}+q} - \varepsilon_{\frac{P}{3}-k-\frac{q}{2}}} = -\frac{m}{2\sqrt{3}\pi}\ln\left(\frac{\Lambda^2 + P^2/6 - m\Omega_+}{P^2/6 - m\Omega_+}\right)$$

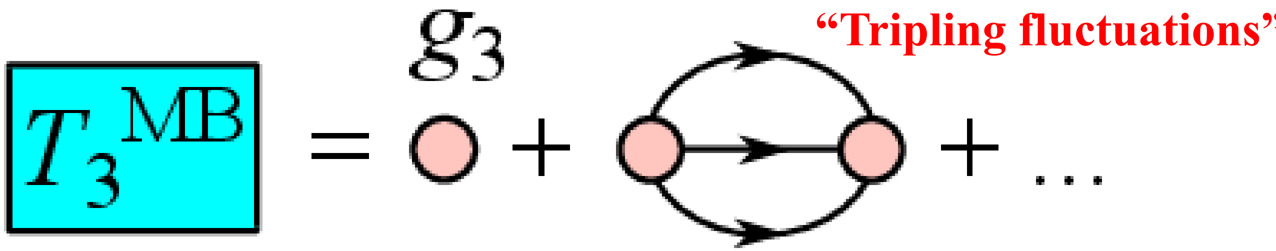
Λ: cutoff

<u>Three-body binding energy</u>

$$\frac{1}{g_3} - \Xi_0(0,\Omega = -\varepsilon_{\mathrm{B}}) = 0 \quad \Rightarrow \quad \varepsilon_{\mathrm{B}} = \frac{\Lambda^2}{m}\exp\left(\frac{2\sqrt{3}\pi}{m g_3}\right)$$

In-medium three-body T-matrix

HT, S. Tsutsui, T. M. Doi, and K. Iida, Phys. Rev. Research **4**, L012021 (2022).



$$T_3^{\mathrm{MB}}(\boldsymbol{P}, i\Omega_n) = \left[\frac{1}{g_3} - \Xi(\boldsymbol{P}, i\Omega_n)\right]^{-1}$$

Ξ: In-medium three-particle (three-hole) propagator

$\Omega_n = (2n+1)\pi T$: Fermion Matsubara frequency

In-medium three-body equation

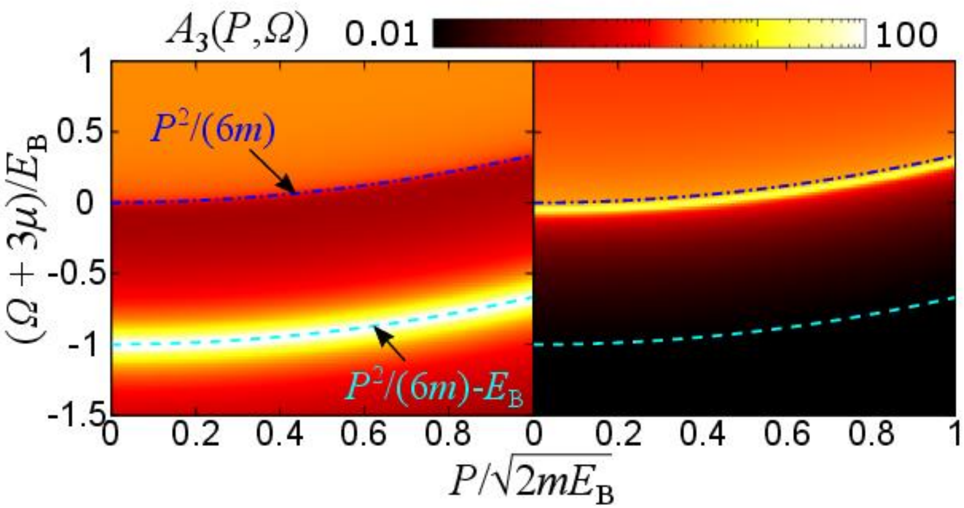
$$\frac{1}{g_3} - \Xi(\boldsymbol{P} = 0, \Omega = -E_{\mathrm{B}}^{\mathrm{M}}) = 0$$

Three-body spectral function

HT, S. Tsutsui, T. M. Doi, and K. Iida, Phys. Rev. Research **4**, L012021 (2022).

In-medium three-body spectra

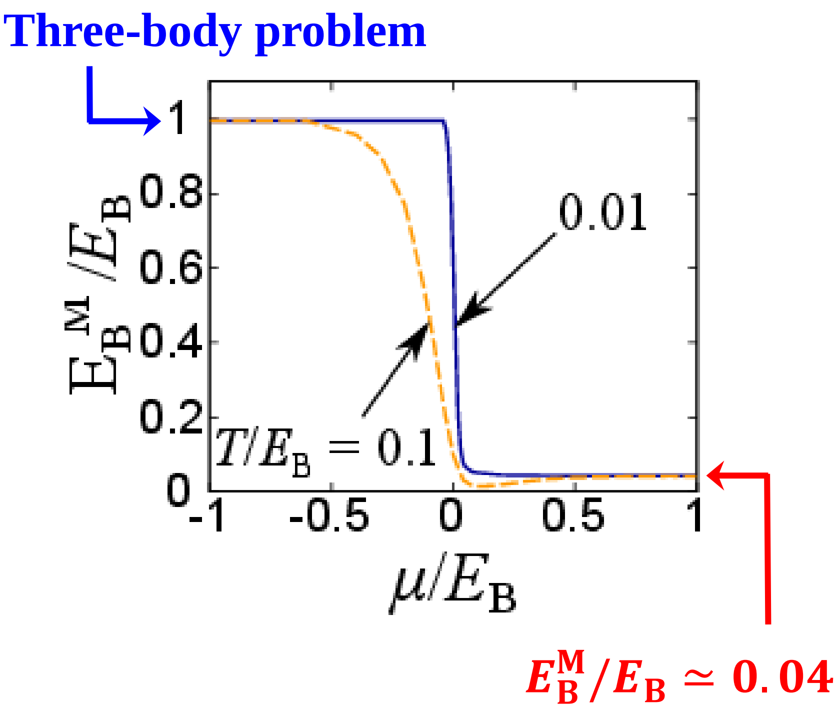
$$A_3(P,\Omega) = -\mathrm{Im}T_3^{\mathrm{MB}}(P,\Omega_+)$$



Low density ($\mu/E_{\mathrm{B}} = -1$)

High-density ($\mu/E_{\mathrm{B}} = 2$)

In-medium three-body binding energy

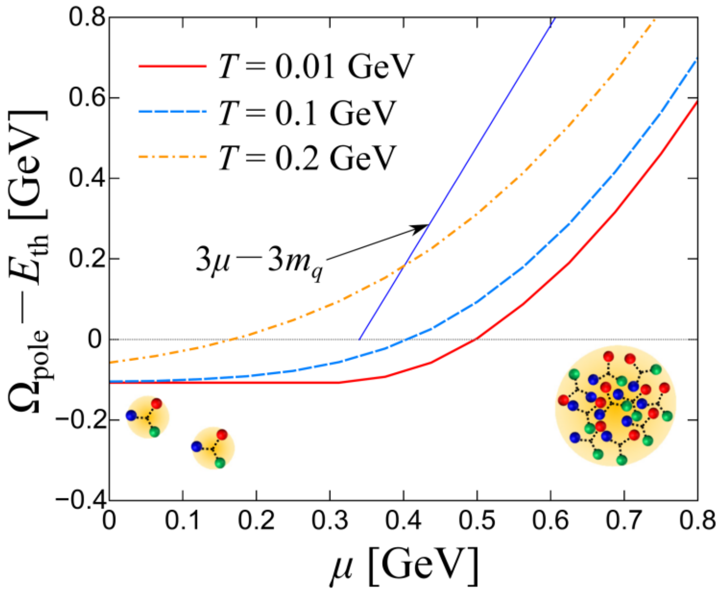


The three-body pole survives even at high density

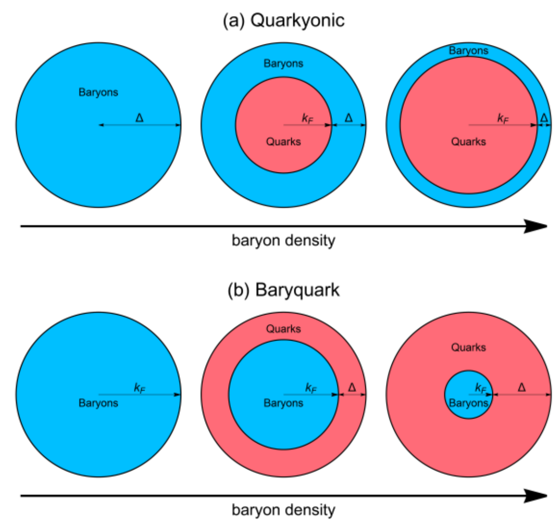
Toy model for hadron-quark crossover

HT, S. Tsutsui, T. M. Doi, and K. Iida, Symmetry **15**, 333 (2023).

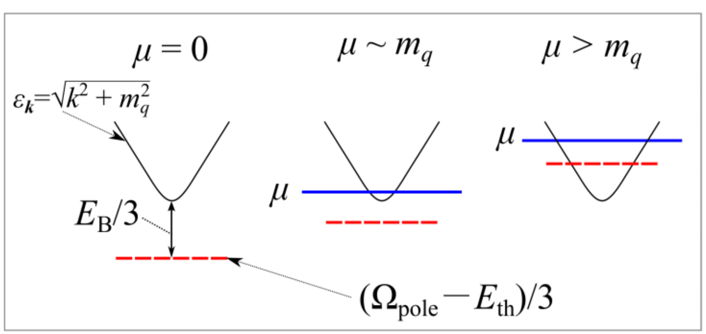
$$H = \sum_{p}\sum_{j} \varepsilon_{p} \psi^{\dagger}_{p,j}\psi_{p,j} + \sum_{k,q,k',q',P} V_{k,q,k',q',P}\psi^{\dagger}_{k,\mathrm{r}}\psi^{\dagger}_{q,\mathrm{g}}\psi^{\dagger}_{P-k-q,\mathrm{b}}\psi_{P-k'-q',\mathrm{b}}\psi_{q',\mathrm{g}}\psi_{k',\mathrm{r}'}$$



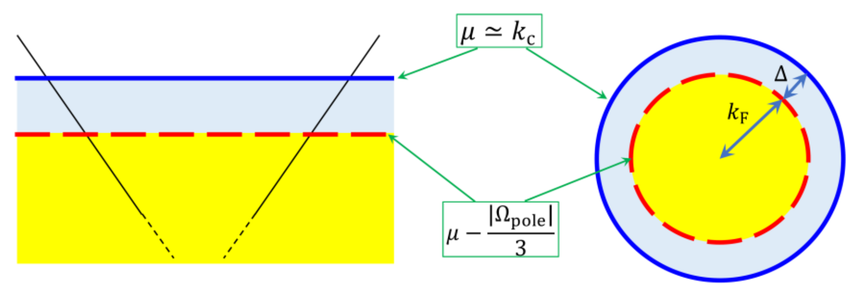
Quarkyonic or Baryquark?



arXiv:2211.14674



Our scenario is close to quarkyonic



$m_q = 0.34$ GeV, $M_B = 0.91$ GeV

Non-relativistic trace anomaly

Trace anomaly equation

$$2\hat{H} - \hat{T}_{xx} = -\frac{g_3^2}{\sqrt{3}\pi}(\psi_{\mathrm{r}}^\dagger \psi_{\mathrm{r}})(\psi_{\mathrm{g}}^\dagger \psi_{\mathrm{g}})(\psi_{\mathrm{b}}^\dagger \psi_{\mathrm{b}})$$

$\hat{T}_{ij}$: energy-momentum tensor

W. S. Dasa, et al., Mod. Phys. Lett. A **34**, 1950291 (2019).

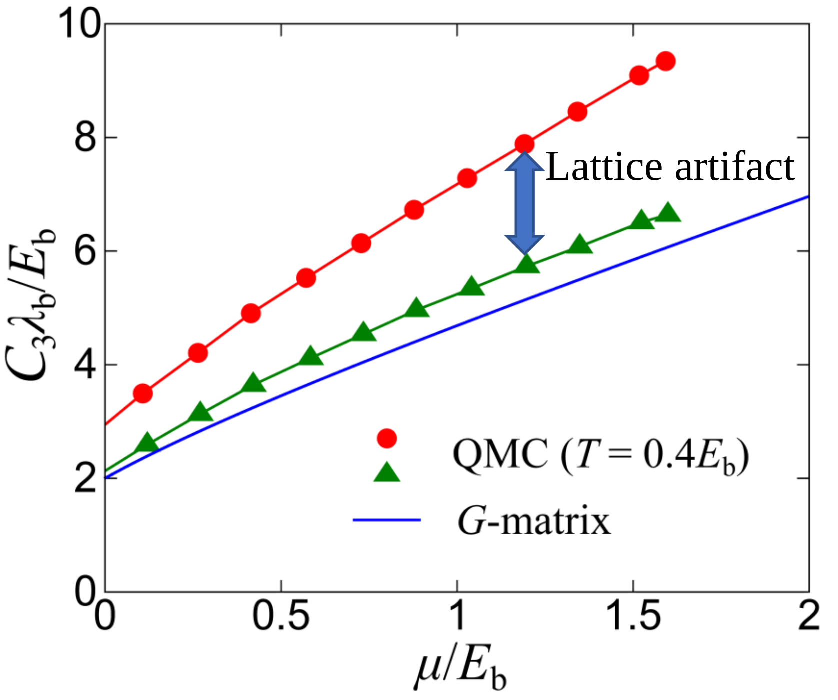
Three-body contact

Statistical average: $2E - P = C_3$

$$C_3 = \frac{8\sqrt{3}}{3\pi}\rho E_{\mathrm{F}} \frac{3E_{\mathrm{F}}/E_{\mathrm{b}}}{\left(1+\frac{3E_{\mathrm{F}}}{E_{\mathrm{b}}}\right)\left[\ln\left(1+\frac{3E_{\mathrm{F}}}{E_{\mathrm{b}}}\right)\right]^2}$$

E: energy density $\quad$ P: pressure

Comparison with QMC results



$\lambda_{\mathrm{b}} = \sqrt{2\pi/mE_{\mathrm{b}}}$: length scale associated eith E_b

QMC: J. McKenny, *et al.*, PRA **102**, 023313 (2020).