QCD Kondo effect for isolated heavy quark: quantum impurity with condensate and resonance

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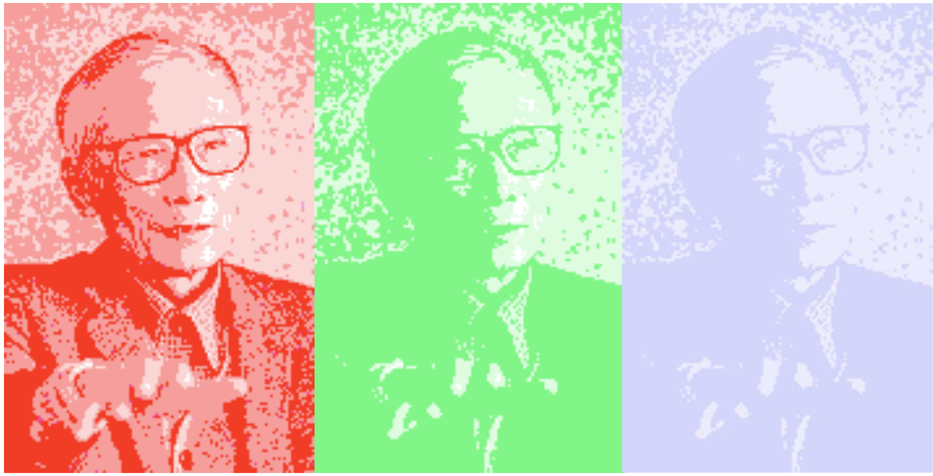
Content

1. Introduction to Kondo effect
2. QCD Kondo effect I: previous studies
3. QCD Kondo effect II: chiral symmetry breaking and isolated heavy quark
4. Conclusion

Content

1. **Introduction to Kondo effect**
2. QCD Kondo effect I: previous studies
3. QCD Kondo effect II: chiral symmetry breaking and isolated heavy quark
4. Conclusion

What's "Kondo effect"?



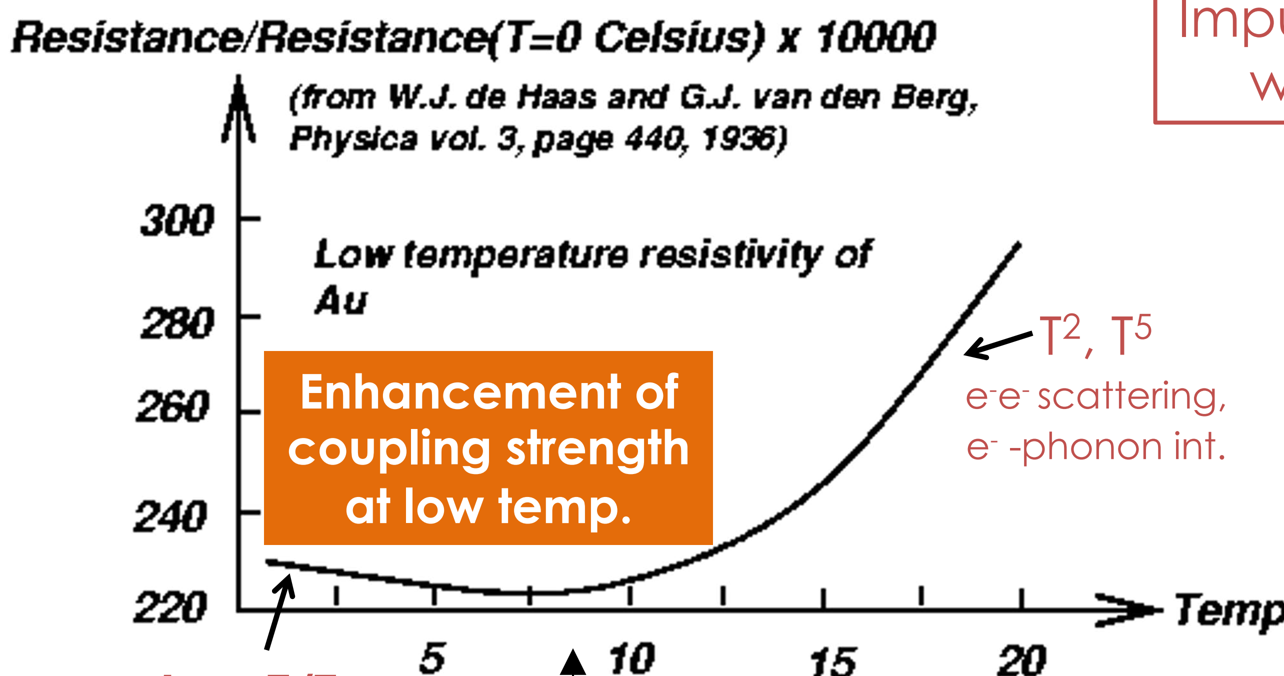
Jun Kondo
(1930-2022)

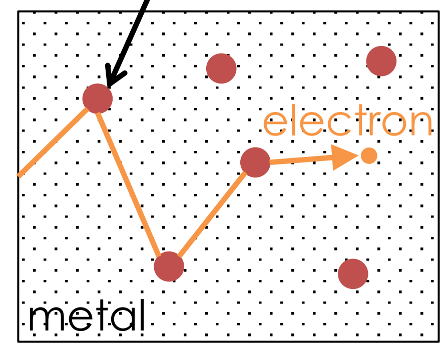
1. Introduction

Original Work: J. Kondo (Prog. Theor. Phys. 32, 37 (1964))

σ: Pauli matrices

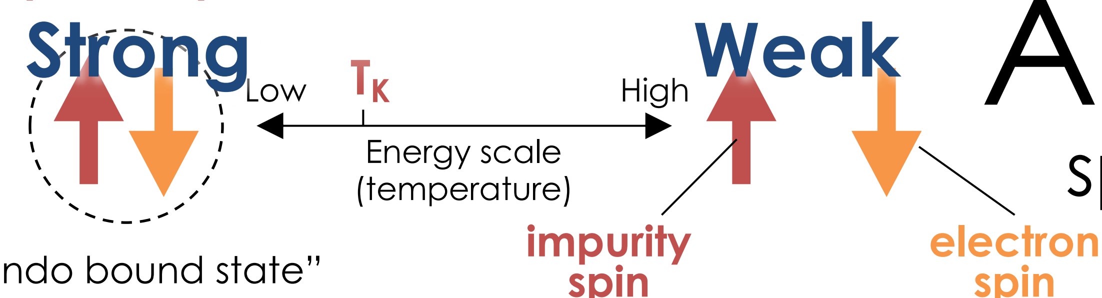
Impurity atom with spin ½ with **σ·σ** interaction





Log T/T_K **(quantum)**

T_K: Kondo temperature

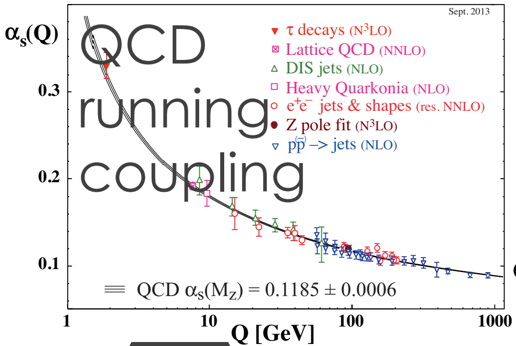


"Kondo bound state"
B.E. ~T_K

Anti-Ferro

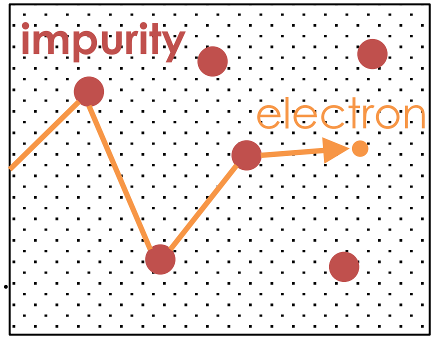
spin antiparallel enhanced

1. Introduction

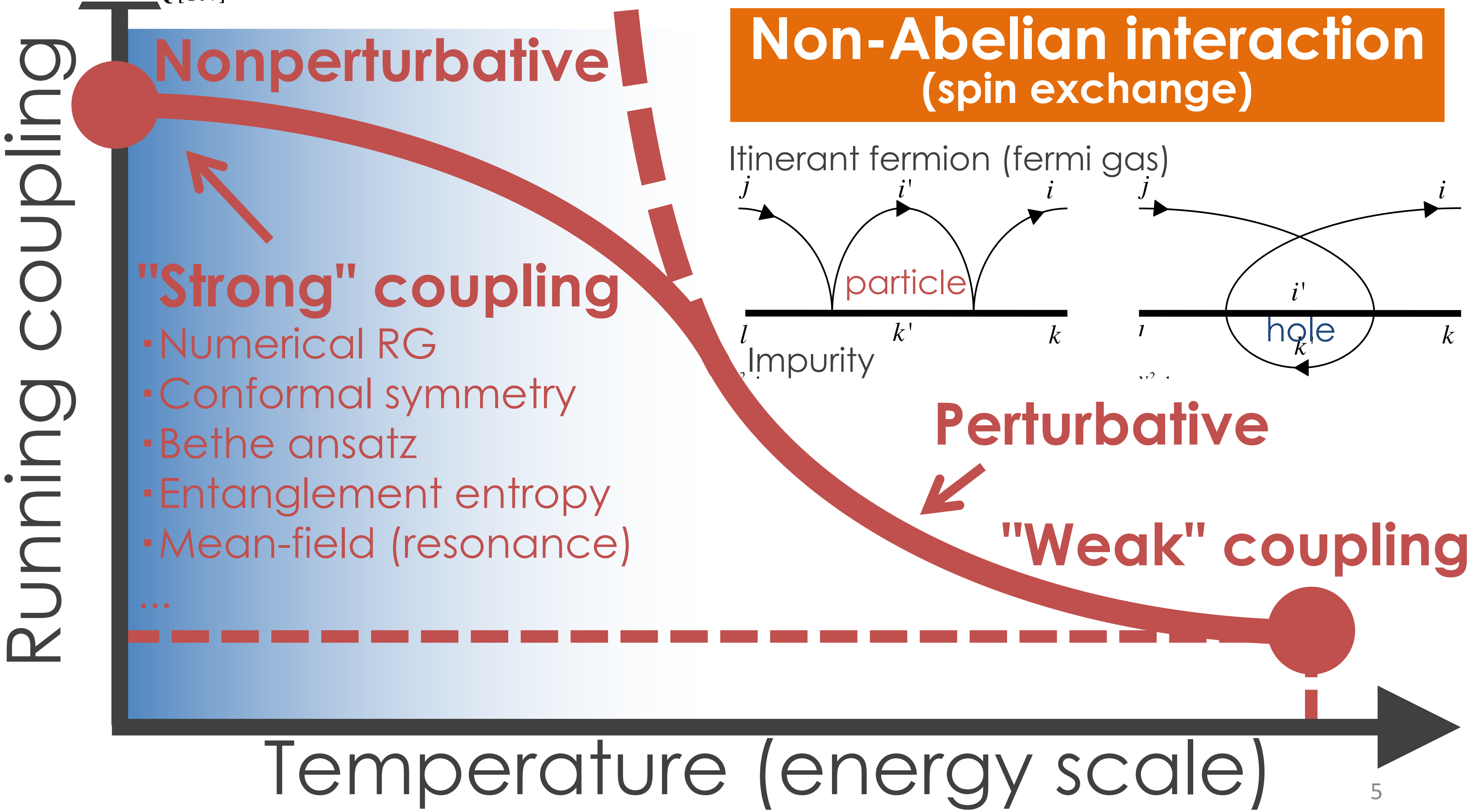


Kondo effect induces running coupling, à la QCD.

erson (1970), Wilson (1975), Coleman (1983), Affleck (1990), …

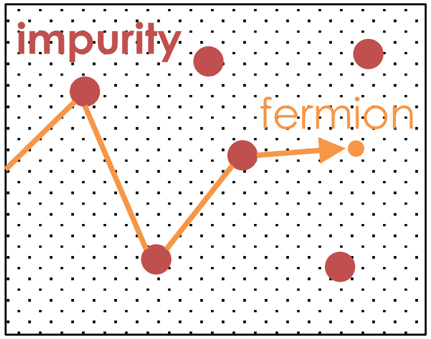


Non-Abelian interaction
(spin exchange)



1. Introduction

Original Work: J. Kondo (Prog. Theor. Phys. 32, 37 (1964))



Heavy impurity

① **Fermi surface**
(degenerate state)

② **Loop effect**
(particle-hole creation)

③ **Non-Abelian int.**
(SU(n) symmetry)

Content

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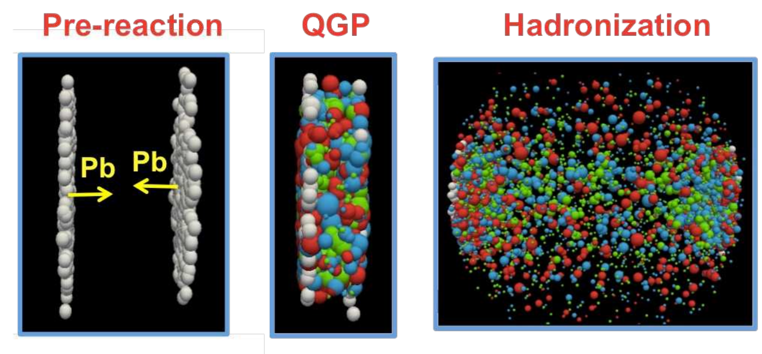
2. QCD Kondo effect I

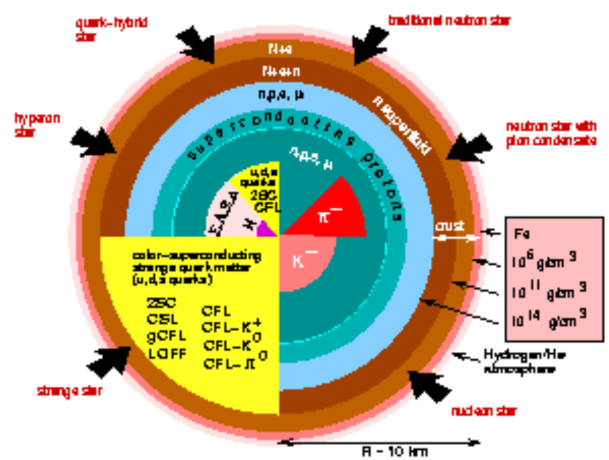
Charm quark in quark matter

Do charm/bottom quarks exist in quark matter?

① Charm quark production at initial (gluon) hard scatterings in relativistic heavy ion collisions (ex. FAIR, NICA, J-PARC).



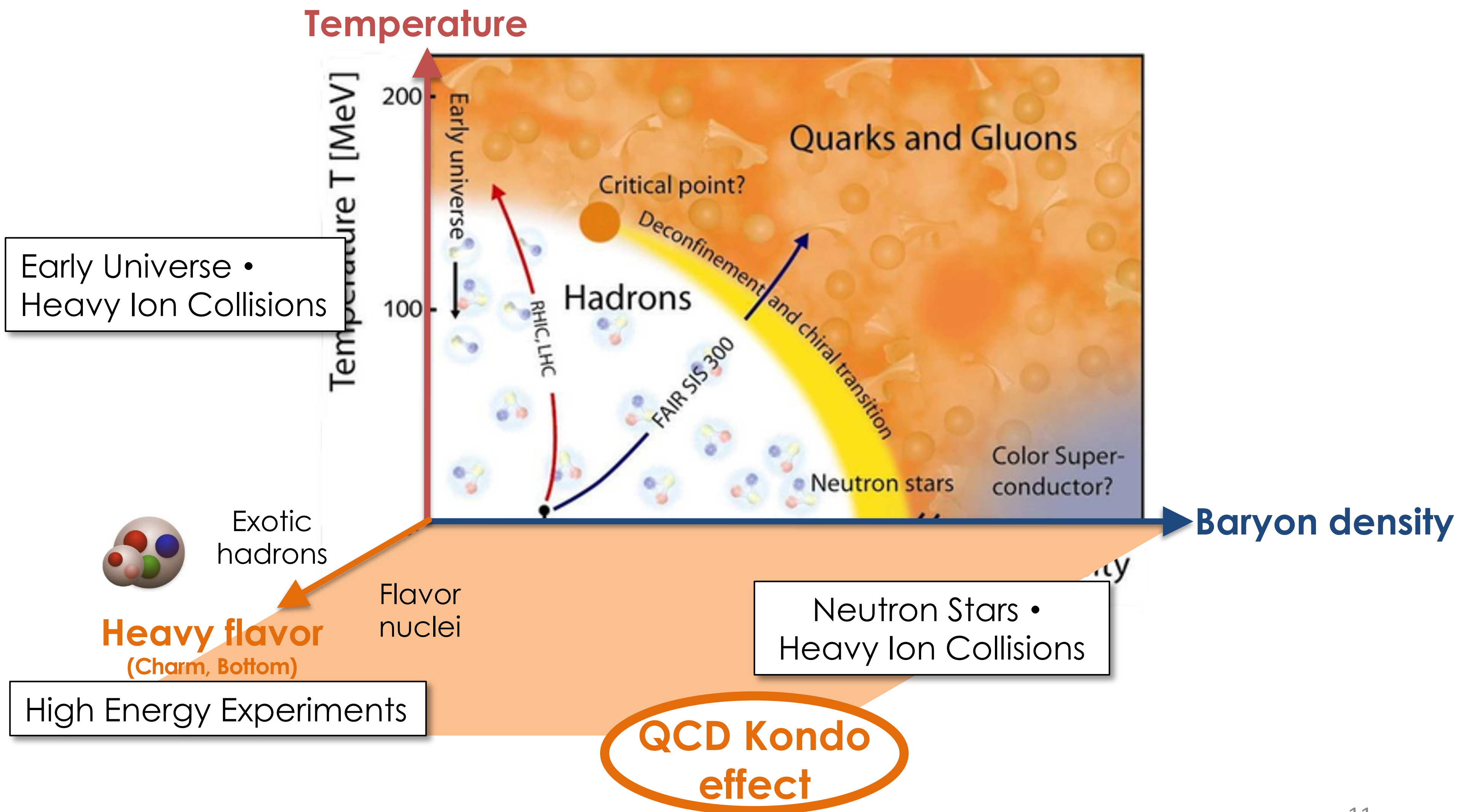
② Flavor change (ex. s→c, d→c) by high energy neutrinos in strange quark matter in neutron stars.



Can QCD Kondo effect exist in our nature?

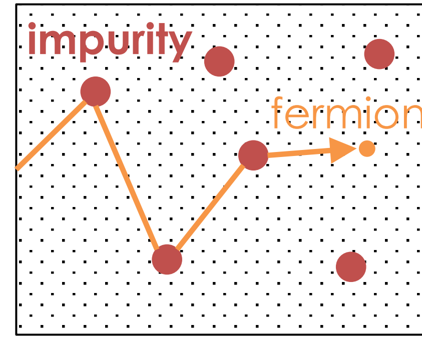
2. QCD Kondo effect I

QCD phase diagram
extended to ***heavy flavor***



2. QCD Kondo effect I

Application of Kondo effect to
Hadron/quark physics



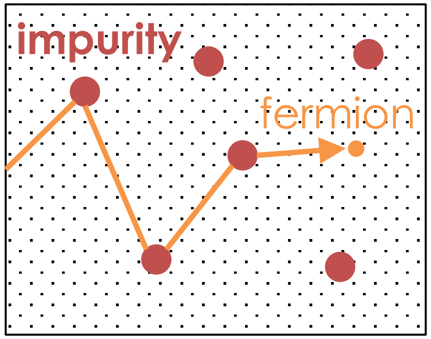
Fermi gas	electron gas	**nuclear matter** (p,n)	**quark matter** (u,d,s)
Heavy impurity	spin-atoms	**Heavy hadron**	**c/b quarks**
Fermi surface (degenerate state)	✔	✔	✔
Loop effect (particle-hole creation)	electron-hole	**nucleon-hole**	**quark-hole**
Non-Abelian int. (SU(n) symmetry)	$SU(2)_{spin}$	$\mathbf{SU(2)_{spin} \times SU(2)_{isospin}}$	$\mathbf{SU(3)_{color}}$

Nuclear matter and quark matter with heavy impurities can exhibit the Kondo effect!

2. QCD Kondo effect I

Application of Kondo effect to

Hadron/quark physics



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2. **QCD Kondo effect**: K. Hattori, K. Itakura, S. Ozaki, S.Y., PRD92, 065003 (2015)
3. **Magnetic catalysis QCD Kondo effect**: S. Ozaki, K. Itakura, Y. Kuramoto, PRD94, 074013 (2016)
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7. **Single heavy-quark QCD Kondo cloud:** S.Y., PLB773, 428 (2017)
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10. **QCD Kondo effect v.s. color superconductivity:** T. Kanazawa, S. Uchino, PRD94, 114005 (2016)
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12. **Chiral condensate vs. Kondo effect:** K. Suzuki, S.Y., K. Itakura, PRD96, 114007 (2017)
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21. **QCD Kondo spin polarization**: D. Suenaga, Y. Araki, K. Suzuki, S.Y., PRD105, 074028 (2022)
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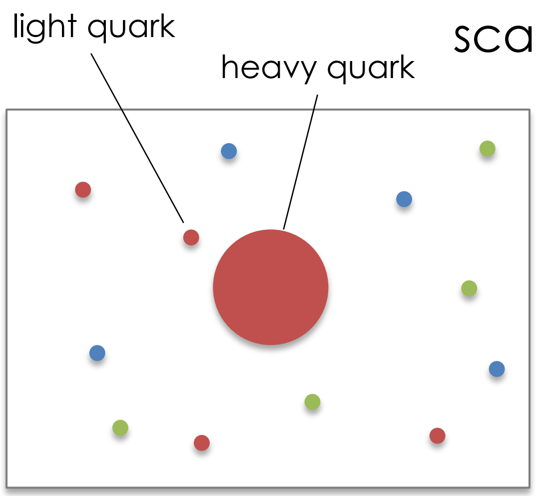
2. QCD Kondo effect I

S.Y., K. Suzuki, K. Itakura,
Nucl. Phys. A983, 90 (2019)

Mean-field approximation

scalar and vector



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Mean-field approximation

S.Y., K. Suzuki, K. Itakura,
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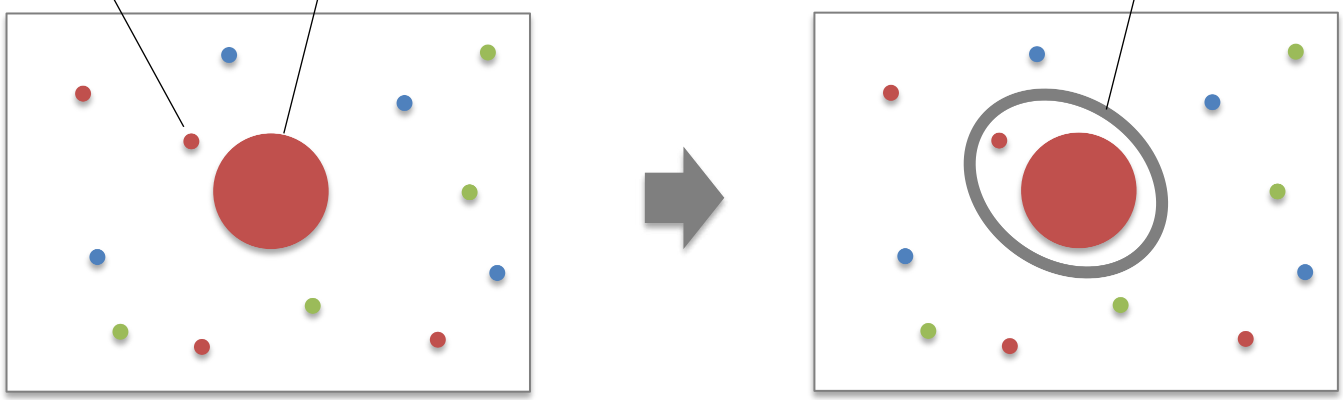


"Kondo cloud"

2. QCD Kondo effect I

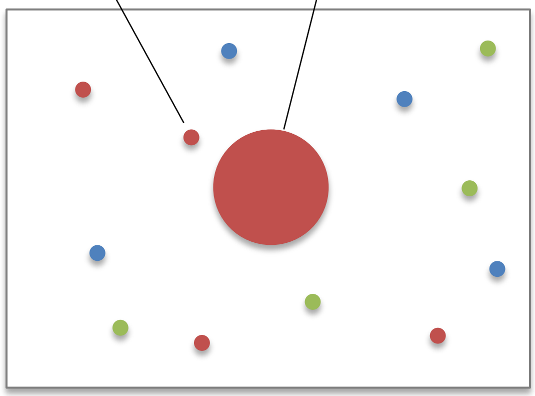
S.Y., K. Suzuki, K. Itakura,
Nucl. Phys. A983, 90 (2019)

Mean-field approximation

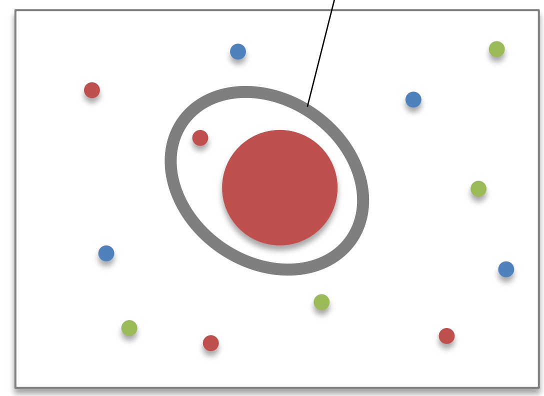
scalar and vector

light quark

heavy quark



heavy-light condensate (Δ)



"Kondo cloud"

Scalar $$\Phi_i = \frac{(N_c^2-1)G_c}{4N_c^2}\bar{\psi}_i\Psi_v$$

Vector $$\vec{\Phi}_i = \frac{(N_c^2-1)G_c}{4N_c^2}\bar{\psi}_i\vec{\gamma}\Psi_v$$

i=1, ..., N_f: light flavor

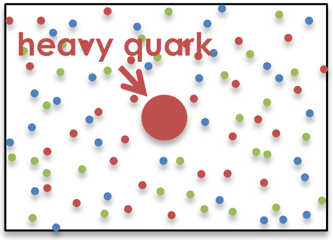
i=1
condensate

$$\langle\Phi_i\rangle = \Delta\delta_{i1}$$

$$\langle\vec{\Phi}_i\rangle = \Delta\frac{\vec{p}}{|\vec{p}|}\delta_{i1}$$

Mean-field (Δ)
hedgehog-type

2. QCD Kondo effect I

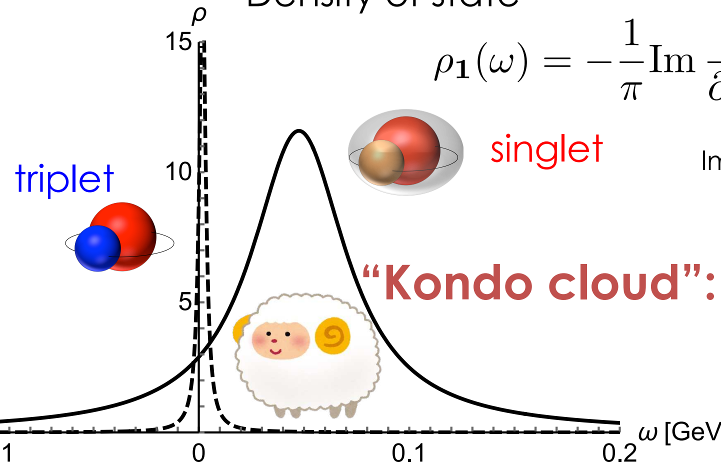


S.Y., Phys. Lett. B773, 428 (2017)

Density of state

$$\rho_{\mathbf{1}}(\omega) = -\frac{1}{\pi}\mathrm{Im}\,\frac{\partial}{\partial\omega}\ln\left(\omega_{+} - \lambda - \sum_{\vec{q}}\frac{2N_f|\Delta_{\mathbf{1}}|^2}{\omega_{+}+\mu-q}\right)$$

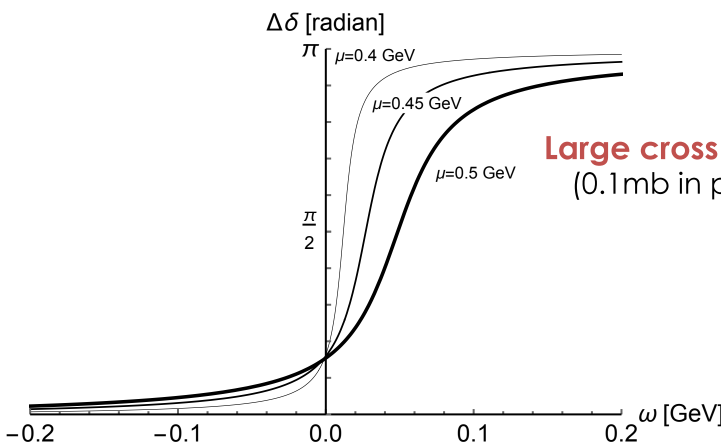
Imaginary part of Green's function



Fermi surface

"Kondo cloud": resonance of light quark and heavy quark

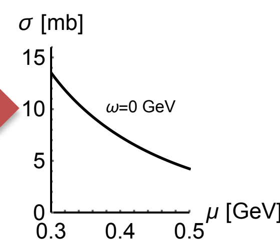
Phase shift: $\Delta\delta(\omega) = \pi\int_{-\Lambda}^{\omega}\rho_{\mathbf{1}}(\omega')\mathrm{d}\omega'$

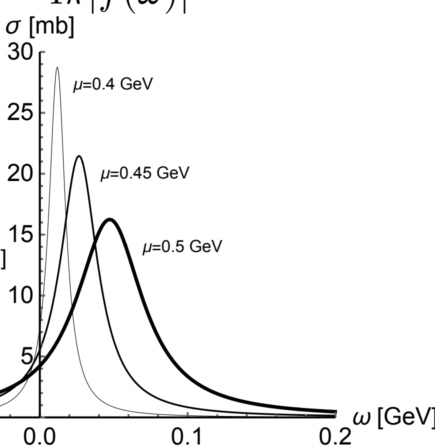


Amplitude: $f(\omega) = e^{i\Delta\delta(\omega)}\sin\Delta\delta(\omega)/k$

Cross section: $\sigma(\omega) = 4\pi|f(\omega)|^2$

Large cross section
(0.1mb in pQCD)





Comparable (strongly correlated system)

Mean free path: $\tau \simeq (\sigma\, n_q/N_f)^{-1}$

$\tau \simeq 1.3 - 0.91$ fm for $\mu = 0.3 - 0.5$ GeV

Mean distance:
$\ell \simeq n_q^{-1/3} = 0.96 - 0.58$ fm

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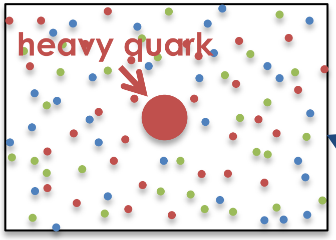
3. QCD Kondo effect II

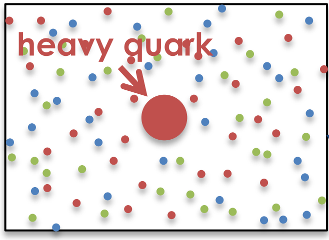


This study: we consider the contributions from **chiral symmetry breaking** in the QCD Kondo effect for **isolated heavy quark**.

Key Point 1: **Chiral-symmetry broken phase**

Key Point 2: **Isolated heavy quark (impurity)**

	impurity bulk matter (many heavy quarks)	isolated impurity (single heavy quark)
Chiral-symmetry restored	S.Y., K. Suzuki, K. Itakura, NPA983, 90 (2019), PRD96, 014016 (2017)	S.Y., PLB773, 428 (2017)
Chiral-symmetry broken	K. Suzuki, S.Y., K. Itakura, PRD96, 114007 (2017)	**This work!**



Light quarks (mass m)

3. QCD Kondo effect II

Lagrangian: current interaction for heavy-light quarks

$$
\begin{aligned}
\mathcal{L}[\psi,\hat{f},\lambda] = &-\bar{\psi}(\gamma\partial + m - \mu\gamma_4)\psi \quad \leftarrow \text{Light quark (mass } m\text{)} \\
&+G\Big(\bar{\psi}\hat{f}\,\hat{f}^\dagger\psi + \bar{\psi}i\gamma_5\hat{f}\,\hat{f}^\dagger i\gamma_5\psi \\
&\quad +\bar{\psi}i\boldsymbol{\gamma}\hat{f}\,\hat{f}^\dagger i\boldsymbol{\gamma}\psi + \bar{\psi}i\boldsymbol{\gamma}\gamma_5\hat{f}\,\hat{f}^\dagger i\boldsymbol{\gamma}\gamma_5\psi\Big)\delta(\boldsymbol{x}) \quad \leftarrow \text{Heavy-light int. (coupling const. } G\text{)} \\
&-\hat{f}^\dagger\partial_\tau\hat{f}\,\delta(\boldsymbol{x}) - \lambda\big(\hat{f}^\dagger\hat{f} - 1\big)\,\delta(\boldsymbol{x}), \quad \leftarrow \text{Heavy quark } (x = 0)
\end{aligned}
$$

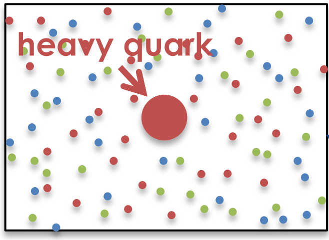
Note 1: Nonzero mass ($m \neq 0$) of light quark for chiral-symmetry breaking.

Note 2: Heavy-quark effective theory (HQET) is adopted.

Note 3: Heavy-quark locates at the original position ($\boldsymbol{x} = \boldsymbol{0}$).

Note 4: The interaction term stems from the NJL-type int.

3. QCD Kondo effect II



Bosonization:

$$Z = \int \mathcal{D}\Phi\mathcal{D}\Phi^\dagger\,\mathcal{D}\Phi_5\mathcal{D}\Phi_5^\dagger\,\mathcal{D}\boldsymbol{\Phi}\mathcal{D}\boldsymbol{\Phi}^\dagger\,\mathcal{D}\boldsymbol{\Phi}_5\mathcal{D}\boldsymbol{\Phi}_5^\dagger\mathcal{D}\lambda$$ ← **Generating functional**

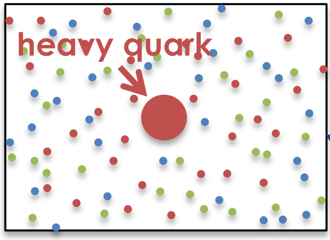
$$\times \exp\left(\mathrm{Tr}\,\ln S^{-1} - \frac{1}{G}\int \mathrm{d}\tau\,\left(\Phi^\dagger\Phi + \Phi_5^\dagger\Phi_5 + \boldsymbol{\Phi}^\dagger\boldsymbol{\Phi} + \boldsymbol{\Phi}_5^\dagger\boldsymbol{\Phi}_5\right) + \int \mathrm{d}\tau\,\lambda\right)$$

$$S(x)^{-1} = \begin{pmatrix} \gamma\partial + m - \mu\gamma_4 & \bar{\Delta}(x)\frac{1+\gamma_4}{2}\delta(\boldsymbol{x}) \\ \frac{1+\gamma_4}{2}\Delta(x)\delta(\boldsymbol{x}) & \frac{1+\gamma_4}{2}(\partial_\tau + \lambda)\delta(\boldsymbol{x}) \end{pmatrix}$$ ← **Inverse propagator (light quark + heavy quark)**

$$\Delta(x) = \Phi(x) + \Phi_5(x)i\gamma_5 + \boldsymbol{\Phi}(x)i\boldsymbol{\gamma} + \boldsymbol{\Phi}_5(x)i\boldsymbol{\gamma}\gamma_5$$ ← **Gap function for QCD Kondo condensate (heavy-light condensate)**

Scalar **Pseudo scalar** **Vector** **Axial vector**

Calculate free energy in mean-field approximation!



Light quarks (mass m)

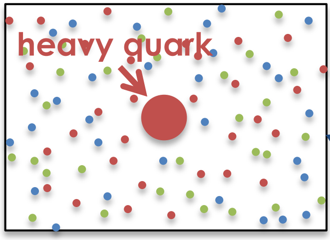
3. QCD Kondo effect II

Ansatz 1: ***Normal*** condensate (w/o *spinor* structure)

$$N_+ : \ \Phi \neq 0 \ \text{ while } \boldsymbol{\Phi} = \mathbf{0}, \ \Phi_5 = 0, \ \boldsymbol{\Phi}_5 = \mathbf{0} \ \leftarrow$$ **Parity +**

$$N_- : \ \Phi_5 \neq 0 \ \text{ while } \Phi = 0, \ \boldsymbol{\Phi} = \mathbf{0}, \ \boldsymbol{\Phi}_5 = \mathbf{0} \ \leftarrow$$ **Parity -**

3. QCD Kondo effect II



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$$N_- : \ \Phi_5 \neq 0 \ \text{ while } \Phi = 0, \ \boldsymbol{\Phi} = \mathbf{0}, \ \boldsymbol{\Phi}_5 = \mathbf{0} \ \leftarrow \textbf{Parity -}$$

Ansatz 2: ***Particle-projected*** condensate (w/ *spinor* structure)

$$P_+ : \boldsymbol{\Phi} = -\frac{\boldsymbol{p}}{E_{\boldsymbol{p}} + m}\Phi \neq \mathbf{0} \text{ while } \Phi_5 = 0, \ \boldsymbol{\Phi}_5 = \mathbf{0} \leftarrow \textbf{Parity +}$$

$$E_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2 + m^2}$$

$$P_- : \boldsymbol{\Phi}_5 = -\frac{i\boldsymbol{p}}{E_{\boldsymbol{p}} - m}\Phi_5 \neq \mathbf{0} \text{ while } \Phi = 0, \ \boldsymbol{\Phi} = \mathbf{0} \leftarrow \textbf{Parity -}$$

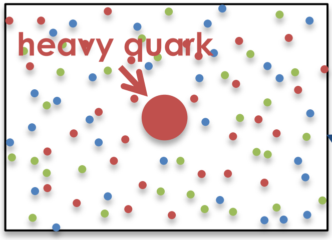
Particle-projection property:

$$P_+ : \hat{f}^\dagger \Delta \psi = \frac{2E_{\boldsymbol{p}}}{E_{\boldsymbol{p}} + m}\hat{f}^\dagger \Phi \Lambda_{\mathrm{P}}(\boldsymbol{p})\psi,$$

$$P_- : \hat{f}^\dagger \Delta \psi = \frac{-2E_{\boldsymbol{p}}}{E_{\boldsymbol{p}} - m}\hat{f}^\dagger i\gamma_5 \Phi_5 \Lambda_{\mathrm{P}}(\boldsymbol{p})\psi.$$

Projection operator for positive-energy state

$$\Lambda_{\mathrm{P}}(\boldsymbol{p}) = \frac{-i\boldsymbol{p}\cdot\boldsymbol{\gamma} + E_{\boldsymbol{p}}\gamma_4 + m}{2E_{\boldsymbol{p}}}$$



Light quarks (mass m)

3. QCD Kondo effect II

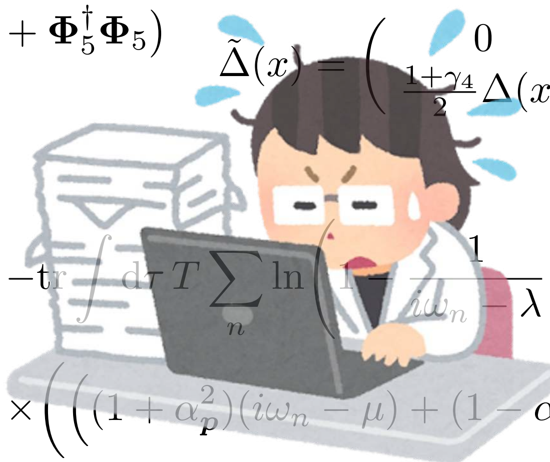
Impurity energy: free energy subtracted by free part $\delta F \equiv F + \mathrm{Tr}\,\ln S_0(x)^{-1}$

$$F = -\mathrm{Tr}\,\ln S_0(x)^{-1} - \sum_{k\geq 1} \frac{(-1)^{k+1}}{k} \mathrm{Tr}\big(S_0(x)\tilde{\Delta}(x)\,\delta(\boldsymbol{x})\big)^k + \frac{1}{G}\int \mathrm{d}\tau\,\left(\Phi^\dagger\Phi + \Phi_5^\dagger\Phi_5 + \boldsymbol{\Phi}^\dagger\boldsymbol{\Phi} + \boldsymbol{\Phi}_5^\dagger\boldsymbol{\Phi}_5\right) - \int \mathrm{d}\tau\,\lambda.$$

$$S(x)^{-1} = S_0(x)^{-1} + \tilde{\Delta}(x)\,\delta(\boldsymbol{x})$$

$$S_0(x)^{-1} = \begin{pmatrix} \gamma\partial + m - \mu\gamma_4 & 0 \\ 0 & \frac{1+\gamma_4}{2}(\partial_\tau + \lambda)\delta(\boldsymbol{x}) \end{pmatrix}$$

$$\tilde{\Delta}(x) = \begin{pmatrix} 0 & \bar{\Delta}(x)\frac{1+\gamma_4}{2} \\ \frac{1+\gamma_4}{2}\Delta(x) & 0 \end{pmatrix}$$



$$T\sum_n F(i\omega_n) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} \mathrm{d}p$$

$$-\sum_{k\geq 1} \frac{(-1)^{k+1}}{k} \mathrm{Tr}\Big(S_0(x)\hat{\Delta}(x)\Big)^k = -\mathrm{tr}\int \mathrm{d}\tau\, T\sum_n \ln\Big(1 - \frac{1}{i\omega_n - \lambda}\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{(i\omega_n-\mu)^2 - E_{\boldsymbol{p}}^2}$$

$$\times\Big(\Big((1+\alpha_{\boldsymbol{p}}^2)(i\omega_n-\mu) + (1-\alpha_{\boldsymbol{p}}^2)m - 2\alpha_{\boldsymbol{p}}|\boldsymbol{p}|\Big)\Phi\Phi^\dagger$$

$$+\Big((1+\alpha_{5\boldsymbol{p}}^2)(i\omega_n-\mu) - (1-\alpha_{5\boldsymbol{p}}^2)m - 2\alpha_{5\boldsymbol{p}}|\boldsymbol{p}|\Big)\Phi_5\Phi_5^\dagger\Big)\Big).$$

$$\times\frac{1}{e^{\beta p_0}+1},$$

$$\int_{-\infty}^{\infty} \mathrm{d}p_0\big(F_+(p_0) - F_-(p_0)\big)$$

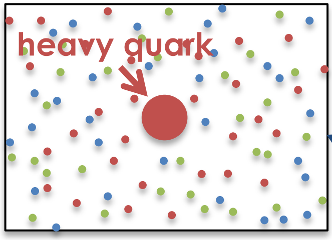
$$\overline{+1},$$

$$\frac{1}{G}\int \mathrm{d}\tau\,\left(\Phi^\dagger\Phi + \Phi_5^\dagger\Phi_5 + \boldsymbol{\Phi}^\dagger\boldsymbol{\Phi} + \boldsymbol{\Phi}_5^\dagger\boldsymbol{\Phi}_5\right) = \frac{1}{G}\int \mathrm{d}\tau\int \mathrm{d}^3\boldsymbol{x}\,\delta(\boldsymbol{x})\left(\Phi^\dagger\Phi + \Phi_5^\dagger\Phi_5 + \boldsymbol{\Phi}^\dagger\boldsymbol{\Phi} + \boldsymbol{\Phi}_5^\dagger\boldsymbol{\Phi}_5\right)$$

$$= \frac{1}{G}\int \mathrm{d}\tau\int \mathrm{d}^3\boldsymbol{x}\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\left((1+\alpha_{\boldsymbol{p}}^2)\Phi^\dagger\Phi + (1+\alpha_{5\boldsymbol{p}}^2)\Phi_5^\dagger\Phi_5\right)$$

$$= \frac{1}{G}\int \mathrm{d}\tau\Big((1+\alpha^2)\Phi^\dagger\Phi + (1+\alpha^2_{5})\Phi^\dagger_5\Phi_5\Big)\Big|$$

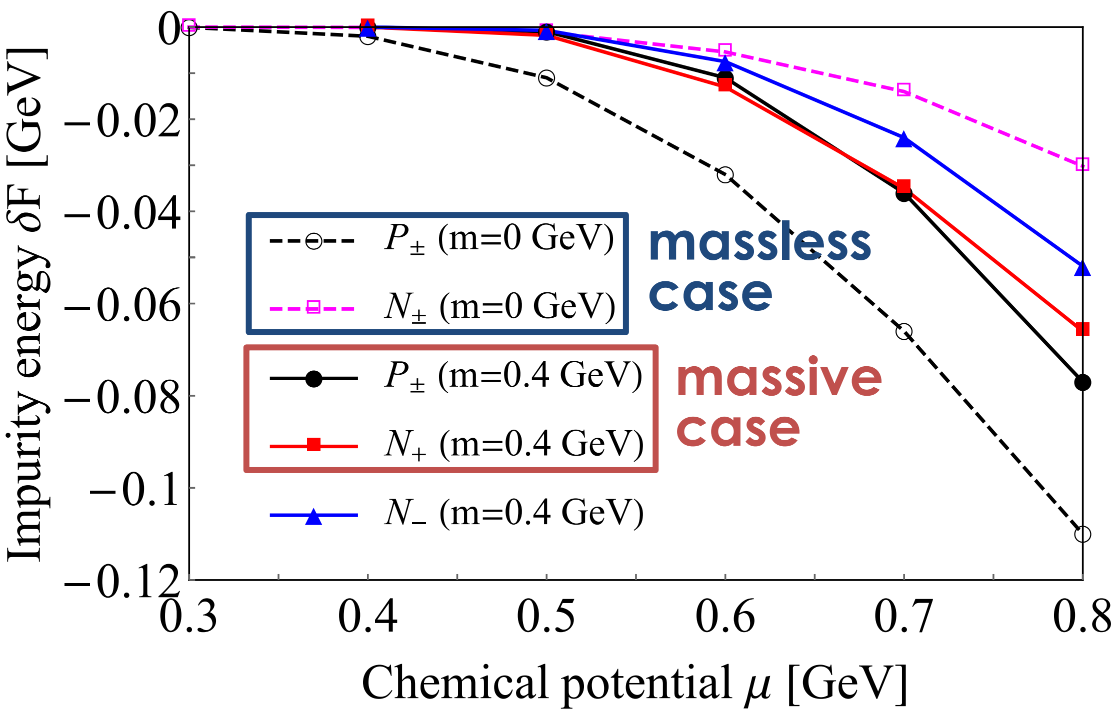
3. QCD Kondo effect II



Impurity energy: free energy subtracted by free part $\delta F \equiv F + \mathrm{Tr}\,\ln S_0(x)^{-1}$

unstable

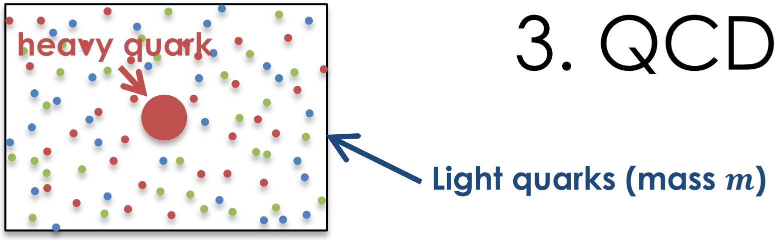
stable



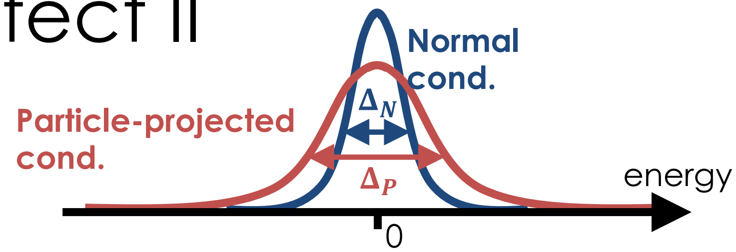
NJL model parameter
$G\Lambda^2 = 4.3$
$\Lambda = 1$ GeV

Result 1: P_+ is most favored in massless case ($m = 0$).

Result 2: N_+ is most favored for smaller chemical potential ($\mu \lesssim 0.7$ GeV) and P_+ is most favored for larger chemical potential ($\mu \gtrsim 0.7$ GeV) in massive case ($m = 0.4$ GeV).



3. QCD Kondo effect II



QCD Kondo resonance: Lorentz-type resonances near Fermi surface

$$\rho_N(\omega) = \frac{1}{\pi} \frac{\Delta_N}{(\omega - \mu - \lambda)^2 + \Delta_N^2}$$ ← **Normal condensate (gap Δ_N)**

$$\rho_P(\omega) = \frac{1}{\pi} \frac{\Delta_P}{(\omega - \mu - \lambda)^2 + \Delta_P^2}$$ ← **Particle-projected condensate (gap Δ_P)**

QCD Kondo condensate ⇔ QCD Kondo resonance width

Our model indicates the ratio: $\frac{\Delta_P}{\Delta_N} = \frac{2}{1 + m/\mu}$ Light quarks mass m, chemical pot. μ

1. When chemical potential is sufficiently large, the ratio becomes $\Delta_P/\Delta_N \approx 2$ indicating that P_+ is more favored than N_+.
2. When chemical potential is sufficiently small, the ratio becomes $\Delta_P/\Delta_N \approx 1$ indicating the same gap for P_+ and N_+. Nevertheless N_+ is more favored because the attraction from *light antiparticles* contribute to N_+.

4. Conclusion

S.Y, D. Suenaga, K. Suzuki, PRD109, 094031 (2024)

1. We discuss the **QCD Kondo effect** for an isolated heavy quark surrounded by massive light quarks in chiral-symmetry broken phase.

2. We consider the **QCD Kondo condensate** for the mixing between heavy quark and light quark.
   (A) For smaller chemical potential, the **normal condensate** *without spinor structure* is favored.
   (B) For larger chemical potential, the **particle-projected condensate** *with spinor structure* is favored.

3. The stability of the QCD Kondo condensate is understood by the **QCD Kondo resonance**.

4. Conclusion

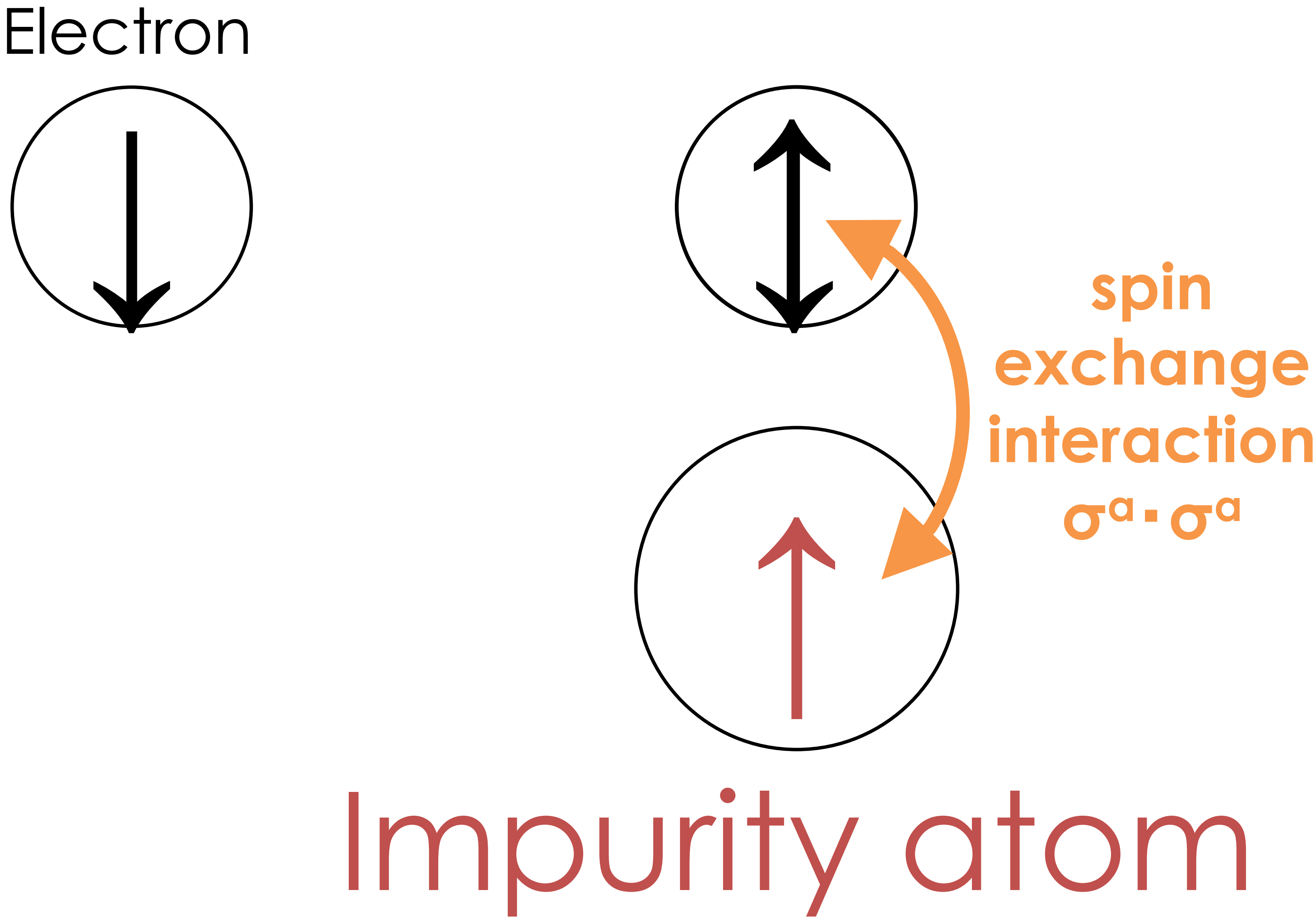
S.Y, D. Suenaga, K. Suzuki, PRD109, 094031 (2024)

Future studies:

- Competition with dynamical chiral symmetry breaking and color superconductivity.
- Transport coefficients.
- Heavy particles and heavy antiparticles.
- Non-Fermi liquid behavior for light flavor $N_f \geq 2$.
- Lattice QCD: chiral chemical potential and strong magnetic field.
- Chiral separation effect and spin polarization in magnetic field: Λ_c (Λ_b) baryon spin polarization in QGP.
- (Dis)continuity between QCD Kondo effect and spin/isospin Kondo effect.
- and more!

Appendix A

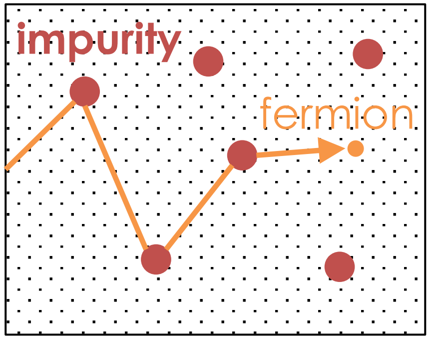
Spin exchange

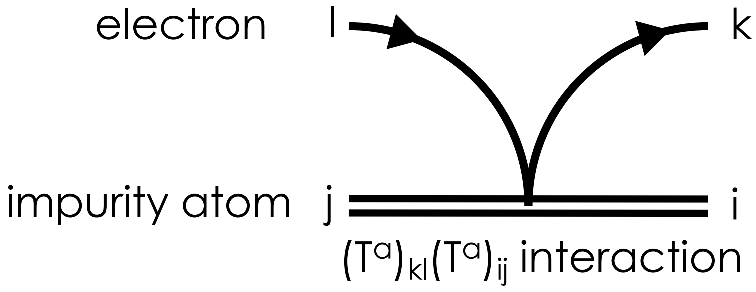


Appendix A

Original Work: J. Kondo (Prog. Theor. Phys. 32, 37 (1964))

Scattering amplitude: **SU(2) → SU(n) symmetry**





k, l, i, j=↑, ↓ in SU(n) (n=2 for spin ½)
T^1, ..., $T^{n\wedge 2-1}$: generators of SU(n)

Appendix A

Original Work: J. Kondo (Prog. Theor. Phys. 32, 37 (1964))

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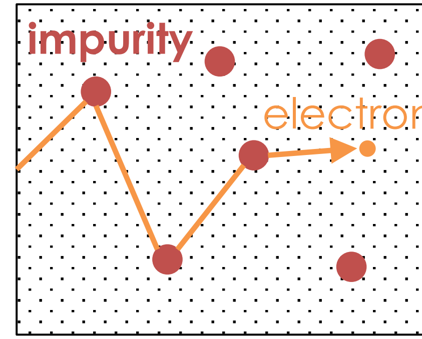
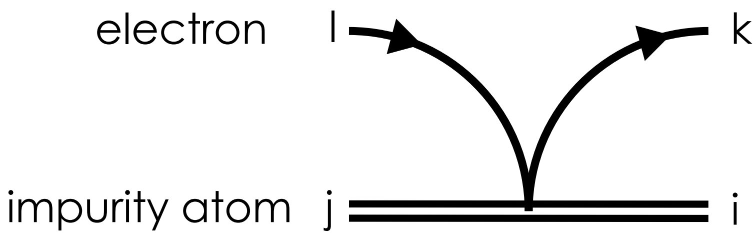
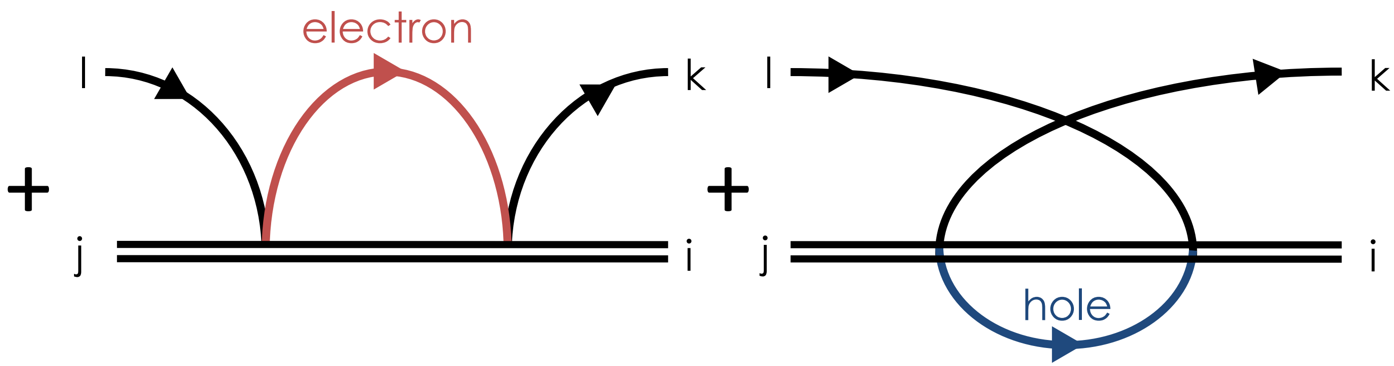




$(T^a)_{kl}(T^a)_{ij}$ interaction

k, l, i, j=↑, ↓ in SU(n) (n=2 for spin ½)
T^1, ..., $T^{n\wedge 2-1}$: generators of SU(n)



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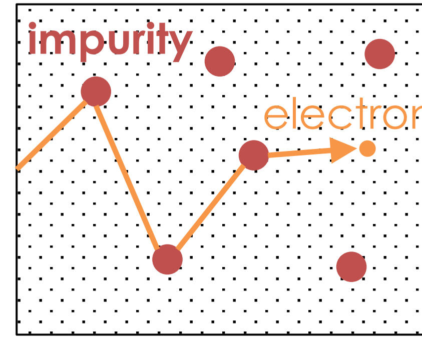
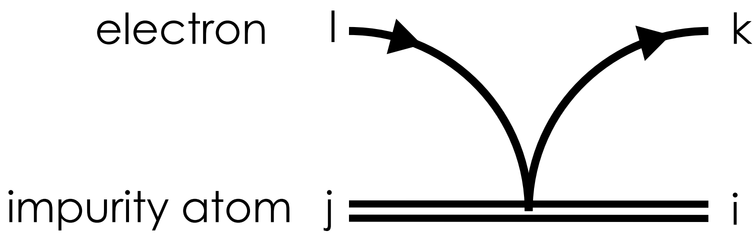
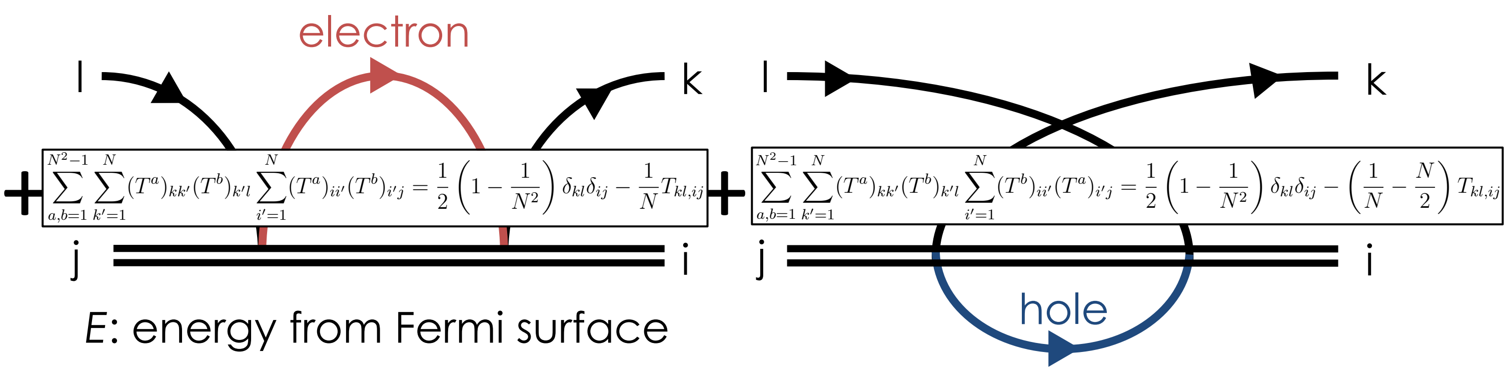




$(T^a)_{kl}(T^a)_{ij}$ interaction

k, l, i, j=↑, ↓ in SU(n) (n=2 for spin ½)
$T^1, \ldots, T^{n^2-1}$: generators of SU(n)



$$+\sum_{a,b=1}^{N^2-1}\sum_{k'=1}^{N}(T^a)_{kk'}(T^b)_{k'l}\sum_{i'=1}^{N}(T^a)_{ii'}(T^b)_{i'j}=\frac{1}{2}\left(1-\frac{1}{N^2}\right)\delta_{kl}\delta_{ij}-\frac{1}{N}T_{kl,ij}$$

$$+\sum_{a,b=1}^{N^2-1}\sum_{k'=1}^{N}(T^a)_{kk'}(T^b)_{k'l}\sum_{i'=1}^{N}(T^b)_{ii'}(T^a)_{i'j}=\frac{1}{2}\left(1-\frac{1}{N^2}\right)\delta_{kl}\delta_{ij}-\left(\frac{1}{N}-\frac{N}{2}\right)T_{kl,ij}$$

E: energy from Fermi surface

$$\int_{E>0}\frac{1}{E}\mathrm{d}E \qquad\qquad \int_{E>0}\frac{1}{-E}\mathrm{d}E$$

Appendix A

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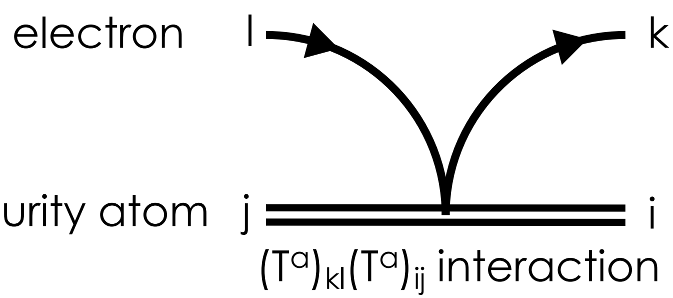
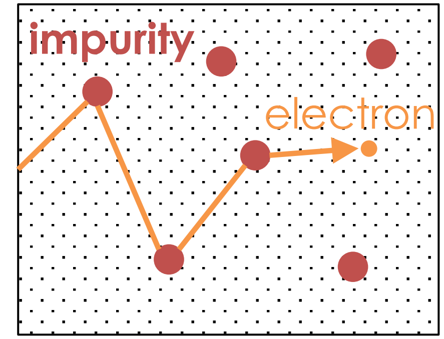
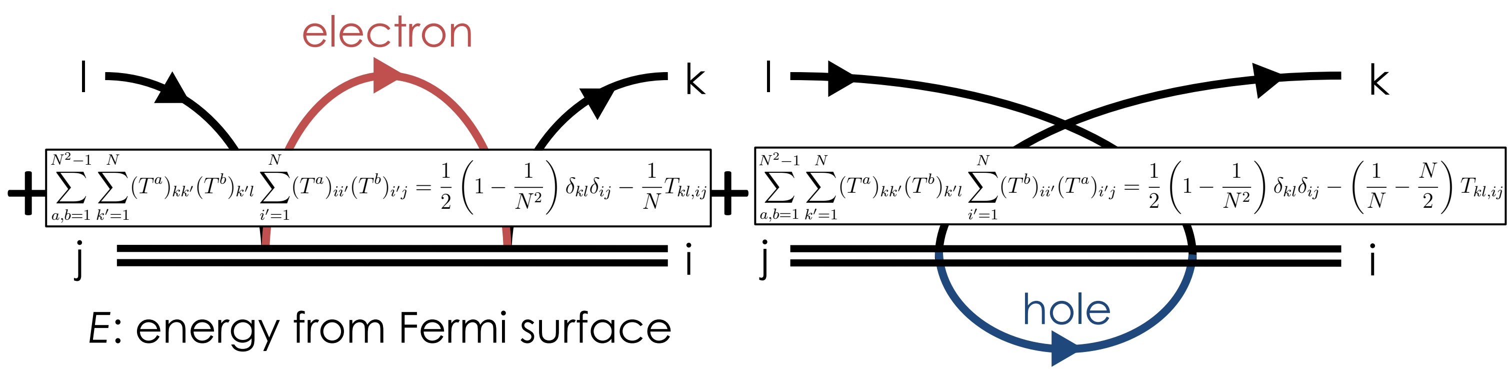
Scattering amplitude: **SU(2) → SU(n) symmetry**





k, l, i, j=↑, ↓ in SU(n) (n=2 for spin ½)
T^1, …, $T^{n\wedge 2-1}$: generators of SU(n)



E: energy from Fermi surface

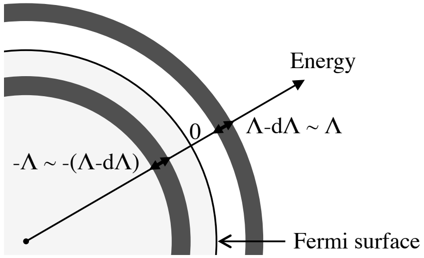
$$\int_{E>0}\frac{1}{E}\mathrm{d}E \times \left(-\frac{1}{n}\right)\sum_a (T^a)_{kl}(T^a)_{ij} \qquad \int_{E>0}\frac{1}{-E}\mathrm{d}E \times \left(\frac{n}{2}-\frac{1}{n}\right)\sum_a (T^a)_{kl}(T^a)_{ij}$$

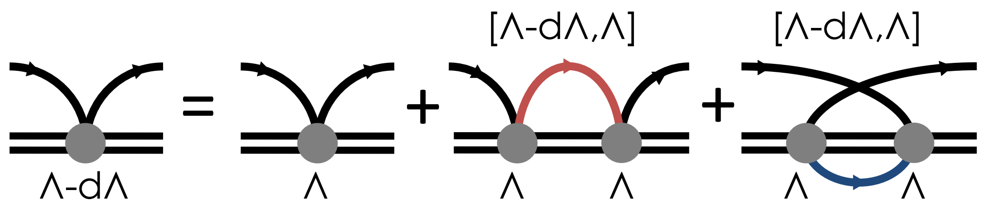
Log E divergence for infrared limit (E→0) for ***any small coupling***

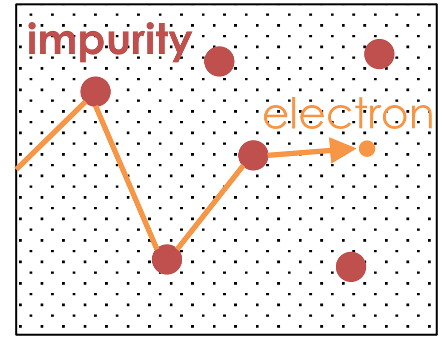
→ Perturbation breaks down!!

Appendix A

Renormalization group







P. W. Anderson,
“poor mans’s scaling”
J. Physics C3, 2436 (1970)

$$M_{kl,ij}(\Lambda - \mathrm{d}\Lambda) = M_{kl,ij}(\Lambda) + G(\Lambda)^2 \rho_0 \frac{n}{2} T_{kl,ij} \int_{\Lambda-\mathrm{d}\Lambda}^{\Lambda} \frac{\mathrm{d}E}{E - i\varepsilon}$$

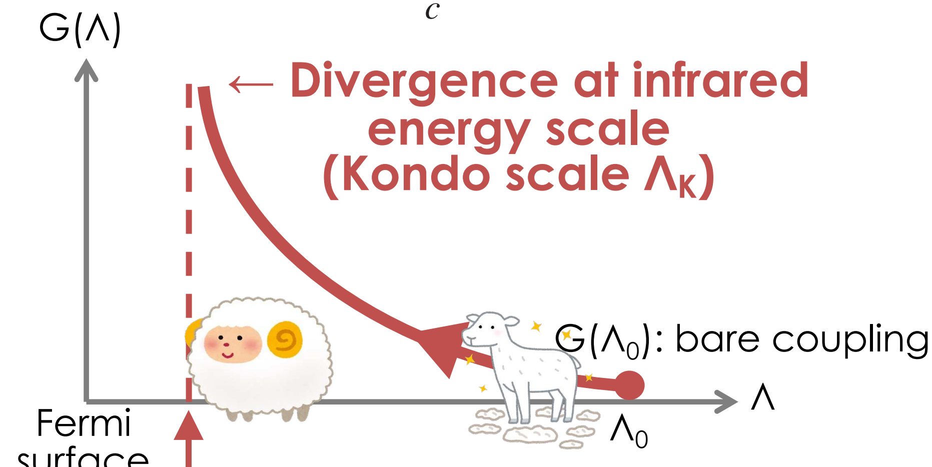
$$M_{kl,ij}(\Lambda) = G(\Lambda) T_{kl,ij}$$

ρ_0: density of state on Fermi surface

$$T_{kl,ij} = \sum_c (t^c)_{kl} (t^c)_{ij}$$ i,j,k,l=1,…,n SU(n)

$$\Lambda \frac{\mathrm{d}}{\mathrm{d}\Lambda} G(\Lambda) = -\frac{n}{2} G^2(\Lambda) \rho_0$$

$$G(\Lambda) = \frac{G(\Lambda_0)}{1 + \frac{n\rho_0}{2} G(\Lambda_0) \log \frac{\Lambda}{\Lambda_0}}$$



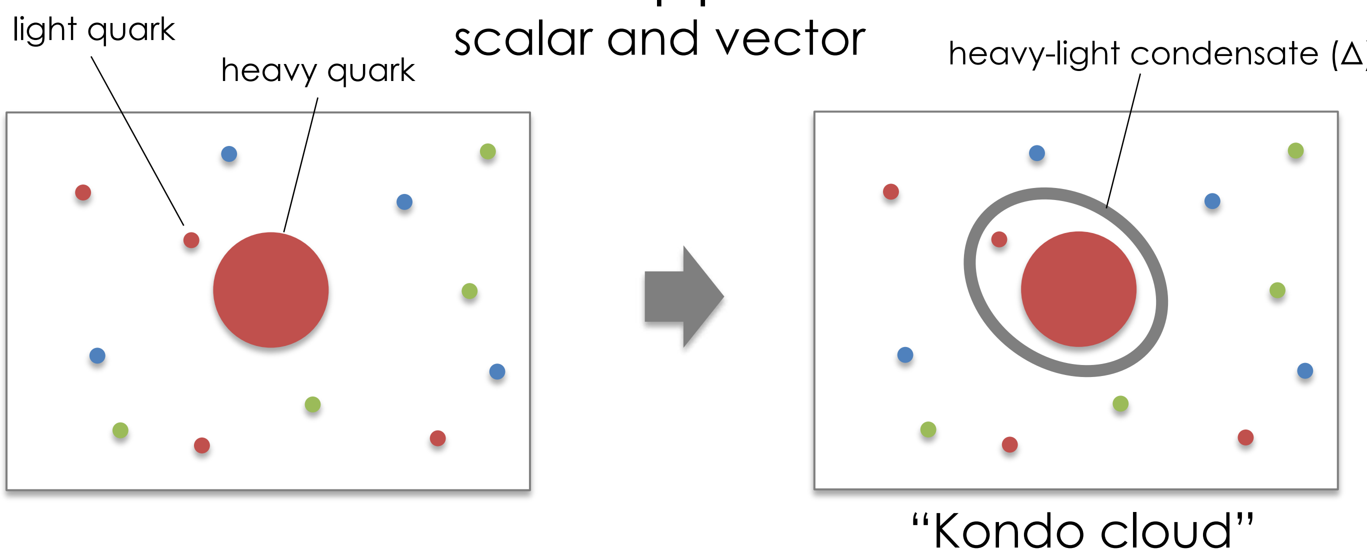
$$\Lambda_K = \Lambda_0 \exp\left(- \frac{2}{n\rho_0 G(\Lambda_0)} \right)$$

Appendix B

Mean-field approximation

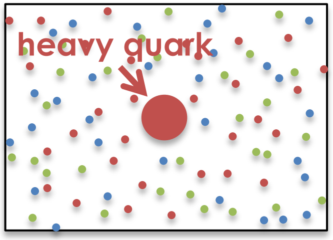
scalar and vector

S.Y., K. Suzuki, K. Itakura,
Nucl. Phys. A983, 90 (2019)



Green's function (mean-field)

$$G(p_0,\vec{p})^{-1} \equiv \begin{pmatrix} \not{p}+\mu\gamma_0 & 0 & \cdots & 0 & \Delta^*(1+\vec{\gamma}\cdot\hat{p}) \\ 0 & \not{p}+\mu\gamma_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ \Delta(1+\vec{\gamma}\cdot\hat{p}) & 0 & \cdots & 0 & p_0-\lambda \end{pmatrix}$$



Appendix B

Energy-gain of *one* heavy quark

S.Y., Phys. Lett. B773, 428 (2017)

Breaking of translational invariance → Only "energy" is concerned.
(no momentum conservation)

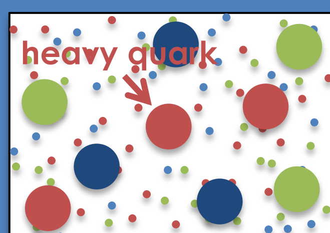
Energy: $\Omega_{\mathbf{1}}(\lambda,\Delta_{\mathbf{1}}) = -\frac{1}{\beta}\int_{-\infty}^{+\infty} \ln\left(1+e^{-\beta\omega}\right) 2N_c\rho_{\mathbf{1}}(\omega)\mathrm{d}\omega + \frac{8N_f|\Delta_{\mathbf{1}}|^2}{G_c}V - \lambda$

Density of state: $\rho_{\mathbf{1}}(\omega) = -\frac{1}{\pi}\mathrm{Im}\,\frac{\partial}{\partial\omega}\ln\left(\omega_+ - \lambda - \sum_{\vec{q}}\frac{2N_f|\Delta_{\mathbf{1}}|^2}{\omega_+ + \mu - q}\right)$

$\Delta_{\mathbf{1}} \propto \delta_{\mathbf{1}}$

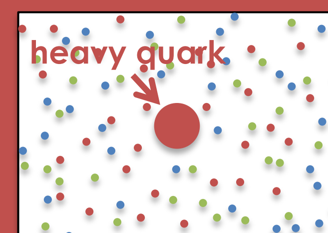
$$= \frac{1}{\pi}\frac{\delta_{\mathbf{1}}}{(\omega-\lambda_{\mathbf{1}})^2+\delta_{\mathbf{1}}^2}$$

"Heavy quark matter"



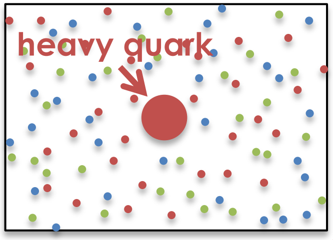
$$\mathcal{H}_{\boldsymbol{k}} = \begin{pmatrix} H^0_{\boldsymbol{k}} & \hat{\Delta}^\dagger_{\boldsymbol{k}} \\ \hat{\Delta}_{\boldsymbol{k}} & \lambda \end{pmatrix}$$

"Single heavy quark"



$$\mathcal{H} = \begin{pmatrix} H^0_{\boldsymbol{k}} & \cdots & 0 & \hat{\Delta}^\dagger_{\boldsymbol{k}} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & H^0_{\boldsymbol{k}'} & \hat{\Delta}^\dagger_{\boldsymbol{k}'} \\ \hat{\Delta}_{\boldsymbol{k}} & \cdots & \hat{\Delta}_{\boldsymbol{k}'} & \lambda \end{pmatrix}$$

$$H^0_{\boldsymbol{k}} = \begin{pmatrix} -\mu & \boldsymbol{k}\cdot\boldsymbol{\sigma} \\ \boldsymbol{k}\cdot\boldsymbol{\sigma} & -\mu \end{pmatrix}$$



Appendix B

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S.Y., Phys. Lett. B773, 428 (2017)

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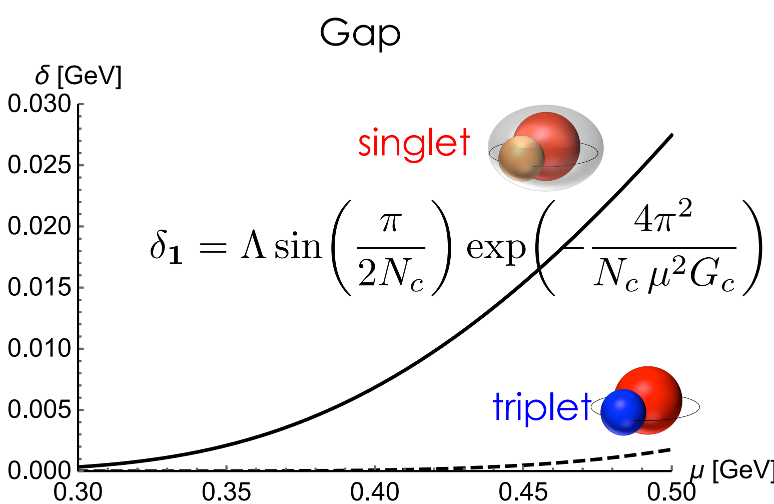
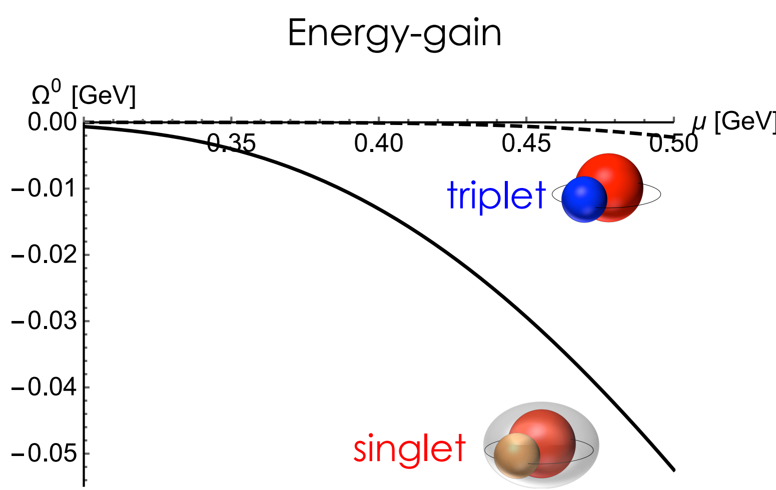
$\Delta_{\mathbf{1}} \propto \delta_{\mathbf{1}}$

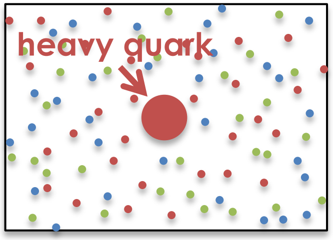
$$= \frac{1}{\pi}\frac{\delta_{\mathbf{1}}}{(\omega-\lambda_{\mathbf{1}})^2+\delta_{\mathbf{1}}^2}$$



$$\delta_{\mathbf{1}} = \Lambda\sin\left(\frac{\pi}{2N_c}\right)\exp\left(-\frac{4\pi^2}{N_c\,\mu^2 G_c}\right)$$



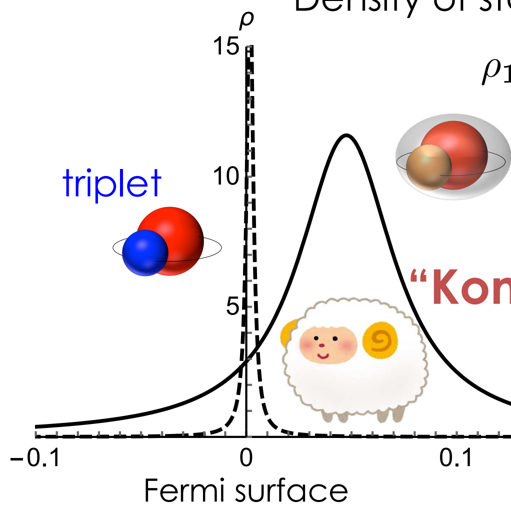
→ What is the property of condensate?



Appendix B

S.Y., Phys. Lett. B773, 428 (2017)

Density of state



singlet

$$\rho_{\mathbf{1}}(\omega) = -\frac{1}{\pi}\mathrm{Im}\,\frac{\partial}{\partial\omega}\ln\left(\omega_{+} - \lambda - \sum_{\vec{q}}\frac{2N_f|\Delta_{\mathbf{1}}|^2}{\omega_{+} + \mu - q}\right)$$

Imaginary part of Green's function

"Kondo cloud": resonance of light quark and heavy quark

Thermodynamic potential (energy gain)

triplet

$$\Omega^{0}_{\bar{\mathbf{3}}} = -\Lambda\frac{2\sqrt{2}}{\pi}\exp\left(-\frac{4\pi^2}{(1+1/N_c)\,\mu^2 G_c}\right)$$

singlet

$$\Omega^{0}_{\mathbf{1}} = -\Lambda\frac{2N_c}{\pi}\sin\left(\frac{\pi}{2N_c}\right)\exp\left(-\frac{4\pi^2}{N_c\,\mu^2 G_c}\right)$$

→ Only singlet condensate survives in large N_c limit ('t Hooft limit).

Appendix C

物理学会誌2018年11月号

チャーミングな近藤効果—非アーベル型相互作用による重いクォーク・ハドロンの不純物物理—

安 井 繁 宏　〈慶應義塾大学自然科学研究教育センター　yasuis@keio.jp〉

素粒子・原子核から物性（電子・原子）までの階層構造を統一的に理解することは，物質構造の普遍性と多様性について重要な知見を我々に与える．異なる物質階層に共通して見られるシステムの例がフェルミガスであり，多様な量子現象が存在することが知られている．その一つが近藤効果である．

近藤効果はフェルミガスにおいて不純物が引き起こす量子効果である．近藤効果の説明のために電子ガスに不純物原子が混入している状況を考えよう．ただし不純物原子はスピンをもつとして，電子ガスと不純物原子の間にスピン交換が行われるとする．このとき電子ガスと不純物原子の相互作用の大きさは媒質効果による影響（ループ効果の繰り込み）を受けて変化し，低エネルギー散乱において対数的に増大する．その崩壊より短い時間スケールの範囲内で平衡状態と見なすことが可能であろう．さて原子核（あるいは核物質）にどのような重い不純物が存在すれば近藤効果が発生するのかを考えよう．近藤効果の条件（i），(ii）は満たされている．(iii）の非アーベル型相互作用をもつ重い不純物として，チャームクォーク（c）あるいはボトムクォーク（b）と軽いクォーク（$q=u, d$）で構成された $\bar{D}, \bar{D}^{*}(\bar{c}q)$ メソンや $B, B^{*}(\bar{b}q)$ メソンを考える．これらは内部自由度として SU(2)×SU(2) 対称性のスピンとアイソスピンをもつので核子と非アーベル型相互作用をする．つまりスピンやアイソスピンに起因する近藤効果が生じると考えられる．

さらにエネルギースケールが高くなると核子に閉じ込められていたクォークが解放されて核物質はクォーク物質に変化する．

—*Keywords*—

非アーベル型相互作用：
内部自由度をもつ粒子が相互作用をするとき内部状態が変化する．相互作用の順番によって粒子の内部状態が異なるものを非アーベル型相互作用であるという．一例が SU(2) 対称性のスピンにおけるパウリ行列である．一般化として SU(N) 対称性を考えることができる．

核物質・クォーク物質：
通常の原子核は有限の大きさをもつが，無限大の体積をもつ原子核を想定して有限体積効果を無視した状態を考えることがある．これを核物質という．核物質の構成要素は核子（陽子・中性子）である．超高密度の核物質では核子内部に閉じ込められていたクォークがほとんど自由な粒子として解放される．この状態を